Mixed-integer linear programming models for the simultaneous unloading and loading processes in a maritime port

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Abstract. This paper discusses the jointly quay crane and yard truck scheduling problems (QCYTSP) with unloading and loading containers from/to vessel(s) in the same time. Yard trucks transport the containers to/from yard locations with all containers that are homogeneous. We propose a mixed integer linear programming model to solve the scheduling problem. We consider in this study, the quay crane interference, containers precedence and safety margin. The main objective is to minimize the total completion time of the vessels.

Keywords: optimization, quay crane, yard truck, scheduling problem, MILP

1 Introduction and literature review

The general process in a container terminal can be described as a sequence of operations from the arrival to the departure of the container's vessels. The container vessel is dedicated to transport containers from a maritime port to another. The container is a parallelepiped metal box designed for the transport of goods by different modes of transport. The quay crane is used to load containers into or unload containers from the vessel. Yard trucks are used for transporting containers from the station of quay cranes to the storage location or vice versa. In the storage location, there is a type of crane called reach-stacked crane, it can be used to load containers into or unload containers from the yard trucks. In this study, we have two container vessels, the first one is dedicated for the containers to be unloaded and transported to the storage location (U-containers), and the second one is dedicated to the containers to be unloaded from the storage location and loaded into the vessel (L-containers). There are two separated storage locations, one for the U-containers and one for the L-containers. More precisely, the unloading process includes the following steps:

1. A quay crane unloads the U-container from the vessel and loads it into a yard truck.
2. A yard truck transports the U-container to the storage location.
3. A reach-stacker crane unloads the U-container from the yard truck and loads it in the storage location.

After this process, the yard truck continues its way to the storage location dedicated for the L-containers. The loading process includes the following steps:

1. A reach-stacker crane collects a L-container from storage location and loads it into the yard truck coming from the storage location for the U-containers.
2. A yard truck transports the L-container to the quay crane station.
3. A quay crane unloads the L-container from the yard truck and loads it into the vessel.

Figure 1 describes the full unloading and loading operations.

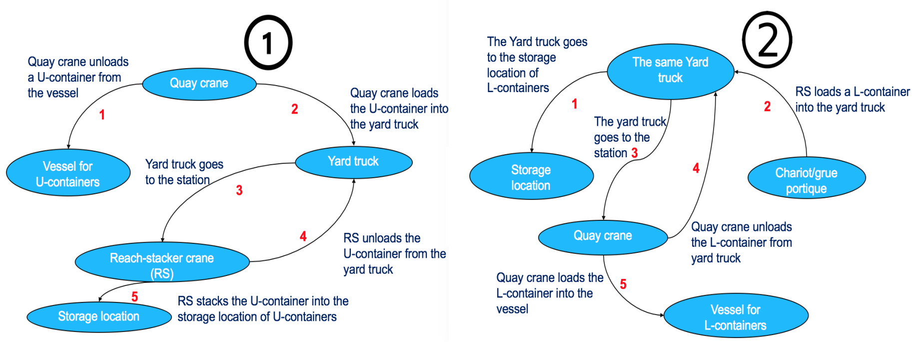

Fig. 1. U-container unloading and L-container loading processes

In our previous studies, [10] Skaf et al. (2018) proposed a mixed-integer linear programming model and a dynamic programming algorithm to solve the quay crane scheduling problem at port of Tripoli-Lebanon. Later, [12] Skaf et al. (2019) proposed a new genetic algorithm to solve the problem, due to the inability to provide results from the two previous exact methods.
After that, [13] Skaf et al. (2019) proposed a mixed-integer linear programming model and a dynamic programming algorithm to solve the scheduling problem for single quay crane and multiple yard trucks at port of Tripoli-Lebanon.
This study is considered new to the literature, but we addressed some researchers who solved the scheduling problem for the quay cranes, the yard trucks or for both of them. [1] Daganzo (1989) studied the quay crane scheduling problem for multiple vessels. He considered that each vessel is divided into many bays, and each bay contains a number of containers. His objective is to reduce the cost of delay using an approximate and an exact method. Furthermore, [2] Peterkofsky and Daganzo (1990) proposed a branch and bound method for the quay crane scheduling problem in the case of quay cranes crossing. After that, [3] Kim and Park (2004) explored the quay crane scheduling problem with non-crossing constraints, and they considered that only one quay crane can work into the vessel. Their objective was to minimize the total completion time.
[5] Lim et al. (2004) considered that each vessel is a job and each quay crane is assigned to this job. They developed a dynamic programming algorithm with a taboo search method to solve the problem. [4] Steeken and Stahlbock (2004) also studied the quay crane scheduling problem and they classified and described the logistic processes and present a new survey for their optimization. [6] Homayouni et al. (2013) proposed a

genetic algorithm to schedule the quay cranes with integration of automated guided vehicles (AGV). Moreover, [7] Diabat and Theodorou (2014) proposed a formulation for the scheduling problem and all assignments for the quay cranes such as quay crane's position. They developed a genetic algorithm to solve this problem. Furthermore, [8] Kaveshgar et al. (2014) proposed a mixed integer programming model for quay cranes and yard trucks scheduling. They also developed a genetic algorithm with a greedy search method. After that, [9] Al-Dhaheri and Diabat (2015) defined the sequence for the unloading operations by fixing a number of quay cranes to perform it. They proposed a mixed-integer programming (MIP) formulation for this problem. Finally, [11] Vahdani et al. (2018) aimed to combine the quay cranes and yard truck assignments among them. For this problem, they proposed a bi-objective optimization model.
This study proposes a mixed-integer programming model solved by CPLEX for jointly quay crane and yard truck scheduling problem where both loading and unloading operations are considered. After that, we generated results and tested our model for small and large instances, and a comparasion with real results from the port of Tripoli-Lebanon.
In section 2 we propose a mixed-integer linear programming model. In section 3, we provide the results of the proposed model. Finally, in section 4, we give a conclusion and a step for future works.

2 Mathematical Formulation

2.1 Assumptions

- The required times for loading and unloading the containers by quay cranes and reach-stacker cranes are known, so as the required times to transport containers and the positions of the containers in the vessels.
- Each quay crane can operate in a single container ship at a time.
- Each vessel can be handled by one or more quay cranes at a time.
- The priority of the containers is taken into account and there maybe a time it waits for quay cranes and yard trucks (they both expect one another).
- Each truck can transport only one container at a time.
- Each reach-stacker crane can unload/load only one container at a time.
- Each container can be transported by only one yard truck at a time.
- Each container can be unloaded/loaded by only one quay crane at a time.
- All containers are homogeneous (same size).
- We do not consider the number of reach-stacker cranes.

2.2 Notations

- Q Set of quay cranes that will unload containers from the vessel.
- $Q^{'}$ Set of quay cranes that will load containers to the vessel.
- T Set of yard trucks.
- C Set of containers to be unloaded from the vessel, c is the number of containers.
- $C^{'}$ Set of containers to be loaded to the vessel, c' is the number of containers.
- p_i Position of container i in the vessel 1, $\forall i \in C$.

- p_i' Position of container i in the vessel 2, $\forall i \in C^{'}$.
- v Yard truck time from vessel 2 to vessel 1.
- v_i' Yard truck time from vessel 1 to the yard location for unloaded containers which exists container i, $\forall i \in C$.
- v_i'' Yard truck time from yard location for unloaded containers which exists container i to the vessel 1, not loaded by any container $\forall i \in C$.
- v_i''' Yard truck time from yard location for containers to be loaded which exists container i to the vessel 2, $\forall i \in C^{'}$.
- λ_{ij} Yard truck time from yard location for unloaded containers which exists container i to yard location for containers to be loaded which exists container j, $\forall i \in C$ and $\forall j \in C^{'}$.
- d_i Quay crane unloading time of container i, $\forall i \in C$.
- d_i' Quay crane loading time of container i, $\forall i \in C^{'}$.
- rs Unloading time of a container by RS.
- rs' Loading time of a container by RS.
- s_0 Distance between quay cranes for safety reason.
- $\Omega 1$ Set of precedence containers to be unloaded.
- $\Omega 2$ Set of precedence containers to be loaded.
- u One unit moving time for the quay crane.
- M Big integer.

2.3 Decision variables

Boolean variables

- $X_{ijq} \begin{cases} = 1 \text{ if quay crane q unloads U-container i before U-container j} \\ = 0 \text{ otherwise}, \forall i \in \{0,...,c\}, \forall j \in \{1,...,c+1\}, \forall q \in Q \end{cases}$

- $Y_{ijq} \begin{cases} = 1 \text{ if quay crane q loads L-container i before U-container j} \\ = 0 \text{ otherwise}, \forall i \in \{0,...,c'\}, \forall j \in \{1,...,c'+1\}, \forall q \in Q' \end{cases}$

- $H_{ii'} \begin{cases} = 1 \text{ if U-container i is matched with L-container } i^{'} \\ = 0 \text{ otherwise}, \forall i \in C, \forall i^{'} \in C' \end{cases}$

- $Z_{ijt} \begin{cases} = 1 \text{ if yard truck t transport U-container i before U-container j} \\ = 0 \text{ otherwise}, \forall i \in \{0,...,c\}, \forall j \in \{1,...,c+1\}, \forall t \in T \end{cases}$

- $W_{ij} \begin{cases} = 1 \text{ if round operation time of U-container i finishes before the starts of} \\ \quad \text{round operation of U-container j by the quay crane} \\ = 0 \text{ otherwise}, \forall i \in C, \forall j \in C \end{cases}$

- $W_{ij}^{'} \begin{cases} = 1 \text{ if round operation time of L-container i finishes before the starts of} \\ \quad \text{round operation of L-container j by the quay} \\ \quad \text{crane} \\ = 0 \text{ otherwise}, \forall i \in C', \forall j \in C' \end{cases}$

Float variables

- E_i The time when the process of U-container i ends, $\forall i \in C$
- E'_i The time when the process of L-container i ends, $\forall i \in C'$
- HA'_i Time when U-container i is ready to be transported by the yard truck, $\forall i \in C$
- HA''_i Time when L-container i is ready to be transported by the yard truck, $\forall i \in C'$
- HA'''_i Time when L-container, which is matched with U-container i, is ready to be transported by the yard truck, $\forall i \in C$
- C_{max} Makespan for both vessels loading and unloading

2.4 Modeling

The following is a mixed-integer linear programming model that we propose for the quay crane and yard truck scheduling :

Objective

$$minimize \quad C_{max} \tag{1}$$

Equation (1) is the objective function which aims to minimize the completion time.

Subject to

$$\sum_{j=1}^{c+1} X_{0jq} = 1 \quad \forall q \in Q \tag{2}$$

$$\sum_{j=1}^{c'+1} Y_{0jq} = 1 \quad \forall q \in Q' \tag{3}$$

$$\sum_{i=0}^{c} X_{i(c+1)q} = 1 \quad \forall q \in Q \tag{4}$$

$$\sum_{i=0}^{c'} Y_{i(c'+1)q} = 1 \quad \forall q \in Q' \tag{5}$$

$$\sum_{q \in Q} \sum_{j=1}^{c+1} X_{ijq} = 1 \quad \forall i \in C \tag{6}$$

$$\sum_{q \in Q'} \sum_{j=1}^{c'+1} Y_{ijq} = 1 \quad \forall i \in C' \tag{7}$$

$$\sum_{j=1}^{c+1} X_{ijq} = \sum_{j=0}^{c} X_{jiq} \quad \forall i \in C, \forall q \in Q \tag{8}$$

$$\sum_{j=1}^{c'+1} Y_{ijq} = \sum_{j=0}^{c'} Y_{jiq} \quad \forall i \in C', \forall q \in Q' \tag{9}$$

Constraints (2), (4), (6) and (8) define the sequence of unloading for U-containers by quay cranes (which ensures that the 1^{st} U-container must be unloaded from the vessel as well as the last U-container, and ensures that all quay crane-container assignments for unloading are made). Constraints (3), (5), (7) and (9) define the sequence of loading for L-containers by quay cranes (which ensures that the 1^{st} L-container must be loaded in the vessel as well as the last L-container, and ensures that all crane-container assignments for loading are made).

$$\sum_{i=1}^{c+1} Z_{0it} = 1 \quad \forall t \in T \tag{10}$$

$$\sum_{i=0}^{c} Z_{i(c+1)t} = 1 \quad \forall t \in T \tag{11}$$

$$\sum_{t\in T}\sum_{j=1}^{c+1} Z_{ijt} = 1 \quad \forall i \in C \tag{12}$$

$$\sum_{j=1}^{c+1} Z_{ijt} = \sum_{j=0}^{c} Z_{jit} \quad \forall i \in C, \forall t \in T \tag{13}$$

Constraints (10), (11), (12) and (13) provide the transport sequence of the U-containers from the vessel by the yard trucks.

$$\sum_{i^{'} \in C'} H_{ii'} = 1 \quad \forall i \in C \tag{14}$$

$$\sum_{i\in C} H_{ii'} = 1 \quad \forall i^{'} \in C' \tag{15}$$

Constraints (14), (15) give all the unique assignment for the pairs of U-containers and L-containers which correspond to each other.

$$E_i >= d_i - M * (1 - \sum_{q\in Q} X_{0iq}) \quad \forall i \in C \tag{16}$$

$$E_i >= E_j - M * (1 - \sum_{q\in Q} X_{jiq}) + (p_i - p_j) * u + d_i \quad \forall i \in C, \forall j \in C \tag{17}$$

Constraints (16), (17) provide the completion time for unloading the U-containers by the quay cranes from the vessel.

$$E'_i >= E'_j - M * (1 - \sum_{q\in Q'} X_{jiq}) + (p'_i - p'_j) * u + d'_i \quad \forall i \in C', \forall j \in C' \tag{18}$$

$$E'_i >= HA''_i + d'_i \quad \forall i \in C' \tag{19}$$

Constraints (18), (19) provide the completion time for loading L-containers by quay cranes into the vessel.

$$HA'_i >= E_i + v'_i + rs \quad \forall i \in C \tag{20}$$

$$HA'_i >= HA'''_j - M * (1 - \sum_{t \in T} Z_{jit}) + v'_i + rs \quad \forall i \in C, \forall j \in C \tag{21}$$

Constraints (20), (21) provide the completion time for transporting the U-containers by the yard trucks.

$$HA''_{i'} >= HA'_i - M * (1 - H_{ii'}) + \lambda_{ii'} + rs' + v'''_{i'} + v \quad \forall i \in C, \forall i' \in C' \tag{22}$$

Constraint (22) provides the completion time for transporting L-containers by yard trucks.

$$HA'''_i >= HA''_{i'} - M * (1 - H_{ii'}) \quad \forall i \in C, \forall i' \in C' \tag{23}$$

Constraint (23) provides the completion time to transport the L-container which is matched with the U-container, by the yard truck.

$$E_j - d_j >= E_i \quad \forall (i, j) \in \Omega 1 \tag{24}$$

Constraint (24) ensures that the operation of each U-container must be completed before another U-container that follows it, if they belong to $\Omega 1$.

$$E'_j - d'_j >= E'_i \quad \forall (i, j) \in \Omega 2 \tag{25}$$

Constraint (25) ensures that the operation of each L-container must be completed before another L-container that follows it, if they belong to $\Omega 2$.

$$M * (1 - W_{ij}) >= E_i - (E_j - d_j) \quad \forall i \in C, \forall j \in C \tag{26}$$

$$M * (1 - W'_{ij}) >= E'_i - (E'_j - d'_j) \quad \forall i \in C', \forall j \in C' \tag{27}$$

$$E_j - d_j - E_i <= M * W_{ij} \quad \forall i \in C, \forall j \in C \tag{28}$$

$$E'_j - d'_j - E'_i <= M * W'_{ij} \quad \forall i \in C', \forall j \in C' \tag{29}$$

$$(p_i - p_j) * (i - j) + M * (W_{ij} + W_{ji}) >= (i - j) * s_0 \quad \forall i \in C, \forall j \in C, i \neq j \tag{30}$$

$$(p'_i - p'_j) * (i - j) + M * (W'_{ij} + W'_{ji}) >= (i - j) * s_0 \quad \forall i \in C', \forall j \in C', i \neq j \tag{31}$$

Constraints (26), (27), (28), (29), (30) and (31) guarantee the non-crossing and the safety margin between the quay cranes.

$$C_{max} >= E'_i \quad \forall i \in C' \tag{32}$$

Constraint (32) indicates the completion time of the vessel who contains the U-containers.

In the previous model, we suppose that the number of U-containers is equal to the number of L-containers. Nevertheless, the numbers of U-containers and L-containers are different. For the case where the number of U-containers is bigger than the number of L-containers, there are no L-containers that matched with the U-containers and the yard truck will return empty from the storage location. So for this reason we will add

$c - c'$ fictive L-containers.
For this case, we propose a model extension and it is formulated as follows:

Objective

$$minimize \quad C_{max} \tag{33}$$

Subject to

$$\text{constraints } (2) \rightarrow (14), (16) \rightarrow (21), (23) \rightarrow (31)$$

$$\sum_{i \in C} H_{ii'} = 1 \quad \forall i' \in \{1, ..., c', ..., c\} \tag{34}$$

Constraint (34) provides all the unique assignments for the pairs including the fictive L-containers.

$$HA''_{i'} >= HA'_i - M * (1 - H_{ii'}) + v''_i$$

$$\forall i \in C \quad \forall i' \in \{c' + 1, ..., c'\} \tag{35}$$

Constraint (35) presents the completion time of the U-containers with the empty movements.

$$C_{max} >= HA'''_i \quad \forall i \in C \tag{36}$$

Constraint (36) defines the makespan of all arriving vessels.
In another way, we swapped the constraint (15) by (34), the constraint (22) by (35) and the constraint (32) by (36).

3 Experimental results

The model is solved using the CPLEX 12.6 solver, and the tests are run on MacBook Pro 2.7 GHz Intel Core i5 with 8GB RAM 1867 MHz DDR3 under OSX 10.11.6.
In this section, we are presenting the results generation and the results obtained for real cases in the port of Tripoli-Lebanon. The makespan is measured in time units (u.t).

3.1 Results for randomly generated instances

Table 1 shows the results of the calculation tests when the numbers of U-containers and L-containers are the same, and when the number of U-containers is greater than the L-containers. For example in instance 24 in Table 1, for 30 U-containers, 25 L-containers, 7 quay cranes (jointly for unloading and loading) and 10 yard trucks, CPLEX cannot provide any result after 3 hours of execution, then we interrupt the execution (N.A. = Interrupt execution (No results)). In this table, we notice that the proposed MILP works for small and medium instances and does not work so well for large instances. So in our next work, we will propose a new exact or metaheuristic methods to improve the execution time and obtain near optimal solutions.

Table 1. Experimental results

Instance	$\|C\|$	$\|C'\|$	$\|Q\|$	$\|Q'\|$	$\|T\|$	CPLEX	Instance	$\|C\|$	$\|C'\|$	$\|Q\|$	$\|Q'\|$	$\|T\|$	CPLEX
1	6	6	1	1	1	1115	14	15	15	2	2	4	562
2	6	4	1	1	2	492	15	16	12	3	2	6	312
3	8	8	1	1	2	748	16	16	14	3	2	6	433
4	8	6	2	2	3	433	17	16	14	3	2	7	327
5	10	10	2	2	4	467	18	18	14	4	3	8	340
6	10	6	2	2	3	527	19	18	16	4	3	10	337
7	10	8	2	2	4	450	20	18	16	4	3	8	229
8	12	8	2	2	4	445	21	20	20	2	3	6	476
9	12	10	2	2	4	545	22	25	20	4	3	10	229
10	12	12	2	2	5	452	23	26	24	4	3	10	225
11	12	10	2	2	5	351	24	30	25	4	3	10	N.A
12	14	12	2	2	5	382	25	30	30	2	2	6	N.A
13	14	12	3	2	6	270							

3.2 Results for real instances from port of Tripoli-Lebanon

Table 2 compares some results from the port of Tripoli-Lebanon, with the obtained results by this model . We emphasize that all port's results are considered in the same values and conditions of the port of Tripoli-Lebanon. As shown in Table 2, our model succeded in improving the completion time of containers, for all the tested instances, by an average 20%. GAP(%) = ((port result - CPLEX result)/port result)*100.

Table 2. Comparison with the real results in port of Tripoli-Lebanon

Instance	$\|\mathbf{C}\|$	$\|\mathbf{C}'\|$	$\|\mathbf{T}\|$	**Port results** (s)	**CPLEX resuts** (s)	**GAP** (%)
P1	2	2	1	783	629	19.67
P2	5	4	2	965	782	18.96
P3	6	6	2	1129	912	19.22
P4	7	4	2	1046	835	20.17
P5	8	7	2	1393	1076	22.76

4 Conclusion

This model investigates the scheduling problem for the quay cranes with yard trucks in an integrated way. We use the dual strategies to reduce the empty movements for the yard trucks. We proposed a mixed-integer linear programming model to minimze the completion time of all containers in the vessels, and thus reducing the docking time of all vessels. From the numerical results, we can see that the proposed model is feasible. For small instances, CPLEX provides results with an acceptable execution time. But for larger instances, CPLEX cannot provide any result. So, in our future studies, we will develop exact or metaheuristic algorithms to compare operational results and thus obtain results for large instances.

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