



## Brief paper

Observer-based boundary control of distributed port-Hamiltonian systems<sup>☆</sup>Jesús Toledo<sup>a,\*</sup>, Yongxin Wu<sup>a</sup>, Héctor Ramírez<sup>b</sup>, Yann Le Gorrec<sup>a</sup><sup>a</sup> FEMTO-ST Institute, Univ. Bourgogne Franche-Comté, ENSMM, CNRS, 24 rue Savary, F-25000 Besançon, France<sup>b</sup> Department of Electronic Engineering, Universidad Técnica Federico Santa María, Avenida España 1680, Valparaíso, Chile

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## ABSTRACT

An observer-based boundary controller for infinite-dimensional port-Hamiltonian systems defined on 1D spatial domains is proposed. The design is based on an early-lumping approach in which a finite-dimensional approximation of the infinite-dimensional system derived by spatial discretization is used to design the observer and the controller. As long as the finite-dimensional approximation approaches the infinite-dimensional model, the performances also do. The main contribution is a constructive method which guarantees that the interconnection between the controller and the infinite-dimensional system is asymptotically stable. A Timoshenko beam model has been used to illustrate the approach.

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## 1. Introduction

Boundary control systems (BCS) (Fattorini, 1968) are a class of control systems where the dynamics are described by partial differential equations (PDEs) with actuation and measurement situated at the boundaries of the spatial domain. Motivated by technological advances, these type of systems have been of great interest for engineers and mathematicians during the last decades since a large class of physical processes can be represented as BCS. This is for instance the case of beams and waves in mechanical systems, heat bars and bed reactors in chemical systems or telegraph equations in electronic systems, among others (Curtain & Zwart, 2012).

Recently, the control of BCS has been addressed by using the framework of port-Hamiltonian systems (PHS) (Le Gorrec et al., 2005; van der Schaft & Maschke, 2002). Boundary controlled PHS (BC-PHS) are an extension of the Hamiltonian formulation of mechanical systems to open multi-physical systems (Duindam et al., 2009). This formalism has been proven to be particularly suitable for the modeling and control of complex physical systems, such as systems described by infinite-dimensional or non-linear models. The stability, stabilization and control synthesis of BC-PHS have

been addressed in Augner and Jacob (2014), Macchelli et al. (2017), Ramirez et al. (2014), Villegas et al. (2009) and Ramirez et al. (2017). More recently, the framework has been extended to deal with robust and adaptive regulation (Humaloja & Paunonen, 2018; Macchelli & Califano, 2018).

In the case of observer-based control design there are generally two approaches. The first one is the late-lumping approach in which the observer is designed from the infinite-dimensional systems (Guo & Xu, 2007; Meurer, 2013). The main problem comes from the infinite-dimensional aspect of the controller structure that needs to be reduced for practical and real-time implementation. The second one is the early-lumping approach. In this case, the system is first approximated and a finite-dimensional observer is implemented on the reduced order system. The main drawback is the spillover effect induced by the use of a reduced order controller on the infinite-dimensional system, leading to high-frequency mode destabilization (Bontsema & Curtain, 1988).

The main result of this paper is the proposition of a systematic synthesis method for observer-based boundary controller design for BC-PHS defined on one dimensional spatial domains. A finite-dimensional PHS approximation of the BC-PHS is used to design a strictly positive real PHS observer-based state feedback. This approximation is considered precise enough such that in the frequency range of interest the modes of the approximated model are close enough to the one of the original system.

The observer is then used to compute the boundary control law for the infinite-dimensional system. Using the passivity properties of power preserving interconnection of PHS it is then possible to guarantee the asymptotic stability of the closed-loop system. The controller hence allows to assign the low-frequency

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modes while guaranteeing stable high-frequency modes, avoiding spillover effects. This paper is organized as follows: Section 2 gives a background on port-Hamiltonian systems. Section 3 gives the main result of this work. Section 4 presents a numerical example, namely a boundary actuated one-dimensional Timoshenko beam.

## 2. Background on port-Hamiltonian systems

### 2.1. Some notation

In this paper,  $M_n(\mathbb{R})$  denotes the space of real  $n \times n$  matrices and  $I$  denotes the identity matrix of appropriate dimensions. By  $\langle \cdot, \cdot \rangle_{L_2}$  or only  $\langle \cdot, \cdot \rangle$  we denote the standard inner product on  $L_2(a, b; \mathbb{R}^n)$  and the Sobolev space of order  $p$  is denoted by  $H^p(a, b; \mathbb{R}^n)$ . A detailed description of the class of boundary control systems under consideration can be found in [Le Gorrec et al. \(2005\)](#) and [Jacob and Zwart \(2012\)](#). In the next section, we recall some basic properties of this class of systems.

### 2.2. Boundary controlled port-Hamiltonian systems

The class of one-dimensional PDEs under study, with inputs and outputs at the spatial boundaries, is given by the following set of equations

$$P \begin{cases} \frac{\partial z}{\partial t}(\zeta, t) = P_1 \frac{\partial}{\partial \zeta}(\mathcal{L}(\zeta)z(\zeta, t)) + (P_0 - G_0)\mathcal{L}(\zeta)z(\zeta, t), \\ u(t) = W \begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix}, \quad z(\zeta, 0) = z_0(\zeta), \\ y(t) = \tilde{W} \begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix}, \quad t \geq 0, \quad \zeta \in [a, b], \end{cases} \quad (1)$$

with  $t > 0$  the time variable and  $\zeta \in [a, b]$  the 1D spatial coordinate, and  $z(\zeta, t) \in \mathbb{R}^n$  the state variable.  $P_1 = P_1^T \in M_n(\mathbb{R})$  is a non-singular matrix,  $P_0 = -P_0^T \in M_n(\mathbb{R})$ ,  $G_0 = G_0^T \geq 0 \in M_n(\mathbb{R})$ ,  $\mathcal{L}(\cdot) \in M_n(L_2(a, b))$  is a bounded and continuously differentiable matrix-valued function satisfying for all  $\zeta \in [a, b]$ ,  $\mathcal{L}(\zeta) = \mathcal{L}^T(\zeta)$  and  $mI < \mathcal{L}(\zeta) < MI$  with  $M > m > 0$  both scalars independent on  $\zeta$ . The state space is  $Z = L_2(a, b; \mathbb{R}^n)$  with inner product  $\langle z_1, z_2 \rangle_{\mathcal{L}} = \langle z_1, \mathcal{L}z_2 \rangle$  and norm  $\|z\|_{\mathcal{L}}^2 = \langle z, z \rangle_{\mathcal{L}}$ , hence  $Z$  is a Hilbert space. The norm  $\|\cdot\|_{\mathcal{L}}$  is usually proportional to the stored energy of the system, hence  $z(\zeta, t)$  is called energy variable and  $\mathcal{L}(\zeta)z(\zeta, t)$  is called co-energy variable. For simplicity, we write  $z$  and  $\mathcal{L}z$  instead of  $z(\zeta, t)$  and  $\mathcal{L}(\zeta)z(\zeta, t)$  unless otherwise stated and arguments of dependent variables may be omitted.

**Definition 1.** Let  $\mathcal{L}z \in H^1(a, b; \mathbb{R}^n)$ . Then, the boundary port variables associated with (1) are the vectors  $f_{\partial}$  and  $e_{\partial} \in \mathbb{R}^n$ , defined by

$$\begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} P_1 & -P_1 \\ I & I \end{pmatrix} \begin{pmatrix} \mathcal{L}(b)z(b, t) \\ \mathcal{L}(a)z(a, t) \end{pmatrix}. \quad (2)$$

Note that, the port-variables are nothing else than a linear combination of the boundary variables. We also define the matrix  $\Sigma \in M_{2n}(\mathbb{R})$  as follows

$$\Sigma = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \quad (3)$$

The following theorem ensures the existence and uniqueness of solutions of (1).

**Theorem 2** ([Le Gorrec et al., 2005](#)). Let  $W$  be a  $n \times 2n$  real matrix. If  $W$  has full rank and satisfies  $W\Sigma W^T \geq 0$  then the system (1) with input

$$u(t) = W \begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix} \quad (4)$$

is a BCS on  $Z$ . Furthermore, the operator  $\mathcal{A}z = P_1 \frac{\partial}{\partial \zeta}(\mathcal{L}z) + (P_0 - G_0)\mathcal{L}z$  with domain

$$D(\mathcal{A}) = \left\{ \mathcal{L}z \in H^1(a, b; \mathbb{R}^n) \left| \begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix} \in \ker W \right. \right\}$$

generates a contraction semigroup on  $Z$ .

Let  $\tilde{W}$  be a full rank matrix of size  $n \times 2n$  with  $[w^T \tilde{w}^T]^T$  invertible and  $P_{W, \tilde{W}}$  given by

$$P_{W, \tilde{W}} = \begin{pmatrix} W\Sigma W^T & W\Sigma \tilde{W}^T \\ \tilde{W}\Sigma W^T & \tilde{W}\Sigma \tilde{W}^T \end{pmatrix}^{-1}. \quad (5)$$

Define the output of the system as the linear mapping  $\mathcal{C} : \mathcal{L}^{-1}H^1(a, b; \mathbb{R}^n) \rightarrow \mathbb{R}^n$

$$y(t) = \mathcal{C}z(\zeta, t) = \tilde{W} \begin{pmatrix} f_{\partial}(t) \\ e_{\partial}(t) \end{pmatrix} \quad (6)$$

Then, for  $u \in C^2(0, \infty; \mathbb{R}^n)$ ,  $\mathcal{L}z(\zeta, 0) \in H^1(a, b; \mathbb{R}^n)$  and  $u(0) = W \begin{pmatrix} f_{\partial}(0) \\ e_{\partial}(0) \end{pmatrix}$  the following balance equation is satisfied

$$\frac{1}{2} \frac{d}{dt} \|z(\zeta, t)\|_{\mathcal{L}}^2 = \frac{1}{2} \begin{pmatrix} u(t) \\ y(t) \end{pmatrix}^T P_{W, \tilde{W}} \begin{pmatrix} u(t) \\ y(t) \end{pmatrix} \quad (7)$$

**Remark 3.** The matrix

$$\begin{pmatrix} W \\ \tilde{W} \end{pmatrix} \Sigma \begin{pmatrix} W \\ \tilde{W} \end{pmatrix}^T = \begin{pmatrix} W\Sigma W^T & W\Sigma \tilde{W}^T \\ \tilde{W}\Sigma W^T & \tilde{W}\Sigma \tilde{W}^T \end{pmatrix} \quad (8)$$

is invertible if and only if  $[w^T \tilde{w}^T]^T$  is invertible.

In this work, we shall consider an early-lumping approach, i.e., the controller and observer are designed on a finite-dimensional approximation of (1). The following assumption is considered

**Assumption 4.** There exists the following finite-dimensional approximation of (1)

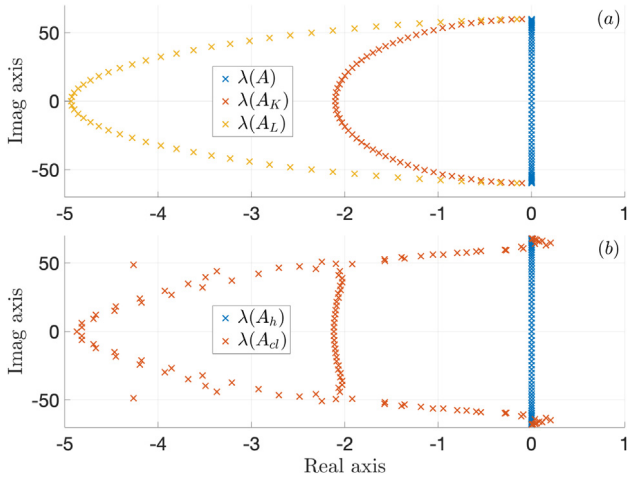
$$P \begin{cases} \dot{x}(t) = (J - R)Qx(t) + Bu(t) \\ y(t) = B^T Qx(t) \end{cases} \quad (9)$$

where  $x \in \mathbb{R}^{n_c}$  with  $n_c$  given by the order of the approximation,  $J = -J^T$ ,  $R = R^T \geq 0$ ,  $Q = Q^T > 0$  all of them in  $M_{n_c}(\mathbb{R})$  and  $B \in \mathbb{R}^{n_c \times n}$ . Furthermore we assume (9) to be controllable and observable. For simplicity, we shall define  $A = (J - R)Q$  and  $C = B^T Q$  and we will refer to the system  $(A, B, C)$  as the approximated model of (1).

**Remark 5.** Approximation schemes which preserve the port-Hamiltonian structure of the original system using mixed finite elements or finite differences on staggered grids for instance, can be found in [Seslija et al. \(2012\)](#) and [Trenchant et al. \(2018\)](#). The achievable closed-loop performances depend on the quality of the approximated model. The order of the approximation  $n_c$  has then to be chosen large enough such that in the frequency range of interest the approximated system poles behave similar to the original ones.

## 3. The observer-based controller

The main objective of this work is to design a finite-dimensional controller that achieves some desired performances on the finite-dimensional system (9) while ensuring closed-loop stability when applied to the infinite-dimensional



**Fig. 1.** (a):  $\lambda(A)$ : discretized model eigenvalues with  $n_c = 59$ ,  $\lambda(A_K)$ :  $A - BK$  eigenvalues and  $\lambda(A_L)$ :  $A - LC$  eigenvalues. (b):  $\lambda(A_h)$ : discretized model eigenvalues with  $n_c = 67$ ,  $\lambda(A_{cl})$ : closed-loop eigenvalues.

system (1). The considered controller is an observer-based state feedback

$$u(t) = r(t) - K\hat{x}(t) \tag{10}$$

where  $\hat{x} \in \mathbb{R}^{n_c}$ ,  $r \in \mathbb{R}^n$  and the Luenberger observer

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t)) \tag{11}$$

with matrices  $K \in \mathbb{R}^{n \times n_c}$ ,  $L \in \mathbb{R}^{n_c \times n}$  to be designed and  $(A, B, C)$  defined in (9). Note that,  $n_c$  is the size of the observer given by the chosen discretization scheme and  $n$  is the number of boundary variables.

Several issues can arise when using an early-lumping approach to design the control, the most critical one being the loss of stability when the controller is applied on the infinite-dimensional system. It is known as the spillover effect (Bontsema & Curtain, 1988). Consider the following illustrative example.

**Example 6.** Consider the 1D wave equation with unitary parameters and Neumann boundary control. The system can be written (see Jacob and Zwart (2012) for more details) in the form (1) with

$$P_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, P_0 = G_0 = 0, \mathcal{L} = I_2.$$

This model is discretized by using finite differences on staggered grids in order to preserve the structure of the system. Consider  $n_c = 59$  elements for the discretization.  $G_0 = 0$  implies  $R = 0$  thus all the eigenvalues of  $A$  are on the imaginary axis as shown in Fig. 1(a).  $(A, B)$  is controllable and  $(A, C)$  observable, hence  $K$  and  $L$  can be designed such that  $A_K = A - BK$  and  $A_L = A - LC$  are Hurwitz. Using for instance the Linear Quadratic Regulator (LQR) method the closed-loop eigenvalues can be assigned as in Fig. 1(a).

The question that naturally arises is if the same control law, i.e. the same choice of matrices  $K$  and  $L$ , preserves the stability when applied on the infinite-dimensional system. The answer in general is no. In this particular case for instance, when increasing the order of the discretized model to  $n_c = 67$ , the closed-loop system turns unstable as shown in Fig. 1(b).

In what follows we start from the achievable closed-loop performances on the finite-dimensional system i.e. an appropriate choice of  $K$ , and design the observer gain such that the Luenberger observer (11) is a strictly positive real PHS. Then we show

that since (10) corresponds to a power preserving interconnection between the infinite-dimensional system and the dynamic boundary controller, the closed-loop system is asymptotically stable.

### 3.1. Some technical results

Before presenting the main result we give the following definitions, lemma, corollary and theorem which are instrumental in the proof.

**Definition 7.** A  $n \times n$  transfer matrix  $G(s)$  is positive real (PR) if  $G(s) + G^T(\bar{s}) \geq 0$  for all  $s$  such that  $\text{Re}(s) > 0$ .

**Definition 8.** A  $n \times n$  transfer matrix  $G(s)$  is strictly positive real (SPR) if there exists a scalar  $\varepsilon > 0$  such that  $G(s - \varepsilon)$  is PR.

**Lemma 9** (Lefschetz–Kalman–Yakubovich Tao & Ioannou, 1988). Assume for the system  $(A, B, C, D)$  that  $(A, B)$  is controllable and  $(A, C)$  is observable. Then, the transfer matrix  $G(s) = C(sI - A)^{-1}B + D$  is SPR if and only if there exist real matrices  $P = P^T > 0$ ,  $\Gamma$ ,  $W_1$  and a scalar  $\varepsilon > 0$  such that

$$PA + A^T P = -\Gamma^T \Gamma - \varepsilon P \tag{12a}$$

$$C - B^T P = W_1^T \Gamma \tag{12b}$$

$$D + D^T = W_1^T W_1 \tag{12c}$$

**Corollary 10.** The system  $(A, B, C, D)$  with  $A = (J - R)Q$ ,  $C = B^T Q$  and  $D = 0$  is strictly positive real if  $J = -J^T$ ,  $R = R^T > 0$  and  $Q = Q^T > 0$ .

**Proof.** From Lemma 9 choose  $P = Q$  and  $W_1 = 0$ , then (12c) is trivial, (12b) is  $C = B^T Q$  and (12a) becomes

$$\Gamma^T \Gamma = 2QRQ - \varepsilon Q \tag{13}$$

then, for  $R > 0$  there exists a constant  $\varepsilon > 0$  such that the right hand side is positive definite, giving a solution for  $\Gamma$ , using for instance Cholesky factorization. See Corollary 7.2.9 in Horn and Johnson (2012). ■

The next theorem assures the stability of (1) interconnected in a power preserving way with a SPR controller.

**Theorem 11** (Villegas, 2007, Ch. 5.1.2). Consider (1) with  $u(t)$  defined according to Theorem 2 and  $y(t)$  such that

$$\frac{1}{2} \frac{d}{dt} \|z(\zeta, t)\|_{\mathcal{L}}^2 = u^T(t)y(t), \tag{14}$$

i.e.  $W \Sigma W^T = \tilde{W} \Sigma \tilde{W}^T = 0$  and  $\tilde{W} \Sigma W^T = I$ . Consider also a finite-dimensional controller with input  $u_c(t)$  and output  $y_c(t)$  such that its transfer matrix is SPR. Then, the closed-loop system with the passive interconnection

$$u_c(t) = y(t) \tag{15a}$$

$$u(t) = -y_c(t) \tag{15b}$$

is well-posed and asymptotically stable.

### 3.2. Equivalent observer based control by interconnection

In what follows we consider the finite-dimensional approximation (9) of (1).

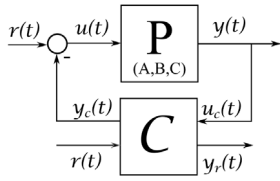


Fig. 2. Control scheme.

**Definition 12.** Control by interconnection with a SPR-PH controller. The considered control scheme is shown in Fig. 2, where the SPR-PH controller is given by

$$C \begin{cases} \dot{\hat{x}}(t) &= (J_c - R_c)Q_c \hat{x}(t) + B_c u_c(t) + Br(t) \\ y_c(t) &= B_c^T Q_c \hat{x} \\ y_r(t) &= B^T Q_c \hat{x}, \end{cases} \quad (16)$$

with  $J_c = -J_c^T$ ,  $R_c = R_c^T > 0$ ,  $Q_c = Q_c^T > 0 \in \mathbb{R}^{n_c \times n_c}$ ,  $B_c \in \mathbb{R}^{n_c \times n}$  and  $B$  defined in (9), and (16) is interconnected in a power preserving way to the system (9), i.e. such that

$$u_c(t) = y(t), \quad u(t) = r(t) - y_c(t) \quad (17)$$

Furthermore we assume that  $((J_c - R_c)Q_c, B_c)$  is controllable.

**Theorem 13.** The control scheme of Definition 12 is asymptotically stable and converges to zero when  $r(t) = 0$ .

**Proof.** Consider the total energy as Lyapunov function

$$V(x, \hat{x}) = \frac{1}{2}x^T Qx + \frac{1}{2}\hat{x}^T Q_c \hat{x}$$

Then from (9) and (16) we have

$$\dot{V}(x, \hat{x}) \leq -\hat{x}^T Q_c R_c Q_c \hat{x},$$

where  $R_c > 0$  from Definition 12. From Lasalle's invariance principle the system converges to the invariant set corresponding to  $\dot{V}(x, \hat{x}) = 0$ , i.e.  $\|\hat{x}\|_2 = 0$ . In this case, from (16), (17) and  $r = 0$  we have  $u_c = y = 0$ , the controller being controllable. The system (9) being observable the only equilibrium point is 0. ■

In what follows, we propose to start from the achievable closed-loop performances obtained by state feedback to build an observer based controller, i.e.  $J_c$ ,  $R_c$ ,  $Q_c$  and  $B_c$  in (16) that is strictly positive real, i.e. satisfies the conditions of Corollary 10.

**Proposition 14.** The interconnection of the system (9) with the observer based controller (10)–(11) is equivalent to the control by interconnection with the SPR-PH controller of Definition 12 if the following matching conditions are satisfied

$$\begin{aligned} (J_c - R_c)Q_c &= A - BK - LC \\ B_c^T Q_c &= K \\ B_c &= L. \end{aligned} \quad (18)$$

**Proof.** The matching equations (18) are directly obtained replacing (10) in (11) and identifying with (16) in order to get a passive and collocated dynamic controller. ■

### 3.3. Main result

The following proposition is the main contribution of this work. It is based on two main assumptions.

**Assumption 15.** The matrix  $K$  has been designed such that  $A - BK$  is Hurwitz by using traditional methods such as LQR design, pole-placement or LMI passivity based control, such as for instance in Prajna et al. (2002).

**Assumption 16.** The matrix  $R_c$  is chosen such that the following matrix

$$H_M = \begin{pmatrix} A_K & 2R_c \\ -C_K & -A_K^T \end{pmatrix} \quad (19)$$

with

$$A_K = A - BK, \quad C_K = -(K^T C + C^T K), \quad (20)$$

has no pure imaginary eigenvalues.

**Remark 17.** A simple choice for  $R_c$  is  $R_c = \alpha I$  for some  $\alpha > 0$  small enough such that the matrix (19) has no pure imaginary eigenvalues.

**Proposition 18.** Under Assumptions 15 and 16, there exists a matrix  $Q_c = Q_c^T > 0$ , solution of the algebraic Riccati equation (ARE)

$$A_K^T Q_c + Q_c A_K + 2Q_c R_c Q_c + C_K = 0, \quad (21)$$

such that the matching equations (18) are satisfied with

$$\begin{aligned} J_c &= \frac{1}{2} [A_K Q_c^{-1} - Q_c^{-1} A_K^T - Q_c^{-1} (K^T C - C^T K) Q_c^{-1}] \\ B_c &= Q_c^{-1} K^T \\ L &= B_c. \end{aligned} \quad (22)$$

Furthermore, the matrix  $A - LC$  is Hurwitz.

**Proof.** From Kosmidou (2007) it is known that if the Hamiltonian matrix (19) has no pure imaginary eigenvalues then there exists a solution  $Q_c = Q_c^T > 0$  for (21). Hence we only need to prove that (21) is compatible with the matching equation (18) for  $J_c$  and  $L$  as in (22). Since  $Q_c$  is invertible and solution of (21) we have

$$\begin{aligned} R_c &= -\frac{1}{2} [Q_c^{-1} A_K^T + A_K Q_c^{-1} + Q_c^{-1} C_K Q_c^{-1}] \\ &= -\frac{1}{2} [Q_c^{-1} A_K^T + A_K Q_c^{-1} - Q_c^{-1} (K^T C + C^T K) Q_c^{-1}] \end{aligned} \quad (23)$$

Then using (22) and (23) we have

$$\begin{aligned} (J_c - R_c)Q_c &= \frac{1}{2} (2A_K Q_c^{-1} - 2Q_c^{-1} K^T C Q_c^{-1}) Q_c \\ &= A_K - Q_c^{-1} K^T C \\ &= A_K - LC \\ &= A - BK - LC \end{aligned} \quad (24)$$

which correspond to (18). From Theorem 13 the closed-loop system

$$\frac{d}{dt} \begin{pmatrix} x \\ \hat{x} \end{pmatrix} = \begin{pmatrix} A & -BK \\ B_c C & (J_c - R_c)Q_c \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix} + \begin{pmatrix} B \\ B \end{pmatrix} r \quad (25)$$

is asymptotically stable. Applying the following transformation

$$\begin{pmatrix} x \\ \tilde{x} \end{pmatrix} = \begin{pmatrix} I & 0 \\ I & -I \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

the closed-loop system (25) can be written

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} x \\ \tilde{x} \end{pmatrix} &= \begin{pmatrix} A_K & BK \\ A_K - B_c C - A_c & A_c + BK \end{pmatrix} \begin{pmatrix} x \\ \tilde{x} \end{pmatrix} \\ &+ \begin{pmatrix} B \\ B \end{pmatrix} r \end{aligned} \quad (26)$$

with  $A_K = A - BK$ ,  $B_c = L$  and  $A_c = (J_c - R_c)Q_c = A - BK - LC$  or equivalently

$$\frac{d}{dt} \begin{pmatrix} x \\ \tilde{x} \end{pmatrix} = \begin{pmatrix} A - BK & BK \\ 0 & A - LC \end{pmatrix} \begin{pmatrix} x \\ \tilde{x} \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} r \quad (27)$$

Since  $A_K$  is Hurwitz, and the closed-loop system asymptotically stable,  $A - LC$  is also Hurwitz. ■

**Theorem 19.** Let us consider the infinite-dimensional system (1) with  $u = -K\hat{x}$  and  $\hat{x}$  solution of the dynamic equation (16) in Proposition 18. The closed-loop system is asymptotically stable.

**Proof.** The proof is a direct application of Theorem 11, Propositions 14 and 18. Choosing  $R_c > 0$  satisfying Assumption 16 from Proposition 18 there exists a matrix  $Q_c = Q_c^T > 0$  solution of (21) such that the finite-dimensional observer defined by (11) is stable and the matching equations (18) are satisfied. Then, from Proposition 14 the control is equivalent to the control by interconnection with a SPR-PH controller. From Theorem 11 the closed-loop system is well posed and asymptotically stable when the finite-dimensional controller is applied to the infinite-dimensional system. ■

**Remark 20.** One special case of Proposition 18 is proposed in Wu et al. (2018) in the context of reduced order control of finite-dimensional PHS. There the matrix  $K$  obtained by a LQR method and the matrix  $Q_c = Q$ .

#### 4. Example: the Timoshenko beam

We consider the boundary control of a Timoshenko beam clamped at the left side and controlled through force and torque at the right side. Both longitudinal and angular velocities at the right side are used for control purposes. The port Hamiltonian formulation of the Timoshenko beam can be found in Macchelli and Melchiorri (2004). It can be written in the form (1) with

$$P_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad P_0 = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{L}(\zeta) = \begin{bmatrix} T(\zeta) & 0 & 0 & 0 \\ 0 & \frac{1}{\rho(\zeta)} & 0 & 0 \\ 0 & 0 & EI(\zeta) & 0 \\ 0 & 0 & 0 & \frac{1}{I_\rho(\zeta)} \end{bmatrix}, \quad G_0 = 0$$

where  $T(\zeta)$  is the shear modulus,  $\rho(\zeta)$  the mass per length unit,  $EI(\zeta)$  the product of the Young's modulus of elasticity  $E$  and the moment of inertia of a cross section  $I$ , and  $I_\rho(\zeta)$  the moment of inertia of a cross section.

The state variables are: the shear displacement, the transverse momentum distribution, the angular displacement and the angular momentum distribution defined respectively by  $z_1(\zeta, t) = \frac{\partial w}{\partial \zeta}(\zeta, t) - \phi(\zeta, t)$ ,  $z_2(\zeta, t) = \rho(\zeta) \frac{\partial w}{\partial t}(\zeta, t)$ ,  $z_3(\zeta, t) = \frac{\partial \phi}{\partial \zeta}(\zeta, t)$  and  $z_4(\zeta, t) = I_\rho(\zeta) \frac{\partial \phi}{\partial t}(\zeta, t)$ , where  $w(\zeta, t)$  and  $\phi(\zeta, t)$  are respectively the transverse displacement of the beam and the rotation angle of a neutral fiber of the beam. Note that,  $T(\zeta)z_1(\zeta, t)$  is the shear force,  $\frac{1}{\rho(\zeta)}z_2(\zeta, t)$  the longitudinal velocity,  $EI(\zeta)z_3(\zeta, t)$  the torque and  $\frac{1}{I_\rho(\zeta)}z_4(\zeta, t)$  the angular velocity.

We choose as inputs and outputs

$$u(t) = \begin{pmatrix} \frac{1}{\rho(a)}z_2(a, t) \\ \frac{1}{I_\rho(a)}z_4(a, t) \\ T(b)z_1(b, t) \\ EI(b)z_3(b, t) \end{pmatrix}, \quad y(t) = \begin{pmatrix} -T(a)z_1(a, t) \\ -EI(a)z_3(a, t) \\ \frac{1}{\rho(b)}z_2(b, t) \\ \frac{1}{I_\rho(b)}z_4(b, t) \end{pmatrix}$$

The total energy of the beam is defined as

$$H(t) = \frac{1}{2} \|z(\zeta, t)\|_{\mathcal{L}}^2 = \frac{1}{2} \int_a^b z^T(\zeta, t) \mathcal{L}(\zeta) z(\zeta, t) d\zeta.$$

and satisfies

$$\frac{d}{dt} H(t) = y^T(t) u(t).$$

#### 4.1. Discretization

The infinite-dimensional system is discretized using finite differences on staggered grids (Trenchant et al., 2018) considering 20 elements per state variable, which leads to  $n_c = 80$ . The finite-dimensional model (9) is then given by

$$J = \begin{bmatrix} 0 & D & 0 & -F \\ -D^T & 0 & 0 & 0 \\ 0 & 0 & 0 & D \\ F^T & 0 & -D^T & 0 \end{bmatrix}$$

$$R = 0, \quad Q = h \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & Q_4 \end{bmatrix}$$

where  $Q_i$ ,  $i \in \{1, \dots, 4\}$  are diagonal matrices containing the evaluation of  $T(\zeta)$ ,  $\frac{1}{\rho(\zeta)}$ ,  $EI(\zeta)$  and  $\frac{1}{I_\rho(\zeta)}$  respectively, at the specific points chosen for the discretization.

$$D = \frac{1}{h^2} \begin{bmatrix} 1 & 0 & \dots & 0 \\ -1 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots \\ 0 & 0 & \dots & 1 \end{bmatrix}, \quad F = \frac{1}{2h} \begin{bmatrix} 1 & 0 & \dots & 0 \\ 1 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots \\ 0 & 0 & \dots & 1 \end{bmatrix},$$

and

$$B = \begin{bmatrix} b_{11} & b_{12} & 0 & 0 \\ 0 & 0 & b_{23} & 0 \\ 0 & b_{32} & 0 & 0 \\ 0 & 0 & b_{43} & b_{44} \end{bmatrix}$$

with

$$b_{11} = \frac{1}{h} \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad b_{12} = \frac{1}{2} \begin{bmatrix} -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$b_{23} = \frac{1}{h} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad b_{43} = \frac{1}{2} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

$$b_{32} = b_{11}, \quad b_{44} = b_{23}.$$

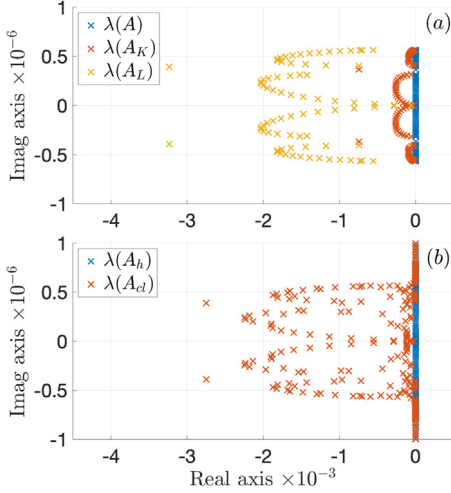
The state variables are

$$x(t) = [x_1^d(t)^T \quad x_2^d(t)^T \quad x_3^d(t)^T \quad x_4^d(t)^T]^T$$

where  $x_i^d(t) \in \mathbb{R}^{20}$ ,  $i \in \{1, \dots, 4\}$  and the  $j$ th component of  $x_1^d$ ,  $x_2^d$ ,  $x_3^d$  and  $x_4^d$  correspond respectively to the approximation of  $z_1((j - 0.5)h, t)$ ,  $z_2(jh, t)$ ,  $z_3((j - 0.5)h, t)$  and  $z_4(jh, t)$ , with  $h = \frac{b-a}{2 \cdot 20 + 1} = 0.0146$ , and  $b - a = 0.3$  m with  $a = 0$ . The beam is clamped at the left side and force and torque actuators at the right side are considered,  $Tz_1(b, t)$  and  $EIz_3(b, t)$  respectively. Hence  $b_{11} = b_{12} = b_{32} = 0$ , which give pairs  $(A, B)$  controllable and  $(A, C)$  observable. In this case  $C = B^T Q$ .

**Table 1**  
Plant parameters.

Parameters	Values	Unit
$T$	$3.4531 \times 10^5$	Pa
$\rho$	0.0643	kg m <sup>-1</sup>
$EI$	37.0116	Pa m <sup>4</sup>
$I_p$	$2.1485 \times 10^{-6}$	Kg m <sup>2</sup>
$[a, b]$	[0, 0.3]	m



**Fig. 3.** (a):  $\lambda(A)$ : eigenvalues of the discretized model with  $n_c = 80$ ,  $\lambda(A_K)$ : eigenvalues of  $A - BK$  and  $\lambda(A_L)$ : eigenvalues of  $A - LC$ . (b):  $\lambda(A_h)$ : discretized model eigenvalues with  $n_c = 200$ ,  $\lambda(A_{cl})$ : closed-loop eigenvalues.

#### 4.2. The observer-based state feedback design

We use the parameters of Table 1.

Two different state feedbacks  $K$  minimizing the cost function

$$J_{LQR} = \int_0^{\infty} \{x^T Q_{LQR} x + u^T R_{LQR} u + 2x^T N_{LQR} u\} dt$$

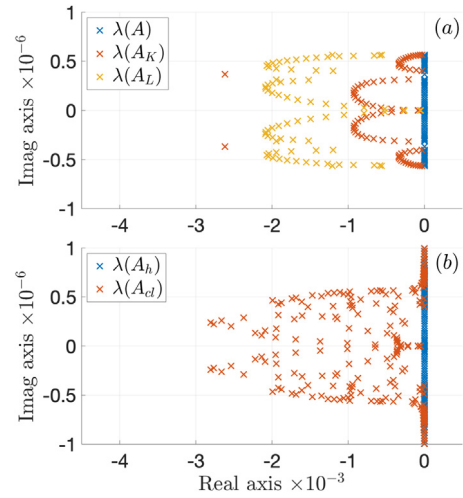
are designed using the Matlab@ Control System Toolbox *lqr.m*. In both designs the ARE algorithm proposed in Lanzon et al. (2008) has been used to solve (21).

For the first design, the matrix  $K$  is performed choosing  $Q_{LQR} = 0.8I_{n_c}$ ,  $R_{LQR} = 10I_4$  and  $N_{LQR} = 0$ , while the matrix  $L$  is designed following Proposition 18 with  $R_c = 10I_{n_c}$ . The eigenvalues of the matrices  $A$ ,  $A - BK$  and  $A - LC$  are shown in Fig. 3(a). The eigenvalues of the closed-loop system using the same controller on a higher order discretization of the beam, choosing  $n_c = 200$ , are given in Fig. 3(b).

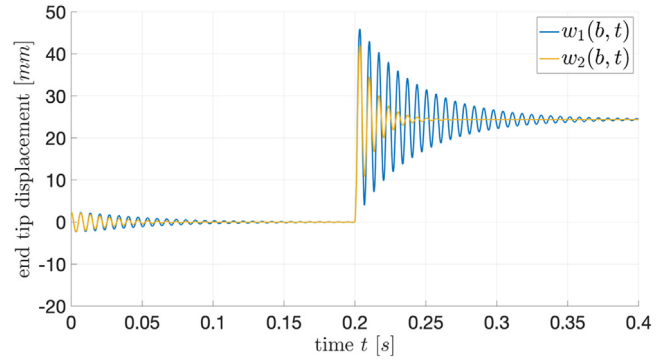
For the second design, the matrix  $K$  is performed choosing  $Q_{LQR} = 0.8I_{n_c}$ ,  $R_{LQR} = 1.33I_4$  and  $N_{LQR} = 0$ , while the matrix  $L$  is designed following Proposition 18 with  $R_c = 4I_{n_c}$ . The eigenvalues of the matrices  $A$ ,  $A - BK$  and  $A - LC$  are shown in Fig. 4(a). The eigenvalues of the closed-loop system using the same controller on a higher order discretization of the beam, choosing  $n_c = 200$ , are given in Fig. 4(b). In both cases, the closed-loop system remains stable and the high frequency modes are not destabilized. Even if  $n_c \rightarrow \infty$ , the closed-loop eigenvalues do not cross the imaginary axis.

#### 4.3. Simulations

Simulations are performed using Matlab over the time interval  $t \in [0, 0.4]$  s. The simulation starts from the initial condition  $z_1(\zeta, 0) = 0.2896 \times 10^{-4}$ ,  $z_2(\zeta, 0) = 0$ ,  $z_3(\zeta, 0) = -0.2702\zeta +$



**Fig. 4.** (a):  $\lambda(A)$ : eigenvalues of the discretized model with  $n_c = 80$ ,  $\lambda(A_K)$ : eigenvalues of  $A - BK$  and  $\lambda(A_L)$ : eigenvalues of  $A - LC$ . (b):  $\lambda(A_h)$ : eigenvalues of the discretized model with  $n_c = 200$ ,  $\lambda(A_{cl})$ : closed-loop eigenvalues.



**Fig. 5.**  $w_i(b, t)$ : end tip displacement for the design  $i = \{1, 2\}$ .

0.0811 and  $z_4(\zeta, 0) = 0$  corresponding to the equilibrium position associated to a force of 10 N applied at the end tip of the beam. The initial condition for the observer is set to zero, i.e.  $\hat{x}(0) = 0$ . An external force  $r(t) = 100$  N is applied at  $t = 0.2$  s at the end of the tip to modify the equilibrium position. In the first simulation the observer-based controller of size 80 and designed on the discretized model for 20 elements is applied to the large scale system obtained considering 50 discretization elements, i.e. 200 state variables. Fig. 5 shows the time responses for the two controllers proposed in sub Section 4.2, where  $w_1(b, t)$  and  $w_2(b, t)$  are the end tip displacements of the beam for the first and second design respectively. Note that, before  $t = 0.2$  s the convergence to the null equilibrium is due to the observer and state feedback dynamics. After the step at  $t = 0.2$  s, the convergence is mostly due to the state feedback as the observer already converged to the system state over the considered range of frequencies.

The estimated values are given by  $\hat{w}_1(b, t)$  and  $\hat{w}_2(b, t)$ , the error is obtained by  $\tilde{w}_i(b, t) = w_i(b, t) - \hat{w}_i(b, t)$  and it is shown in Fig. 6 for both designs.

In Fig. 7 is given the evolution of the norm of the error between the system state, that has been used for control design, and the observer state, with respect to time when considering different initial conditions. Fig. 7 illustrates the convergence rate of the observer on the reduced order system.

Since the controller is designed based on a finite-dimensional approximation  $P$  of the system, but at the end, has to be implemented on the infinite-dimensional system  $\mathcal{P}$ , it is interesting

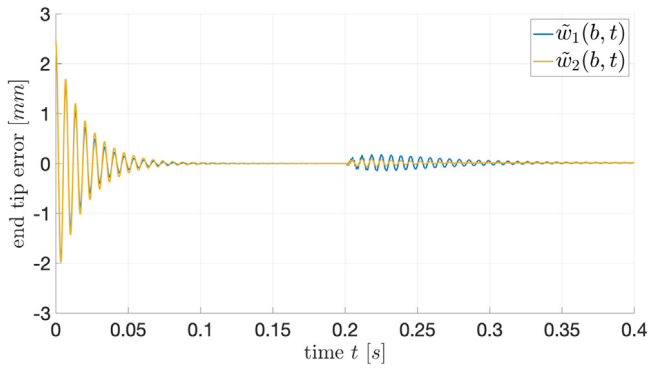


Fig. 6.  $\tilde{w}_i(b, t)$ : observer error of the end tip displacement for the design  $i = \{1, 2\}$ .

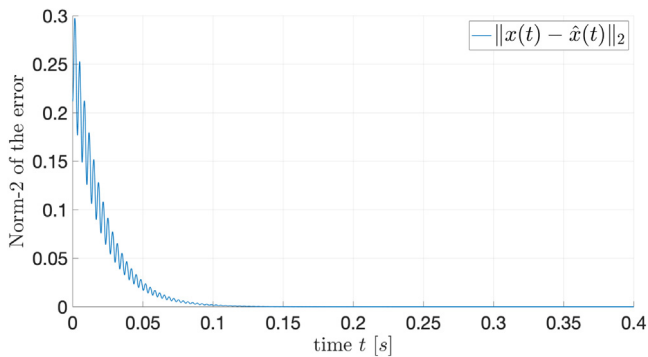


Fig. 7. Evolution of the norm of the error between the observer and the discretized model to initial conditions.

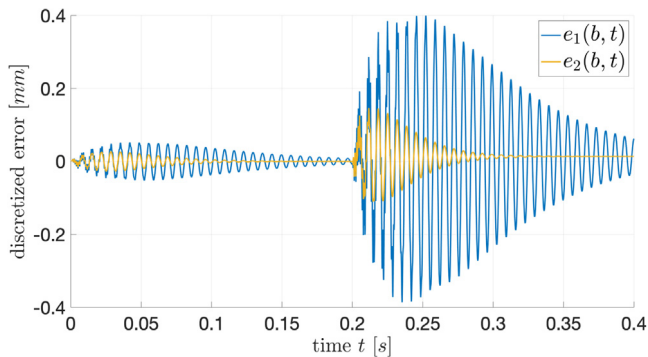


Fig. 8. Discretization error  $e_i(b, t) = w_i^{\mathcal{P}}(b, t) - w_i^{\mathcal{P}}(b, t)$  for the design  $i = \{1, 2\}$ .

to compare the behavior of both closed-loop systems. For that purpose, we denote by  $w^{\mathcal{P}}(b, t)$  the end tip displacement when applying the controller to the finite-dimensional model  $\mathcal{P}$  used for the design (20 elements) and by  $w^{\mathcal{P}}(b, t)$  the end tip displacement when applying the controller to a higher dimensional model stemming from a fine approximation of the infinite-dimensional model  $\mathcal{P}$  (obtained for 50 elements). The approximation error  $e_i(b, t) = w_i^{\mathcal{P}}(b, t) - w_i^{\mathcal{P}}(b, t)$  is shown in Fig. 8 for both designs. We can notice that the approximation error increases for high frequencies signals, meaning performances cannot be guaranteed over all frequencies. Yet the error remains small with respect to the higher order approximation.

Finally, a new simulation is done using the second design for  $t \in [0, 0.2]$  with the same initial conditions than before and an external force step applied at  $t = 0.1$  s. The deformation of

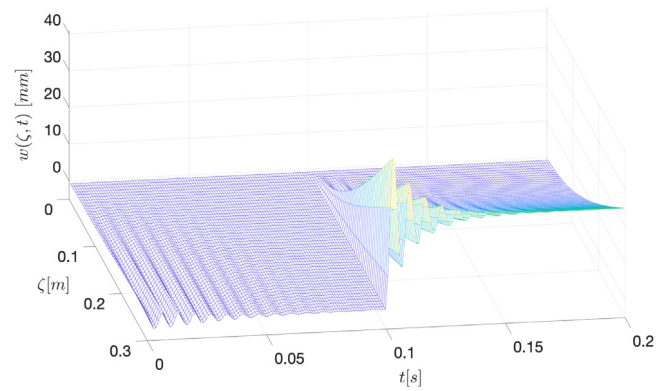


Fig. 9. Displacement of the beam for time  $t \in [0, 0.2]$  and space  $z \in [0, 0.3]$ .

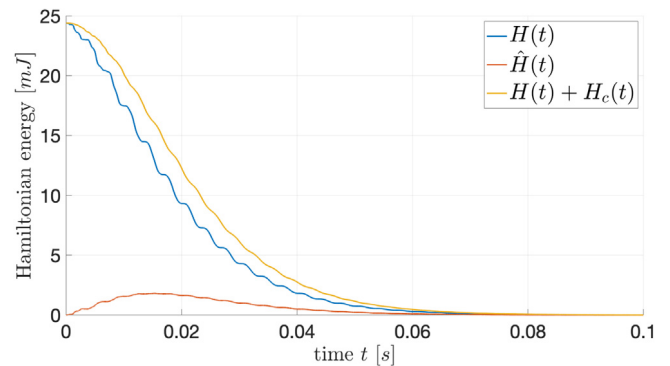


Fig. 10. Evolution of the system and observer energy functions with respect to initial conditions.

the beam along the space and over time is shown in Fig. 9. The oscillations occurring during the first 0.1 s are due to the observer convergence since the system and the observer do not have the same initial conditions.

Fig. 10 shows the evolution of the system and observer energies with respect to time when considering different initial conditions. We can see that the observer energy function converges to the plant energy function, and at the same time the control brings the closed-loop energy function to zero.

## 5. Conclusion

An observer based boundary controller has been proposed for a class of boundary controlled PHS defined on a 1D spatial domain. The design is based on an early-lumping approach in which a finite-dimensional PHS approximation of the infinite-dimensional system is used to design the observer and the controller. The main contribution is a constructive method that guarantees that the finite-dimensional dynamic boundary controller is a strictly positive real PHS. This guarantees that the interconnection between the controller and the infinite-dimensional system is asymptotically stable. As soon as the finite-dimensional approximation of the system that is used for the observer design is close to the infinite-dimensional system over the considered range of frequencies, the closed-loop performances on the infinite-dimensional system are close to the ones obtained on the finite-dimensional approximation. The stabilization of a Timoshenko beam with force and torque actuators and collocated measurements (velocities) has been used to illustrate the approach.

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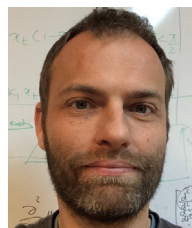
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