Non-locality and Entanglement Detection with Mermin polynomials for Grover's algorithm and Quantum Fourier Transform

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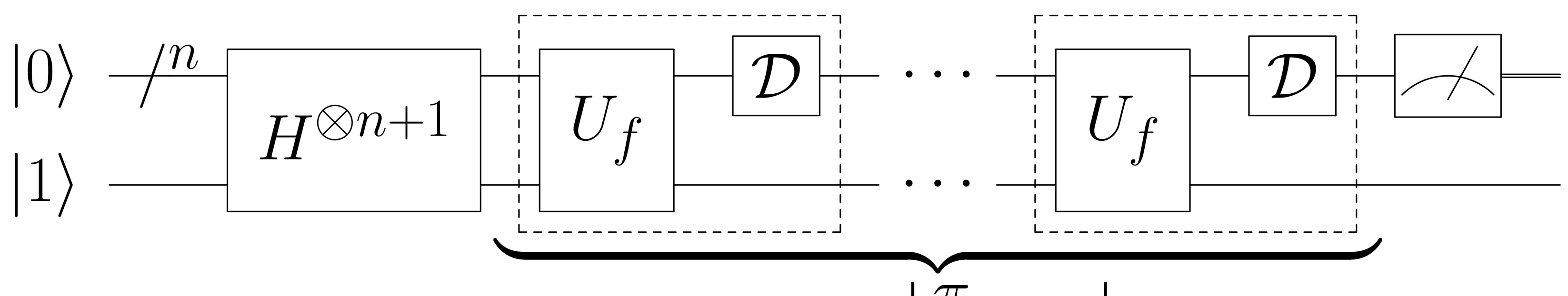
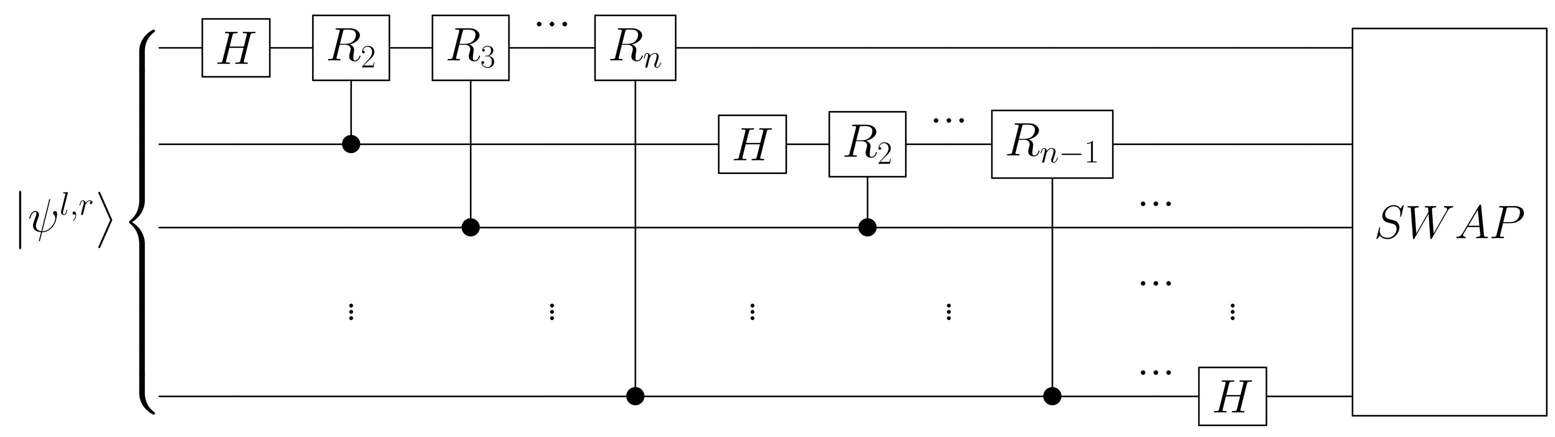
Grover's quantum algorithm



repeated $k_{opt} = \left\lfloor \frac{\pi}{4}\sqrt{2^n} \right\rfloor$ times

- Search of an item $|\mathbf{x_0}\rangle$ in a list, based on an oracle
- Complexity: $\mathcal{O}(\sqrt{2^n})$ (against $\mathcal{O}(2^n)$ for classical search algorithms)

Quantum Fourier Transform



Generally performed on periodic states defined by: $|\varphi^{l,r}\rangle = \frac{1}{\sqrt{A}}\sum_{i=0}^{A-1}|l+ir\rangle$

- Quantum analogue of the discrete Fourier transform
- Complexity: $\mathcal{O}(n^2)$ (against $\mathcal{O}(n2^n)$)

Mermin's polynomials

$$\begin{cases} M_1 = a_1 \\ \forall n \geq 2,\ M_n = \frac{1}{2}M_{n-1}\otimes(a_n + a'_n) + \frac{1}{2}M'_{n-1}\otimes(a_n - a'_n) \end{cases}$$

(a_i) and (a'_i) are one-qubit observables with eigenvalues in $\{-1, 1\}$

$\langle\varphi|M_n|\varphi\rangle > 1$ implies that $|\varphi\rangle$ is non-local.

Example:
With $(a_1, a_2, a'_1, a'_2) = (Z, X, (Z+X)/\sqrt{2}, (Z-X)/\sqrt{2})$, M_2 corresponds to the operator used for Bell inequalities.

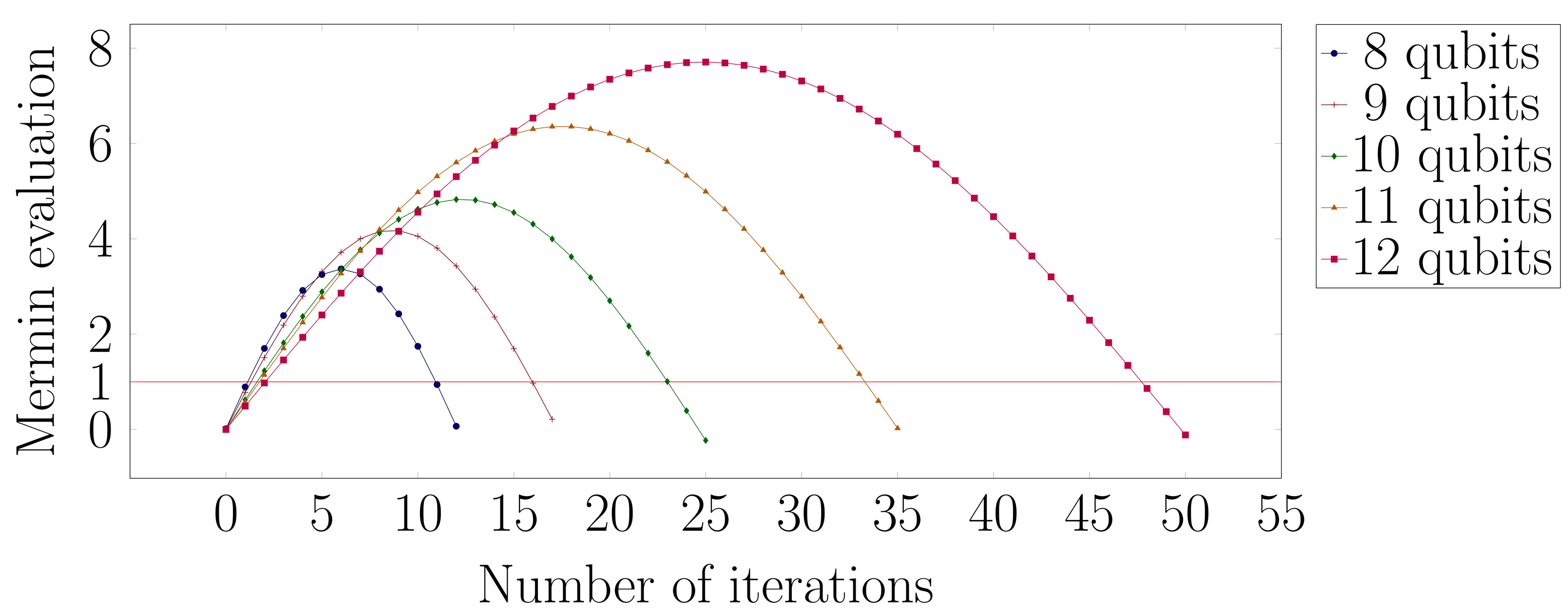
Grover's algorithm evaluation

Proposition ([JH19]):

- The states in Grover's algorithm are $|\varphi_k\rangle = \alpha_k|\mathbf{x_0}\rangle + \beta_k|+\rangle^{\otimes n}$, with $(\alpha_0, \beta_0) = (0, 1)$ and $(\alpha_{k_{opt}}, \beta_{k_{opt}}) \approx (1, 0)$.
- For k close to $k_{opt}/2$, $|\varphi_k\rangle$ comes close to a state $|\varphi_{ent}\rangle = (|\mathbf{x_0}\rangle + |+\rangle^{\otimes n})/K$ maximizing $\langle\varphi|M_n|\varphi\rangle$.

Evaluation method: find M_n such that $\langle\varphi_{ent}|M_n|\varphi_{ent}\rangle$ is maximal.

Computing $\langle\varphi_k|M_n|\varphi_k\rangle$ for every k with this M_n **positively** answers the following question:
"Is Grover's algorithm using entanglement to achieve quantum speedup?".

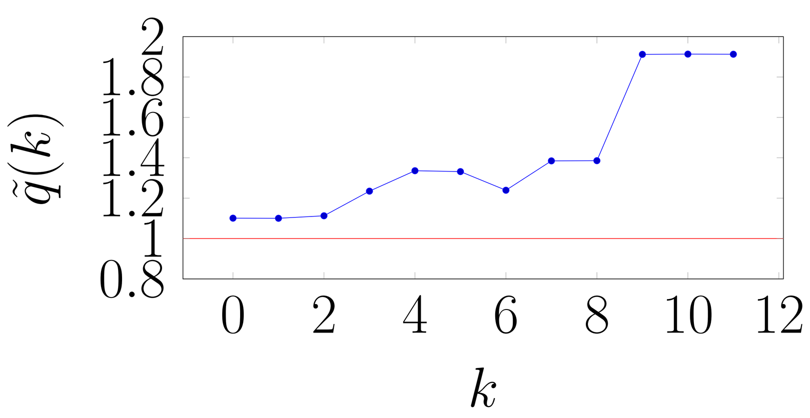


QFT evaluation

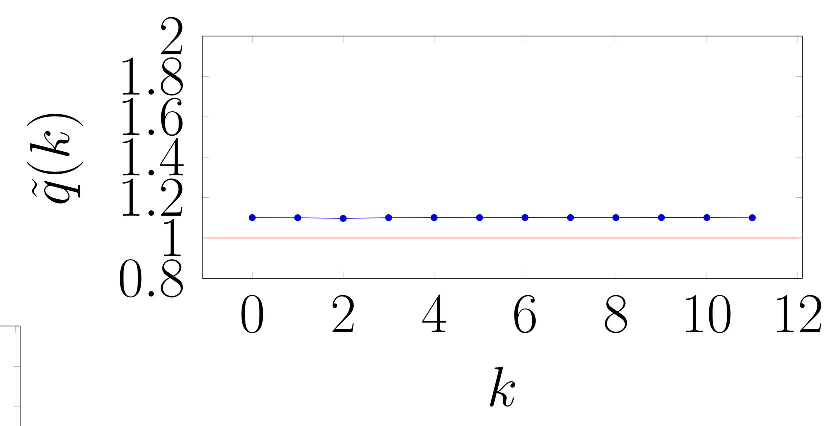
$q : |\varphi\rangle \mapsto \max_{M_n}\langle\varphi|M_n|\varphi\rangle$ is a measure of entanglement.

Evaluation method: for each state $|\varphi_k\rangle$, find $\widetilde{M_n}$ such that $\langle\varphi_k|\widetilde{M_n}|\varphi_k\rangle$ is maximal.

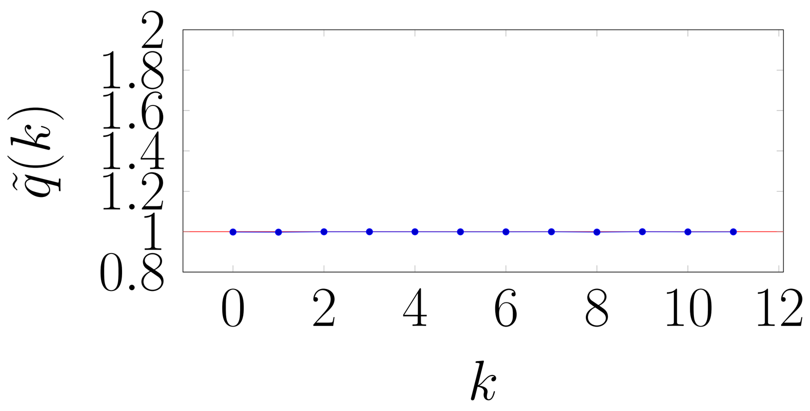
The corresponding experimental approximation $\tilde{q}$ of q allows us to distinguish between three types of QFT runs in our experiments with $n = 4$:



Entangled states and variable measure (here for $(l, r) = (9, 1)$).



Entangled states and constant measure (here for $(l, r) = (2, 2)$).



Separable states (here for $(l, r) = (8, 5)$).

One can also check some key points such as the fact that entanglement evaluation doesn't change during LOCC operations (H gates in this case).

References

[dJH+20] **H. de Boutray, H. Jaffali, F. Holweck, A. Giorgetti, and P.-A. Masson. Non-locality and Entanglement Detection with Mermin polynomials for Grover's algorithm and Quantum Fourier Transform. *Submitted in September 2020*, previous version at arXiv:2001.05192.**

[JH19] H. Jaffali and F. Holweck. Quantum Entanglement involved in Grover's and Shor's algorithms: The four-qubit case. *Quantum Information Processing*, 18(5):133, May 2019.

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This work is supported by the French Investissements d'Avenir program, project ISITE-BFC (contract ANR-15-IDEX-03), and by the EIPHI Graduate School (contract ANR-17-EURE-0002). The computations have been performed on the supercomputer facilities of the Mésocentre de calcul de Franche-Comté.