Acoustic topological circuitry in square and rectangular phononic crystals

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Acoustic analog of quantum valley Hall effect


- (Bloch) vortex waves at the K point hold the topological valley invariant
- Edge modes guided at a domain wall are formed by combining crystals with angles $\pm\alpha$
Three \textbf{K} Points: no backscattering in 3 directions

Acoustics

Lu et al., Nat. Phys. 13 (2017) 369

Elastic waves


Optics

Khanikaev et al., Nat. Mater. 12 (2012) 223
Space groups of the 2D crystal and the 1D waveguide

Frieze group: p11g
(A and B are images in a glide reflection)
Dirac points in the square lattice? 3-way splitter?


The sign of the Chern number associated with Dirac cones dictates if waves go along a domain wall or not
Square inclusion in square-lattice

A pair of Dirac points is needed along each direction

Domain walls with square-lattice crystal

$f = 7.3 \text{ Hz}$
3-way splitter with water waves?

Simulation without dissipation  Simulation with dissipation  Experiment


$f = 7.3 \text{ Hz}$
Square lattice topological crystal: acoustic waves

\[ a = 3 \text{ mm}, \ b = 1.5 \text{ mm} \]
\[ f \sim 500 \text{ kHz} \]

Measuring the phononic band gap

Measuring the domain wall waveguide

Coupling of acoustic and elastic waves

\[
\nabla \cdot (\rho^{-1}_1 \nabla p) = \frac{1}{\kappa} \frac{\partial^2 p}{\partial t^2}
\]

\[
\nabla \cdot [\mathbb{C} : \nabla u] = \rho_2 \frac{\partial^2 u}{\partial t^2}
\]

**Acoustoelastic coupling:**

\[
T = \mathbb{C} : \nabla u
\]

\[
T_{ij} n_j = -p n_i
\]

\[
n \cdot (\rho^{-1}_1 \nabla p) = -n \cdot \frac{\partial^2 u}{\partial t^2}
\]

\[a = 3 \text{ mm, } b = 1.5 \text{ mm}\]
Dispersion of guided waves

Trouble is... there’s a third Dirac point!

\[ \alpha = 0^\circ \]

\[ \Gamma N \]

\[ \alpha = 20^\circ \]

\[ \alpha = 0^\circ \]

\[ \alpha = 20^\circ \]
Breaking diagonal symmetry removes the third Dirac point

It also results in a slight frequency shift of the main Dirac points
And a slight reduction in band gap width

Dispersion of the guided waves shifts

3-way splitter with the rectangular-lattice

And finally, a ‘trident’ circuit!

Summary and conclusion

- Dirac points exist not only in the hexagonal (graphene) lattice, but with other symmetries, including the square and the rectangular lattices.
- We have implemented topological phononic crystal circuits both with water waves and ultrasonic acoustic waves.
- The rotated square inclusion in square-lattice crystal has ‘spurious’ Dirac points that can be removed considering the rectangular lattice.
- Square and rectangular lattices are compatible with 3-way splitters and more general phononic circuits.