Impact of decision horizon on post-prognostics maintenance and missions scheduling: a railways case study

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ABSTRACT
In this paper, we propose a study of the decision horizon duration for rolling stock mission assignment and maintenance planning in a prognostics and health management (PHM) context. The aim is to determine the best decision horizon duration that allows the construction of a suitable schedule that assigns railways vehicles to missions and integrates required maintenance operations according to the current and future health of the vehicles. A genetic algorithm is used to minimize the overall cost of the joint schedule as a function of the decision horizon. The results are compared to three proposed heuristics to study the influence of the resolution method on the decision horizon duration. The best decision horizon duration is given for each used method for an illustration case.

KEYWORDS
Prognostics and health management, Post-prognostic decision-making, Decision horizon study, Rolling stocks

1. Introduction

An effective public transportation system is vital for the economic well-being and the good quality of life of every major city. Public transport can be divided into two categories: (i) rail transport systems (e.g., railways systems, tramways, and metros) and (ii) road transport systems (e.g., intercity buses and urban buses). The requirements on bus, rail, underground, and tram systems are growing with efficiency and reliability as the key factors. Therefore, the optimization of public transportation systems is being fairly well researched. The public transit planning can be decomposed into five successive processes: (i) network design, (ii) line planning, (iii) timetable planning, (iv) rolling stock assignment, and (v) crew planning and rostering. To ensure the reliability, availability and operational safety of the overall transportation system, it is necessary to inspect and maintain its components (i.e., structure, vehicles, ...) after a certain period of service. Thus, the maintenance activities should be considered while assigning the rolling stock units. In this work, we focus on rail transport systems. These systems can be divided into three classes: (i) geographically distributed fixed systems (e.g., rail infrastructure, rail track sections, switch or power supply), (ii) rolling systems with variable geographical positions (e.g., trains), and (iii) rolling systems with fixed

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positions at the end of each mission (e.g., tramways, metros). In this work, we aim to study the problems of mission assignment and maintenance planning for rolling stocks of the third category.

Rolling stock operational scheduling is usually done manually by dispatchers [1]. Although the problem could be considered as classic, it presents several constraints to take into consideration (e.g. maintenance requirements, service constraints, and passenger allocation). Therefore, it takes a lot of effort and time to provide a near-optimal schedule manually. Recently, some optimization techniques and decision support systems are being used to help manage the mission assignment problem. The maintenance of railway rolling stock is preventive in nature that includes: inspection, adjustment, lubrication, and replacement of critical components. These interventions are usually made at regular time or mileage intervals. Nowadays, rolling stock maintenance strategies witnessed an evolution. Various strategies have been studied including systematic and periodic preventive protocols, reliability-centered, and condition-based maintenance (CBM). Lately, the maturation of prognostics and health management methodology lead to a maintenance break with the predictive strategy [2].

Prognostics and health management (PHM) aims at studying the conditions of a fielded engineering system, analyzing its behavior (i.e., predict the component responsible of the failure and estimate when it will fail) and taking the suitable decisions that best overcome an undesirable future event or mitigate its effects. The post-prognostic decision process of PHM deals with three categories of decisions; (i) maintenance decisions [3], (ii) operational decisions (e.g., automatic control [4], mission assignment [5], production scheduling [6], and logistic planning [7]), and (iii) mixed decisions [8], [9].

The rail infrastructure systems, like the tracks, have been studied in the PHM context: Letot *et al.* [10] used a cost model to optimize the scheduling of tamping interventions for rail-tracks. Durazo-Cardenas *et al.* [11], conceived a maintenance decision support system for railways based on big data fusion and systems engineering.

The rail switch systems have also been the subject of PHM studies: Camci in [12] presented a novel generalisation of the travel salesman problem and define a new travel maintainer problem that was applied on rail switch systems. The author used a genetic algorithm and a particle swarm optimization algorithm to determine a maintenance schedule that minimizes an elaborate cost function. Villarejo *et al.* [13] developed a hybrid model for a prognostics and health management framework on the railway system. Verbert *et al.* [14], designed a two-level maintenance strategy optimization for railway networks (i.e., tracks and switches).

Traction railway power supply is another system that has been studied in a PHM context. Lin *et al.* [15] used a partially observable Markov decision process to plan maintenance interventions for a traction power supply based on its remaining useful life (RUL). In [16], the authors developed a novel framework that combines PHM and active maintenance to better manage the health and the maintenance activities of high-speed railways traction power supply system.

In the context of PHM, this paper study the decision horizon duration for rolling stock mission/task assignment from a predefined timetable jointly with maintenance scheduling. In this study, rolling stock units are considered as multi-component systems. The degradation of these components is modeled by a stochastic process. The condition of each rolling unit is monitored and its health state is assessed. The prognostics method assesses the rail vehicle's ability to achieve a task based on its predicted future state. The method consists of estimating the final degradation level of the rolling equipment (e.g., tram or train) under the operating conditions of the considered task.

However, one rolling stock unit can be able to fulfill more than one task. Therefore, a decision algorithm is developed to validate the assignment that complies best with the objective function.

The rest of this paper is organized as follows. A literature review on joint mission assignment and maintenance planning for rolling stock is given in section 2. The section 3 is dedicated to the problem statement. In section 4, the used scheduling methods (i.e., the genetic algorithm and the heuristics) are presented. A numeric example is given in section 5. Plus the obtained results from the different methods are presented and discussed in the same section. An overview of the results and some future works are finally given in the concluding section.

2. Related works

In the context of applying PHM methodology on rolling stock, several works have studied condition monitoring [17] and prognostics [18, 19]. In this work, we focus on the decision-making process.

The problem of maintenance planning for rolling stocks (e.g. trains or tramways) have been well addressed in the literature. Budai *et al.* [20] presented a heuristics to schedule preventive maintenance activities while minimizing the needed time for the interventions. Wang *et al.* [21] used a branch and bound algorithm to solve multiple criteria maintenance strategy selection problem. Cheng et al. [22] evaluated and selected a suitable maintenance strategy using an analytic network process technique. The authors, then, estimated the quantities of spare parts for the components and their corresponding replacement intervals.

Moreover, the joint problem of mission/task assignment or vehicle scheduling and maintenance planning has been addressed in the railways context. For example, Giacco *et al.* [23] proposed a mixed-integer linear programming model to solve the joint problem of rolling stock routing and maintenance planning. The authors aimed to minimize the total number of used trains and the number of empty rides and to maximize the distance traveled by each train between two similar maintenance operations. Lai *et al.* [24] used also a mixed-integer programming method to optimize the trips scheduling for a fleet of trains while taking into consideration their needs in terms of maintenance inspections. The presented study is limited to cyclic maintenance inspections of 3-days to 3-months frequencies. Andres *et al.* [25] determined a suitable train routing while scheduling the necessary maintenance interventions using a mixed-integer linear programming model that minimizes the global cost. Lin *et al.* [26] proposed a binary non-linear programming model to solve train mission-assignment and maintenance planning for high-speed trains. The problem consists of assigning well-conditioned trains to each trip while scheduling maintenance on the accumulated mileage or time from the last inspection. Although several frequencies of inspections have been proposed, the authors considered the train as a single component with systematic preventive maintenance requirements. The planning horizon of this study is fixed to one week even though in some cases the duration of the horizon is significantly smaller than the inspection frequency. Zhong *et al.* [27] scheduled rolling stock units based on predefined timetables while considering maintenance restrictions. The authors proposed a two-stage heuristics. At the first stage, a mixed integer programming method solves the assignment problem while ignoring maintenance constraints. In the second stage, the obtained schedules are checked for feasibility when the maintenance restrictions are added. Mira *et al.* [28] developed an integer linear programming model that integrates preventive maintenance planning within the rolling stock operation schedule. The objective of their work is to find the sequence of trips that minimizes the cost and satisfies the predefined maintenance needs. Both works [27] and [28] considered maintenance interventions as special trips (i.e. trips without passengers) plus trips are assumed to have different starting and ending points.

Most of the published works on integrating maintenance in the rolling stock operation schedules deal with preventive maintenance or systematic cyclic inspections. Although condition monitoring and prognostics and health management technologies are presented in this field, their application and integration in the decision-making on rolling stock either with variable or fixed geographical positions are still under study. To our knowledge, there is a lack of publications on PHM application on rolling stock. Herr *et al.* [9] presented a linear program to optimize jointly task assignment and the maintenance scheduling of trains. Given predefined train timetables, the algorithm best matches train with the scheduled missions and plan maintenance interventions when needed. The assignment of rolling stock units and the schedule of its maintenance intervention is based on prognostic information to maximize the use of each train in terms of useful life. In this study, the trips of the timetables have different duration and different starting dates. Later, Herr *et al.* [29] fine-tuned the problem definition. In the second paper, the authors considered a set of identical daily trips and the maintenance interventions have been restricted to one intervention per rolling unit during the planning horizon. The problem is then solved for different fleet sizes and horizon duration to demonstrate the influence of these parameters on the performance of the linear programming method. Except for the works of Herr *et al.*, we could not find other works that integrated health information in the management of rolling stock units (trains, trams or metros).

Rolling stock scheduling and maintenance planning are done manually by dispatchers most of the time and for a small horizon of 7 to 10 days [27]. This problem is limited with several constraints e.g. the number of components considered in the rolling unit, the maintenance sites, and the depot sites. Therefore, most of the works that studied this problem assumed that a train can be considered as a single-component system. They also only considered systematic preventive maintenance. Rolling stocks operation and maintenance management is a challenging field. Rolling units are considered among the systems that have a long life spin. This motivated researchers to apply PHM methodology to the rolling systems.

With the use of PHM, several new challenges are created. Due to their movements, rolling units experience different environmental and operational conditions that would affect differently the degradation of the components. Moreover, a rolling unit is composed of several subsystems, each have its characteristics and degradation model. When applying PHM, these components should be studied to find which are beneficial. Therefore, selecting the right components for the PHM application is also a challenge for the PHM community.

Even though the problem described by the works of Herr *et al.* seems fairly realistic, it presents several major assumptions. First, the authors have optimized the joint problem from a maintenance point of view. However, the joint problem is a multi-objective problem since it mixes two disciplines maintenance and operational. Moreover, the authors considered rolling stock units as single-component systems. When in reality, trains are composed of several critical components. Plus, each component presents its health state and its deteriorating speed. Furthermore, the degradation model of the trains is considered deterministic and linear. However, the prognostic algorithm generally provides a distribution of possible future states due to the existence and the propagation of uncertainties. Finally, as mentioned by the authors the generated problems are characterized by a great unnecessary maintenance capacity.

In this paper, we aim to reinforce the problem stated by Herr *et al.* to make it more realistic and to fill the pointed out gaps. Therefore, We integrate prognostic information in the process of decision-making to schedule jointly maintenance and operation for rolling stock systems. The considered problem studies rolling units as multi-components with dynamic degradation models.

All the previously mentioned works focused on solving the joint problem regardless of the decision horizon duration. These works omitted the decision horizon duration in the definition of their problem and the used resolution methods. In PHM context, the prognostics algorithm accuracy are supposed to be in function of the prognostics horizon (i.e., how far ahead predictions are made) [30]. Due to uncertainty sources [31], the larger the prognostic horizon is the greater the uncertainties are present in the prediction phase. Thus, the resulted RUL has a higher error rate and is more inaccurate. Since prognostics is performed to provide new important information to the decision-making process to better manage the system life cycle, we wonder if the decrease of the prediction's accuracy over time, implies the decrease of the decision-making process performances. Therefore, we propose in this paper to study the decision horizon influence on the resolution of the joint problem. We aim to find the best decision horizon duration when using prognostic-based approaches that integrate a dynamic PHM framework as defined by Bougacha *et al.* in [32].

3. Problem description

This work deals with the study of the decision horizon duration impacts on the joint problem of vehicle scheduling and predictive maintenance planning of a fleet of railways vehicles. In other words, we propose to find the best decision-making frequency that minimizes the total cumulative cost on a simulation horizon. Therefore, we start by defining the problem of jointly assigning missions to railways vehicles and the planning of their maintenance. The vehicle scheduling problem focuses on assigning timetabled tasks to the fleet of available vehicles. However, other constraints need to be considered while creating a vehicle schedule (e.g., vehicle type and maintenance inspection). Before assigning a mission, the prognostics algorithm assesses the rail vehicle's ability to achieve the task. The decision-making algorithm considers this information while solving the vehicle scheduling problem. Predictive maintenance is scheduled to find a compromise between early maintaining the rolling equipment, risking its failure, and missing tasks due to fleet unavailability.

3.1. Equipment model

Each rolling stock unit is considered as a serial multi-component system i.e. if one of these components fails the whole system fails. In PHM context, we distinguish two types of components; (i) preventive components that are difficult to monitor, their degradation cannot be modeled or observed, or it would be more strategic to perform systematic cyclic maintenance; and (ii) predictive components that are equipped with sensors, they present big volume of historical data, and their degradation behavior is at least partially observable. For these components, PHM technology is applied. Therefore each rolling stock unit m is defined as a series of K predictive components $(c_k, k = 1, \ldots, K)$ and L preventive components $(c_l, l = K + 1, \ldots, K + L)$ and can be presented as

$$m = (c_1, ..., c_K, c_{K+1}, ..., c_{K+L}) \tag{1}$$

For preventive components, the maintenance dates are defined for a certain value of mileage coverage. Each of the preventive components has its mileage threshold noted Θ_l. Consider, at instant t, $\theta_{m,l}(t)$ being the mileage traveled by component l of unit m since its last maintenance operation. Preventive component l is scheduled for maintenance only when its mileage reaches a predefined threshold Φ_l as

$$\theta_{m,l}(i) \geq \Phi_l \tag{2}$$

All the rolling stock units are supposed to be of the same type. They are however differentiated by their components degradation level estimated by a condition assessment process. A prognostics process assesses the health state and estimates the remaining useful life of predictive components. For example, we consider the heating, ventilation, and air-conditioning (HVAC) subsystem as a component. Moreover, for parts of large numbers (e.g., wheels or doors), they are grouped into components according to some criterion (e.g., by wagon or by side).

The health state of a predictive component k of unit m is described through a variable $H_{m,k} \in [0, 1]$ for $(1 \leq k \leq K)$. The degradation of any predictive component is considered to be monotonically increasing over time as a result of an accumulation of small positive independent increments. Moreover, such a degradation process has been widely modeled by stochastic processes in literature. Readers could refer to the survey paper by Van Noortwijk [33]. To simulate the real evolution of degradation and generate health data, we consider that the degradation of the predictive components could be modeled by a homogeneous Gamma process. Therefore, the health state of a predictive component k $\{ H_{m,k}(t),\ t \geq 0 \}$ is considered a homogeneous Gamma process $\Gamma(\nu_k(t),\ \mu_k)$ with shape parameter $\nu_k(t) = \alpha_k * t$ and scale parameter μ_k and the following properties:

- $H_{m,k}(t' = 0) = 0$
- $H_{m,k}(t)$ has independent increments
- For $t > 0$ and a small non-negative time increment $h > 0$ during which unit m is serving a mission, $H_{m,k}(t + h) - H_{m,k}(t)$ follows a gamma distribution $\Gamma(\nu_k(t+h) - \nu_k(t), \mu_k)$ with shape parameter $(\nu_k(t + h) - \nu_k(t))$ and scale parameter μ_k

Components of the same type are considered identical in their degradation dynamics. Thus, they have the same parameter of the gamma distribution (α_k, ν_k). However, they are differentiated by their degradation level obtained from the prognostics. Component k of unit m is considered as good as new when its health state $H_{m,k} = 0$ and $H_{m,k} = 1$ indicates that it has reached its end of life and failed. To ensure passenger safety and comfort and avoid failures, components' end of life are considered upon a certain threshold denoted $\Delta_{m,k} = \Delta_k \in [0, 1[$. The maintenance of the component is only considered when its degradation reaches a threshold denoted $\Lambda_{m,k} = \Lambda_k \in [0, \Delta_k[$ as

$$H_{m,k}(i) \geq \Lambda_k \tag{3}$$

This threshold can be defined as part of the decision process or provided by the prognostics in the form of a remaining useful life (RUL) value. These thresholds are usually given by experts. In their definition, economic and safety margins are taken into account to reduce the risks of breakdowns in operations on one hand and avoid very early maintenance interventions on the other. The relation between the variable $H_{m,k}$, the thresholds Δ_k and Λ_k and the launch of a maintenance operation is illustrated in Figure 1.

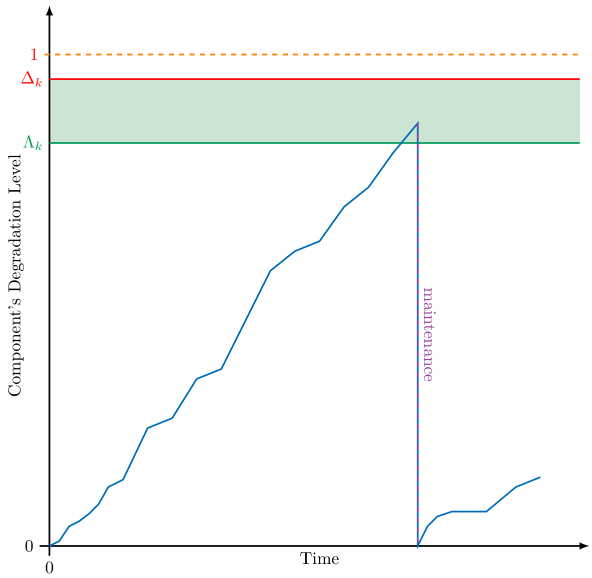

Figure 1. Health state illustration.

3.2. Vehicle scheduling problem

The vehicle scheduling problem addresses the task of assigning vehicles to cover the mission in a timetable. Therefore, given a set $\mathcal{P}$ of planned tasks (with $Card(\mathcal{P}) = P$) and a set $\mathcal{M}$ of rolling stock units (with $Card(\mathcal{M}) = M$), the solving algorithm builds an assignment between the M units and the P missions based on equipments' health state and their ability to fulfill missions. The scheduling is supposed to be done over a rolling horizon of a duration $DH = I \times \Delta T$ where ΔT is the time unit (a day for instance) and in this case I is the number of time units. For each time unit, denoted i with $1 \leq i \leq I$ the set of missions should be fulfilled by the railway vehicles. Each mission p corresponds to a planned route which is composed of a set of trips starting and ending in a depot. Each mission is characterized by a severity coefficient s_p that represents the length of the journey (i.e., number of miles the rolling stock unit should travel in that mission), the number of stops, and the chosen path (i.e., if it is a high-speed track or normal track). Thus, it influences the degradation of the components by changing the scale parameter of the gamma process which can be expressed as $\mu_k = s_p \times \mu_k^{'}$ with $\mu_k^{'}$ a characteristic of the component's type.

The task p is associated with a degradation rate $\delta_{p,k} \in [0, 1[$ that can vary from one component to another. This wear rate is provided by a prognostic process. For each component k of a given rolling stock unit m at instant $t = i \times \Delta T$ with a health state $H_{m,k}(i)$, its degradation level after executing its assigned mission p must be lower than its failure threshold Δ_k. This constraint is expressed as

$$H_{m,k}(i) + \delta_{p,k} < \Delta_k,\ \forall\ k \in \{1, ..., K\},\ \forall\ i \in \{1, ..., I\} \tag{4}$$

Also each task p is characterized with a distance d_p to be covered by the rolling vehicle m at which the mission will be assigned to. Therefore, the assigning of mission p to rolling stock unit m should also verify that the covered distance will not exceed the threshold for preventive maintenance. This constraint is represented as

$$\theta_{m,l}(i) + d_p < \Theta_l,\ \forall\ l \in \{K+1, ..., K+L\},\ \forall\ i \in \{1, ..., I\} \tag{5}$$

It is assumed that if the mileage of a preventive component l exceeds its threshold, the component fails, thus causing the failure of the vehicle. Lets denote the variable $\beta_{p,m}(i) \in \{0, 1\}$. Where $\beta_{p,m} = 1$ if mission p is assigned to unit m. Otherwise, it is equal to zero. These variables define a set of constraints. A first constraint represents the mission coverage constraint, where each task p is covered by at most one railway vehicle during any period i of the decision horizon. It is expressed as

$$\sum_{m=1}^{M} \beta_{p,m}(i) \leq 1,\ \forall\ i \in \{1, ..., I\},\ \forall\ p \in \{1, ..., P\} \tag{6}$$

A second constraint implies that one rolling stock unit m can fulfill at most one task during any period i of the decision horizon. It is defined as

$$\sum_{p=1}^{P} \beta_{p,m}(i) \leq 1,\ \forall\ i \in \{1, ..., I\},\ \forall\ m \in \{1, ..., M\} \tag{7}$$

Moreover, unit m, during any period i, can either be in maintenance, at rest (i.e., no mission is assigned to it) or assigned to a task p.

$\omega_m(i) \in \{0, 1\}$ represents the maintenance state of unit m during period i. If unit m is in maintenance then $\omega_m(i) = 1$. Otherwise, $\omega_m(i) = 0$.

$\pi_m(i) \in \{0, 1\}$ capture if rail vehicle m is at rest during period i (i.e., $\pi_m(i) = 1$ if m is neither in maintenance nor in operation. Therefore, the state of rolling stock unit m during a period i can be limited with constraint as

$$\begin{aligned} \pi_m(i) + \omega_m(i) + \sum_{p=1}^{P} \beta_{p,m}(i) = 1,\ \forall\ i \in \{1, ..., I\}, \\ \forall\ m \in \{1, ..., M\} \end{aligned} \tag{8}$$

Lets denote $C_{lost,p}$ the penalty cost of missing mission p during a period. All missions have the same priority. Thus, they all have the same missing penalty $C_{lost,p} = C_{lost} \forall p \in \{1, ..., P\}$. Moreover, we excluded the operational cost of achieving a mission from the objective function under the assumption that the mission assignment costs the same no matter the rail vehicle or the mission assigned to it.

3.3. Maintenance problem

Lets note $\sigma_{m,k}(i) \in \{0,1\}$ and $\sigma_{m,l}(i) \in \{0,1\}$ with $k \in \{1,...,K\}$ and $l \in \{K+1,...,K+L\}$ the variables that determine if component k respectively l of rolling stock unit m are scheduled for maintenance during period i. If $\sigma_{x,m}(i) = 1 \ \forall \ x \in \{1,...,K,K+1,...,K+L\}$ means that component x, whether it is predictive or preventive, is scheduled for maintenance. Otherwise, the component is not maintained during period i. Each component's type is characterized with a maintenance cost. Thus, lets note CR_k and CR_l the replacement cost of predictive component k and preventive component l respectively. Early maintenance of any component generates an additional cost to the replacement one. This is the penalty on the lost mileage of the maintained component noted $LP_x \ \forall \ x \in \{1,...,K,K+1,...,K+L\}$. Therefore, at period i, the maintenance cost of predictive component k of unit m, noted $C_{m,k}(i) \ \forall \ k \in \{1,..,K\}$, and preventive component l of unit m, noted $C_{m,l}(i) \ \forall \ l \in \{K+1,..,K+L\}$ are defined as

$$C_{m,k}(i) = CR_k + (\Delta_k - H_{m,k}(i)) * LP_k, \ \forall \ i \in \{1,...,I\}, \\ \forall \ m \in \{1,...,M\} \tag{9}$$

$$C_{m,l}(i) = CR_l + (\Theta_l - \theta_{m,l}(i)) * LP_l, \ \forall \ i \in \{1,...,I\}, \\ \forall \ m \in \{1,...,M\} \tag{10}$$

Lets note $f_m(i) \in \{0,1\}$ the variable that describes if unit m fails during period i. If during a task one of the components of unit m (predictive or preventive) fails and causes the system to fail (i.e., $f_m(i) = 1$), a corrective maintenance cost noted C_{cor} is generated. Vehicle's failure causes a disturbance of the timetable schedule, a delay for all scheduled missions on the same line, and efforts to move the failed rolling unit to a spare track. Thus, corrective maintenance penalty should take into consideration all these activities and their cost. Therefore, the corrective maintenance cost is defined in a way that missing some missions is less costly than risking the asset failure as

$$C_{cor} >> C_{lost} \tag{11}$$

Maintenance resources are considered limited, i.e., the number of maintenance operators and their shifts are limited. This limitation is presented through the maximum number of components $ML_{Comp} > 0$ that can be maintained during period i. Moreover, the maintenance workshop is supposed to have a limited number of tracks. Therefore, only a certain number of railway vehicles noted $ML_{Equip} > 0$ can be maintained simultaneously during period i. The maintenance planning process should take into consideration and satisfy these limits. Therefore, lets define the constraints that represent the limit in the number of rolling stock units and the limit in the number of components. These limits are expressed as

$$\sum_{m=1}^{M} \omega_m(i) \leq ML_{Equip}, \ \forall \ i \in \{1,...,I\} \tag{12}$$

$$\sum_{m=1}^{M}(\sum_{k=1}^{K}\sigma_{m,k}(i)+\sum_{l=K+1}^{K+L}\sigma_{m,l}(i))\leq ML_{Comp},\ \forall\ i\in\{1,...,I\} \tag{13}$$

In this paper, we considered only perfect maintenance activities (i.e. replacement). In other words, when a component is scheduled for maintenance, it is replaced with a new component thus restoring the health state of the component to as good as new.

3.4. The joint problem

Classically, the objective function of a joint maintenance and operation optimization could be defined as the minimization of the total cost including maintenance cost and missing task cost over the duration of the decision horizon noted DH. This decision horizon is often divided into I time periods (e.g., days, hours, or weeks). The total cost over a decision horizon DH is defined as

$$\begin{aligned} Total\_Cost(DH) = \sum_{i=1}^{I}[\sum_{p=1}^{P} C_{lost} * (1 - \sum_{m=1}^{M}\beta_{p,m}(i)) \\ + \sum_{m=1}^{M}(f_m(i) * C_{cor}) \\ + \sum_{k=1}^{K}\sigma_{m,k}(i) * C_{m,k}(i) \\ + \sum_{l=K+1}^{K+L}\sigma_{m,l}(i) * C_{m,l}(i)] \end{aligned} \tag{14}$$

The first term in the total cost function is the cost of all missed missions over the period. The second term represents the maintenance cost including corrective maintenance penalty in case of failure, and predictive and preventive components if they are maintained.

Thus, the objective function of the classic joint problem can be written as

$$min\ Total\_Cost(DH) \tag{15}$$

Solution of this joint problem is valid if it proposes a schedule of operational and maintenance activities that satisfies all constraints defined in equations 6, 7, 8, 12, and 13. Moreover, if this solution, also, satisfies equations 4 and 5, it is feasible.

3.5. The decision horizon study

In reality, the joint problem is going to be optimized several times during the operations of the assets. And what railways companies are looking for is the optimization of the overall cost in the long term. The main contribution of this paper is to find the decision horizon duration DH that optimizes the resolution of the joint problem over a simulation horizon SH. In other words, the treated problem by this paper is finding the suitable DH value that minimizes the cumulative total cost over the simulation horizon SH obtained by repeating the optimization of the joint problem for I periods. This simulation horizon is covered by N steps of decision-making (i.e. joint problem resolution) over the rolling horizon DH. The number of steps, noted N, is defined in a way that verifies $SH = N * I * \Delta T$. Where ΔT is the duration of a time period (e.g. duration of one day).

Therefore, the objective function of the decision horizon study problem is defined as

$$min \sum_N Total\_Cost(DH) \tag{16}$$

With $\sum_N Total\_Cost(DH)$ is defined as the cumulative total cost over the simulation horizon SH.

To summarize, this concept is presented in Figure 2. The simulation horizon SH is divided into N equal parts of duration we call decision horizon DH. For each duration, we solve the joint problem to obtain the total cost. The sum of these total costs produces the cumulative total cost. The aim of the paper is to study the influence of the decision horizon on this cumulative total cost and minimize it by finding the suitable decision horizon.

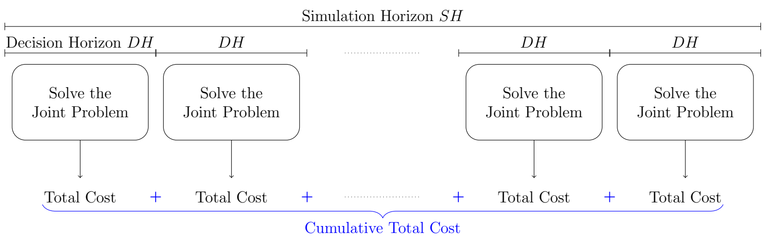

Figure 2. Illustration of the cumulative total cost.

4. Methodology

To study the influence of decision horizon, we should solve the joint problem of mission assignment and maintenance planning in rolling stock systems. This problem is a combinatorial discrete optimization problem. The use of prognostics information like degradation level and/or remaining useful life of the systems adds some non-linearity to the problem (e.g., by using a non-linear degradation model). Therefore, we used a genetic algorithm and some heuristics to solve this problem. These methods are described in this section.

4.1. Genetic algorithm

Genetic algorithm (GA) is a well used mature method based on heuristic rules to produce improved approximations of the objective function over several iterations. GA search techniques are based on biological systems rules for natural survival in a different environment. The algorithm starts with a set of initial solutions called population, in which each solution (i.e., an individual of the population) is called a chromosome.

Through successive generations (i.e., iterations), these chromosomes evolve as the result of crossover and mutation operators. Each chromosome is evaluated using some measure of fitness. The new generation is created by selecting some chromosomes from the previous generation (i.e., survival chance) and new chromosomes resulted from the genetic operators (i.e., crossovers and mutations) [34]. Therefore, to implement GA, several components should be considered:

- the genetic representation of the solution,
- the fitness function,
- the method to generate initial population,
- the genetic operators (mutation and crossover), and
- the survival rules.

Even though GA does not guarantee the global optimum solution, it is a commonly used method in cases of combinatorial, high instances or non-linear optimization problems. This paper does not provide new GA operators (i.e., crossover and mutation); rather, existing operators are used. The required components of the GA implementation are presented in this subsection.

4.1.1. Solution representation

The coding of GA chromosomes is a key step when using such kind of approach. As detailed in the model the variables of the optimization problem are $\beta_{p,m}(i)$ and $\sigma_{x,m}(i)$ $\forall p \in \{1,..,P\}, m \in \{1,..,M\}$ and $\forall x \in \{1,..,K+L\}$. Then, we propose a coding consisting of a 2D matrix of integers, in which columns represent the M rolling stock units while rows represent the decision horizon DH (i.e., I periods). This coding is a sequence of alleles, in which each allele is the schedule of a rolling stock unit m $(1 \leq m \leq M)$ over the decision horizon (i.e., I periods). Element $e_{i,m}$ of the 2D-array (i.e., element of the m^{th} column and the i^{th} row) represents the planned activity for unit m during period i $(1 \leq i \leq I)$. The value of any element can be negative or positive, depending on the activity it represents. The possible values for an element $e_{i,m}$ according to the scheduled activity are defined as

$$e_{i,m} = \begin{cases} p & \text{if } \beta_{p,m}(i) = 1 \\ z_{i,m} & \text{if } \omega_m(i) = 1 \\ 0 & \text{if } \pi_m(i) = 1 \end{cases} \tag{17}$$

For scheduled missions, the value is equal to the identification number of the mission. For maintenance activities, the value of the element is a negative integer $z_{i,m}$ that represents a coding of the identification number of the components to be maintained as

$$z_{i,m} = (-1) \times \sum_{y=1}^{Card(Maint_{m,i})} (10^{a \times y} \times Maint_{m,i}[y]) \tag{18}$$

with a the number of digits in $K+L$ and $Maint_{m,i}$ the set of components' identification number that are scheduled for maintenance expressed as

$$Maint_{m,i} = \{x \in \{1,...,K+L\} \mid \sigma_{x,m}(i) = 1\} \tag{19}$$

To represents the unit at rest, the value of the corresponding element is set to zero.

Time Periods	Railway Equipments $m=1$	$m=2$	$m=3$	$m=4$	$m=5$
$i=1$	3	2	0	1	4
$i=2$	0	5	8	6	7
$i=3$	12	0	9	11	10
$i=4$	13	16	14	−0105	15

Figure 3. An example of solution representation.

An example of solution representation is given in Figure 3. $e_{4,4} = -0105$ which means that components number 1 and 5 of unit 4 are scheduled for maintenance during the period 4.

4.1.2. Initial generation

GA is based on improving solutions from one generation to another over a predefined number of iterations. Therefore, this kind of algorithm requires an initial generation (or set) of valid and/or feasible solutions. To build the first generations' individuals, a heuristic algorithm (Algorithm 1) is used. The idea is to sort the set of rolling stock vehicles according to their degradation level which is defined by the degradation level of their most degraded components. Then the algorithm schedules maintenance for the most deteriorated components (that satisfies the maintenance condition Equation 3). It assigns randomly the available tasks of the period i to the rest of the vehicles. The outcome of the i^{th} period is simulated and the algorithm moves to schedule the next period until it reaches the end of the decision horizon. This heuristic is executed several times until the initial population is constructed.

Algorithm 1: Creation of the initial generation.

```
while i ≤ DH do
    Sort vehicles according to their degradation
    while (∑_m(∑_k σ_{m,k} + ∑_l σ_{m,l}) ≤ ML_Comp) AND
      (∑_m ω_m(i) ≤ ML_Equip) do
        Schedule maintenance for most deteriorated component
    Assign randomly tasks to the rest of vehicles
    Simulate the outcome of the schedule
    i++
```

4.1.3. Mutation

The mutation is a genetic operator that provides spontaneous changes in the chromosome's genes. Usually, mutations are designed to alter one or more genes of a chromosome. Given the representation of the solution, it is necessary to redefine the mutation operator. Two mutation operators are used in this GA:

- **Simple mutation:** First, the index of a column m (i.e., a vehicle) is randomly selected. Then, the schedule of the designated vehicle is altered. For each period i, if there are unassigned missions at period i (i.e., Equation 20), then one of them is randomly assigned to $e_{i,m}$, else a second vehicle m_2 is randomly selected and the elements $e_{i,m}$ and e_{i,m_2} are interchanged.

$$\sum_{p=1}^{P}\sum_{m=1}^{M}\beta_{p,m}(i) \leq P \tag{20}$$

- **Exchange mutation:** For this mutation, two positions of the units m_1 and m_2 (columns) to mutate are given. Then, for each period the schedule of the two vehicles is interchanged (Equation 21).

$$e_{i,m_1} \rightleftharpoons e_{i,m_2},\ \ \forall\ i \in \{1, ..., I\} \tag{21}$$

4.1.4. Crossover

The Crossover is a genetic operator that operates on two individuals (called parents) at the same time. It combines the features of these parents to generates offspring. The GA uses a single-point order crossover adapted for a 2D-array representation of the solution. Basically, for each period the first part of alleles from the first parent drops down to the first child and remaining values are placed in the child in the order which they appear in the second parent. An example of this crossover is given in Figure 4 for 6 vehicles two days example.

Parent 1	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
$i=1$	6	3	2	1	5	4
$i=2$	10	12	8	7	9	11

Parent 2	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
$i=1$	1	4	6	3	2	5
$i=2$	11	10	12	9	8	7

Child 1	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
$i=1$	6	3	2	5	1	4
$i=2$	10	12	8	9	7	11

Child 2	$m=1$	$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
$i=1$	1	4	6	5	3	2
$i=2$	11	10	12	7	9	8

Figure 4. Example of single-point order crossover.

4.1.5. Genetic operators results

The used representation of chromosomes does not allow to directly use the genetic operators (mutation and crossover). Therefore, before applying these operators, the chromosomes undergo a special treatment to take out maintenance activities. This treatment consists of placing rolling stock units at rest if they are scheduled for maintenance.

Since the maintenance plans are excluded from the genetic operators, the resulted chromosomes from mutation and crossover represent mission assignments only. Hence, a reverse operation is required to schedule maintenance activities. A heuristic is defined to fix the offspring of genetic operators by scheduling maintenance and respecting the different constraints to obtain a feasible solution. This heuristic tries also to schedule the still unassigned mission to the available rolling units that could be missed by the genetic algorithm. Thus, it extends the search space of the problem.

4.1.6. Fitness evaluation

Individuals in the GA are evaluated to measure their fitness toward an objective function. The fitness of a solution is defined as the value of the objective function. In this case, the fitness evaluation uses the definition of the total cost as in Equation 14. The total cost is obtained by simulating the outcome of the chromosome. The effects of the execution of missions and maintenance activities are simulated on the health indicators of each rail vehicle. The constraints are checked to detect the potential failures and corrective maintenance activities. And the total cost is computed progressively.

Each rail vehicle m is represented through a series of simulated health indicators $H^s_{m,k}$ and $\theta^s_{m,l}$. For each period i, these indicators are updated according to the scheduled activity for the vehicle. For operational activities, the mission's degradation rate and its distance are added to the adequate simulated health indicators. Equation 22 represents the update of vehicle m indicators at period i during which mission p is assigned to the vehicle (i.e., $e_{i,m} = p$). Maintenance activities are assumed to be perfect. Therefore, the simulation of maintenance is equivalent to reset the concerned component's simulated health indicator to zero. For example, if during period i component k of rail unit m is scheduled for maintenance (i.e., $e_{i,m} = -k$) then $H^s_{m,k}(i) = 0$. At the end of the activity's simulation, its corresponding cost is computed. Moreover, at the end of each period i, the cost of each activity is summed and the penalties on missing tasks are added if necessary. The fitness of a solution equals the sum of the periods' costs during the decision horizon DH.

$$\begin{cases} H^s_{m,k}(i) = H^s_{m,k}(i-1) + \delta_{p,k} & \forall\ k \in \{1, ..., K\} \\ \theta^s_{m,l}(i) = \theta^s_{m,l}(i-1) + d_p & \forall\ l \in \{K+1, ..., K+L\} \end{cases} \tag{22}$$

4.1.7. Construction of new generation

The method to construct a novel generation is an important step in a GA. The transition from one generation to another is a driving force of the genetic search and evolutionary progress. New generations are created from previous generation survivors and genetic operators' offspring. In this context, a novel generation is built by; (i) a percentage $X_{Survival}\%$ of best chromosomes from the previous generation, (ii) a percentage $X_{Mutation}\%$ of best chromosomes of the mutation's offspring, and (iii) a percentage $X_{Crossover}\%$ of best chromosomes of the crossover's offspring. The choice of these parameters influences the speed of convergence of the GA. In a way, that satisfies a constraint defined as

$$X_{Survival}\% + X_{Mutation}\% + X_{Crossover}\% = 100\% \tag{23}$$

4.1.8. Overall algorithm

The overall GA is described in Algorithm 2. Lets note that parents selection in case of crossover is done according to the roulette wheel method [35].

Algorithm 2: GA for rolling stock task assignment and maintenance planning.

```
Create Initial_Generation;
Input_Generation ← Initial_Generation;
while number_of_generations < Generation_Limit do
    Evaluate Individuals of the Input_Generation;
    Sort Input_Generation ;
    Survival of the best X_Survival% Individuals of the Input_Generation;
    Mutation_Res ← [ ];
    Crossover_Res ← [ ];
    for Individual ∈ Input_Generation do
        Generate a Random Numbers a, b and c;
        if a < Simple_Mutation_Probability then
            Generate a random number pos for position;
            Simple_Mutate Individual at position pos;
            Add the offspring to Mutation_Res;
        if b < Exchange_Mutation_Probability then
            Generate two random numbers pos1 and pos2 for positions;
            Exchange_Mutate Individual at positions pos1 and pos2;
            Add the offspring to Mutation_Res;
        if c < Crossover_Probability then
            Generate one random numbers pos for position;
            Select another parent from Initial_Generation;
            Crossover Individual and the other parent at positions pos;
            Add the offsprings to Crossover_Res;
    Evaluate individuals of the Mutation_Res;
    Sort Mutation_Res;
    Selection of the best X_Mutation% individuals of the Mutation_Res;
    Evaluate Individuals of the Crossover_Res;
    Sort Crossover_Res;
    Selection of the best X_Crossover% individuals of the Crossover_Res;
    Input_Generation ← Output_Generation;
    number_of_generations++;
```

4.2. Heuristics

Heuristics are strategies developed from previous experiences with the same problem. These approaches do not guarantee an optimal solution, but they are considered as practical methods to generate "good enough" solutions that are sufficient to reach an immediate goal, in reduced execution time.

For the resolution of the joint optimization of vehicle scheduling and maintenance planning, two heuristics are presented in this section.

4.2.1. Heuristic H1

Heuristic H1 is inspired by a "common sense" method of scheduling tasks and maintenance of rolling stocks. The heuristic consists of a succession of iterations corresponding to a decision for each period of the horizon DH. For each period i ($1 \leq i \leq I$), the rolling stocks are sorted according to their RUL (in increasing order). Then, for each unit m, a set $\mathcal{D}_{m,i}$ of possible decisions is constructed. this set is composed of put at rest action, maintenance intervention (containing all component eligible for maintenance) and all valid mission assignments (see Eqs. 4 and 5). Each decision of this set is associated with a regret value which is computed as a function of the rest of the decision horizon (periods $i+1$ to I). The regret function depends on the nature of the decision to be taken and a set of rules that describe different scenarios that can occur.

Three rules have been used in this heuristics. For each rolling unit, they define a provisional schedule for the rest of the horizon based on the potential decision for the current period. The potential decisions, in this study, are: maintenance, resting, mission performing. The rules are the following:

- **Rule 1:** for a maintenance decision, the rolling unit should be assigned to missions for the rest of the decision horizon.
- **Rule 2:** for a stay at rest, the rolling unit should be assigned to missions for the rest of the decision horizon.
- **Rule 3:** for mission performing, the rolling unit is put in maintenance or at rest during the next period ($i+1$) then, it should be assigned to missions for the rest of the decision horizon.

The outcome of the different rules is used to compute a regret value. This regret is computed for each rolling unit m and each possible decision $dc \in \mathcal{D}_{m,i}$. The regret value of one decision at period i is computed as the sum of two penalty values:

- **Maintenance penalty:** for each $dc \in \mathcal{D}_{m,i}$, the decision dc is supposed to be done, then the regret is an evaluation of the maintenance cost for the rest of the schedule. For that, the number of maintenance interventions per component is computed for predictive and preventive components as

$$N\_M(m,k) = \lfloor \frac{H_{m,k}(i+1) + \sum_{j=i+1}^{DH} \max_{p \in P_j}(\delta_{p,k})}{\Lambda_k} \rfloor \tag{24}$$

$$N\_M(m,l) = \lfloor \frac{\theta_{m,l}(i+1) + \sum_{j=i+1}^{DH} \max_{p \in P_j}(d_p)}{\Phi_l} \rfloor \tag{25}$$

- We assume in these equations that the unit will always carry out the most severe missions for the rest of the decision horizon. The maintenance regret (noted Reg_m) is then defined as

$$
\begin{aligned}
Reg_m(dc,i) = &\sum_{k\in K} N\_M(m,k) * (CR_k + \frac{\Delta_k - \Lambda_k}{2} LP_k)) \\
&+ \sum_{l\in L} N\_M(m,l) * (CR_l + LP_l * \frac{\Theta_l - \Phi_l}{2})
\end{aligned} \tag{26}
$$

- **Operation penalty:** There are two parts in the operational regret (noted Reg_o). The first one depends on the workload of maintenance operators. If this load is too high then some mission would be missed. the penalty corresponds to this evaluating cost, the first part of the equation. The second part of the penalty concerns only maintenance decisions and putting rolling units at rest decision. If the number of remaining units $SortedEquip$ is lower than the number of remaining tasks $RestMissions$, some missions cannot be achieved therefore causing cost penalty, the second part of the equation. The overall operational regret is expressed as

$$
\begin{aligned}
Reg_o(dc,i) = &\tau * \frac{\sum_{x=1}^{K+L} N\_M(m,x)}{ML_{Comp}} * C_{lost} \\
&+ (\omega_m(i) + \pi_m(i)) * MissionsToLose * C_{lost}
\end{aligned} \tag{27}
$$

With:

$$
\begin{aligned}
MissionsToLose = &card(RestMissions) \\
&- card(SortedEquip)
\end{aligned}
$$

Heuristic H1 provides a joint schedule of maintenance and tasks for a rolling decision horizon. The schedule is constructed based on the rolling stock unit's ability to carry out a task, the defined rules and the regret computation of possible actions. The process of decision-making of heuristic H1 for a decision horizon is presented in algorithm 3.

4.2.2. Heuristic H2

Heuristic H2 is based on the health state of the rolling stocks. The aim is to guarantee a certain periodicity between the RULs of the vehicles in a way that the maintenance operation of different rolling units is well distributed. In this purpose, three sets of vehicles are defined according to their health state:

- **Rolling units in good health:** The vehicles in this set ($\mathcal{GH}$) can carry out a certain number of missions before needing any maintenance activities. To belong to this category, the RUL of the vehicle should be higher than a threshold Th_{GH}.
- **Rolling units in medium health:** Once the RUL of a vehicle in Good Health falls behind Th_{GH} but still exceeds Th_{MH}, they are placed in the Medium Health Rolling Units set ($\mathcal{MH}$).
- **Rolling units in poor health:** Vehicles in this set ($\mathcal{PH}$) have a RUL that falls behind Th_{MH}. These rolling units can carry on some missions depending on their severity.

The number of units in each set (i.e., N_{PH}, N_{MH}, and N_{GH}) and the threshold of the sets (i.e., Th_{GH} and Th_{MH}) are the major decision variables for this heuristic.

Algorithm 3: Heuristic H1.

while $i \leq I$ **do**
  $RestMissions \leftarrow [1..P]$;
  $SortedEquip \leftarrow$ Rolling units sorted in increasing order of their RUL;
  while $SortedEquip \neq$ **do**
    $m \leftarrow$ first element of $SortedEquip$;
    $SortedEquip \leftarrow SortedEquip \setminus m$;
    $\mathcal{D}_{m,i} \leftarrow [\ ]$;
    if $\exists k \in \{1, .., K\}/H_{m,k} \geq \Lambda_k$ ***Or*** $\exists l \in \{K+1, .., K+L\}/\theta_{m,l} \geq \Phi_l$ **then**
      Add maintenance action to $\mathcal{D}_{m,i}$;
    Add Rest to $\mathcal{D}_{m,i}$;
    for $p \in RestMissions$ **do**
      if $((H_{m,k} + \delta_{p,k}) < \Delta_k \forall k \in K)$ ***And*** $((\theta_{m,l} + d_p) < \Theta_l \forall l \in L)$ **then**
        Add task p to $\mathcal{D}_{m,i}$;
    Compute Regret for all decisions in $\mathcal{D}_{m,i}$;
    Choose the decision with the lowest regret value;
    Update $H_{m,k}(i+1) \forall k \in K$;
    Update $\theta_{m,l}(i+1) \forall l \in L$;
    Update $RestMissions$;
  i++

The assignment of missions is done in a way that tries to keep more or less the same number of vehicles in each category as defined by the parameters. Rolling Units in Poor Health are maintained to re-balance the Good Health and the Medium Health categories.

Two versions of heuristic H2 are developed in this paper, according to which service is prioritized over the other.

- **H2_v1:** In this version of the heuristic H2, the maintenance activities are prioritized over the task assignment. Therefore at the beginning of the algorithm, the most deteriorated vehicles of $\mathcal{PH}$ are placed into maintenance without checking their ability to carry on any of the missions. Once the maintenance activities are scheduled, the task is assigned while trying to appoint the most degraded unit to the hardest task it is capable of achieving it. This strategy allows the reduction of the remaining useful life wasted with maintenance activities. This heuristic is illustrated in algorithm 4. In this algorithm, two procedures are used HealthSetsBalance and MissionAssignment.

- **H2_v2:** Compared to H2_v1, this version prioritizes task assignment over maintenance planning. All rolling units sets (i.e., $\mathcal{PH}$, $\mathcal{MH}$, and $\mathcal{GH}$) are sorted according to their RUL. Most deteriorated vehicles of the poor health set $\mathcal{PH}$ are checked to fulfill tasks before being sent to maintenance. In this heuristic, maintenance decisions are made while considering the possibility of having a bottleneck in the maintenance workshop in the next period. Therefore, the number of vehicles $WillNeedMaint$ that would probably require a maintenance intervention in the next period is computed. If these vehicles are more than the maintenance limit ML_{Equip} in the next period $(i+1)$, then some units will be

Algorithm 4: Heuristic H2_v1.

input : $\mathcal{PH}, \mathcal{MH}, \mathcal{GH}$
input : N_{PH}, N_{MH}, N_{GH}

$MaintenanceQueue \leftarrow [\ ]$;
while $i \leq DH$ **do**
  $RestMissions \leftarrow \mathcal{P}$;
  $MaintenanceQueue \leftarrow \{m \in \mathcal{PH} \mid$
    $\exists\ k \ \in \{1, .., K\}$ where $H_{m,k} \ \geq \ \Lambda_k$
    or $\exists\ l \ \in \ \{K+1, .., K+L\}$ where $\theta_{m,l} \ \geq \ \Phi_l$
    or $\nexists p \in RestMissions \mid m$ is able to achieve $p\}$;
  Sort $MaintenanceQueue$ according to increasing RUL values;
  while *Maximum* ML_{Equip} *and* ML_{Comp} *are not met* **do**
    Plan Maintenance for the first unit of $MaintenanceQueue$;
  Sort $\mathcal{GH}$, $\mathcal{MH}$ and $\mathcal{PH}$ according to increasing RUL values;
  Sort $RestMissions$ according to decreasing s_p values;
  HealthSetsBalance($\mathcal{GH}, \mathcal{MH}, \mathcal{PH}$ **and** $RestMissions$**)**;
  MissionAssignment($\mathcal{GH}, \mathcal{MH}, \mathcal{PH}$ **and** $RestMissions$**)**;
  Update all $H_{m,k}$ and $\theta_{m,l}$ $\forall m, k$ and l;
  Update $\mathcal{PH}, \mathcal{MH}$ and $\mathcal{GH}$;
  i++;

Procedure HealthSetsBalance($\mathcal{GH}, \mathcal{MH}, \mathcal{PH}$ and $RestMissions$)

Sort $\mathcal{GH}$, $\mathcal{MH}$ and $\mathcal{PH}$ according to increasing RUL values;
Sort $RestMissions$ according to decreasing s_p values;
forall $\mathcal{X} \in \{\mathcal{GH}, \mathcal{MH}, \mathcal{PH}\}$ **do**
  while *card*$(\mathcal{X}) \geq N_X$ ***And*** $RestMissions \neq \emptyset$ **do**
    $m \leftarrow$ first element of $\mathcal{X}$;
    forall $p \in RestMissions$ **do**
      if $m \in \mathcal{X}$ ***And*** m *is able to achieve* p *(Eqs. 4, 5)* **then**
        Assign p to m;
        $RestMissions \leftarrow RestMissions \setminus p$; $\mathcal{X} \leftarrow \mathcal{X} \setminus m$;

scheduled to maintenance during the current period (i) to avoid this bottleneck. Else, the heuristic assigns to each of these vehicles the hardest mission they can achieve. Tasks assignment, in this version, is similar to the previous version using the two procedures HealthSetsBalance and MissionAssignment. The choice of the task is based on their severity and the need to move a vehicle from one category to another. The H2_v2 heuristic is presented in algorithm 5.

Procedure MissionAssignment($\mathcal{GH}, \mathcal{MH}, \mathcal{PH}$ and $RestMissions$)

forall $p \in RestMissions$ **do**
  if $\mathcal{PH} \neq \emptyset$ **then**
    Assign last vehicles m of $\mathcal{PH}$ to p; $\mathcal{PH} \leftarrow \mathcal{PH} \setminus m$;
  else if $\mathcal{MH} \neq \emptyset$ **then**
    Assign last vehicles m of $\mathcal{MH}$ to p; $\mathcal{MH} \leftarrow \mathcal{MH} \setminus m$;
  else if $\mathcal{GH} \neq \emptyset$ **then**
    Assign last vehicles m of $\mathcal{GH}$ to p; $\mathcal{GH} \leftarrow \mathcal{GH} \setminus m$;
  else
    p is a missed mission;

5. Results and discussion

5.1. Numeric example

The numeric example in this paper is inspired by the problem presented by Herr *et al.* in [9]. The problem treats the assigning of a set of $P = 15$ daily missions on a fleet of $M = 18$ rolling vehicles over a simulation horizon $SH = 300$ *days* where the periods are considered as days. The value of the simulation horizon is set to 300 *days* to guarantee that every component of the trains is at least maintained once during this horizon.

Three types of missions are considered in this application. The task characteristics are presented in Table 1. Each rolling unit is composed of $K = 13$ predictive components and $L = 4$ preventive components. he characteristic of these components is defined in Tables 2 and 3 for predictive and preventive components respectively. All predictive components ($k \in \{1..13\}$) have the same failure threshold $\Delta_k = 0.95$ and maintenance threshold $\Lambda_k = 0.7$. All preventive components ($l \in \{14..17\}$) have the same thresholds $\Theta_l(\%) = 95\%$ and $\Phi_k(\%) = 85\%$ of the mean mileage between maintenance.

Each day only $ML_{Equip} = 2$ vehicles are allowed in the maintenance workshop, with a maximum total of $ML_{Comp} = 4$ components to be maintained. LP_k and LP_l are set in a way that one lost mile of any component would cost 2u.m (unit of money). To summarize, all the used variables are presented in Table 4.

The decision horizon duration DH is defined as a divisor of the simulation horizon SH so that the number of decision-making steps N is an integer (i.e., $N \in \mathbb{Z}$).

In general, when dealing with train operational scheduling it is unusual to keep the same schedule after 60 days. Therefore, we only consider decision horizons that are a divisor of SH and have a duration of at most 60 days. In this case, the possible values for DH are defined as

$$DH \in \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 25, 30, 50, 60\} \tag{28}$$

5.2. Methods comparison

All the results are expressed in terms of the cumulative total cost over the simulation horizon which consists of the sum of the total cost over the decision horizons (see

Algorithm 5: Heuristic H2_v2.

input : $\mathcal{PH}, \mathcal{MH}, \mathcal{GH}$
input : N_{PH}, N_{MH}, N_{GH}
$MaintenanceQueue \leftarrow [\,]$;
while $i \leq DH$ **do**
  $RestMissions \leftarrow \mathcal{P}$;
  Sort $RestMissions$ according to decreasing s_p values;
  Sort $\mathcal{GH}$, $\mathcal{MH}$ and $\mathcal{PH}$ according to increasing RUL values;
  $MaintenanceQueue \leftarrow \{m \in \mathcal{PH} \mid$
    $\nexists p \in RestMissions \mid m$ is able to achieve $p\}$;
  while *Maximum* ML_{Equip} *and* ML_{Comp} *are not met* **do**
    Plan Maintenance for the first units of $MaintenanceQueue$;
  $p \leftarrow$ first element of $RestMissions$;
  $WillNeedMaint \leftarrow \{m \in \mathcal{PH} \mid$
    $\exists\ k \ \in \{1, .., K\}$ where $H_{m,k} + \delta_{p,k} \ \geq \ \Lambda_k$
    or $\exists\ l \ \in \ \{K+1, .., K+L\}$ where $\theta_{m,l} + d_p \ \geq \ \Phi_l\}$;
  if $Card(WillNeedMaint \cup MaintenanceQueue) \ \leq \ ML_{Equip}$ **then**
    for $m \in WillNeedMaint$ **do**
      forall $p \in RestMissions$ **do**
        if $m \in WillNeedMaint$ ***And*** *m is able to achieve p (Eqs. 4, 5)* **then**
          Assign p to m;
          $RestMissions \leftarrow RestMissions \setminus p$;
          Remove m from $WillNeedMaint$;
  else
    while *Maximum* ML_{Equip} *and* ML_{Comp} *are not met* **do**
      Plan Maintenance for the first units of $WillNeedMaint$;
  HealthSetsBalance($\mathcal{GH}, \mathcal{MH}, \mathcal{PH}$ **and** $RestMissions$);
  MissionAssignment($\mathcal{GH}, \mathcal{MH}, \mathcal{PH}$ **and** $RestMissions$);
  Update all $H_{m,k}$ and $\theta_{m,l}$ $\forall m, k$ and l;
  Update $\mathcal{PH}, \mathcal{MH}$ and $\mathcal{GH}$;
  i++;

Table 1. Characteristics of the missions.

Type	Severity (s_p)	Length (mi)	Number
1	0.85	110	5
2	1	130	5
3	1.3	170	5

Table 2. Rolling stocks predictive components characteristics.

Type	Number	α_k	μ_k	CR_k
T_A	1	0,00346	0.002	100
T_B	2	0,0031	0.00178	150
T_C	2	0,01246	0.00166	75
T_D	8	0,00798	0.00208	100

Equation 16). This cost includes the cost of missed missions, corrective maintenance, missed mileage before maintenance, and normal maintenance. For example, for $DH = 10days$ we have $N = 30$ and the obtained cumulative total cost corresponds to the sum of the 30 total costs. With each total cost is obtained by solving the joint problem on a decision horizon of a duration equal to $10days$.

Results obtained from all the methods are presented in Figure 5. The results of heuristic H1 are presented in the dashed orange line. The H1 curve is similar to an exponentially decreasing function. The curve is almost stable around the (2800 k u.m) value. This can be explained by the fact that in H1 each vehicle is selecting the i^{th} decision based on the regret value. The regret values are computed for the rest of the decision horizon. Therefore, the longer the decision horizon gets the better the regret computation is, and thus the better the management of the vehicles. Although the cost obtained by H1 is improved from a longer decision horizon, the values are still far from the optimal solution. For this reason, H1 is not further studied in the rest of the paper.

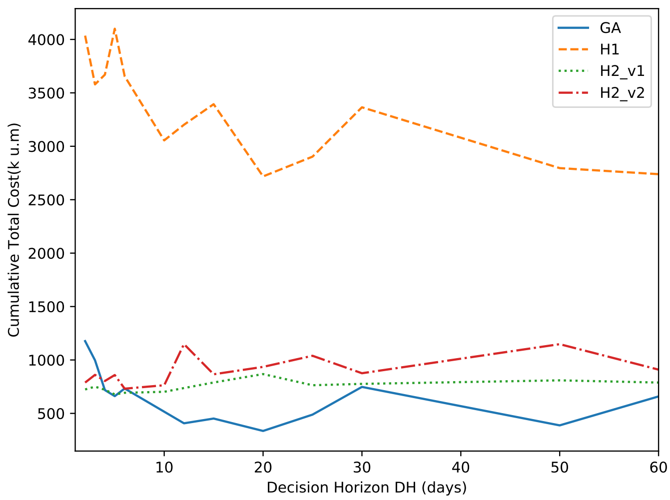

Figure 5. Results of the different methods.

Results obtained by heuristic H2 (version 1 and 2) are represented by respectively dotted green line and dash-dot red line. We note that these results are almost stable regardless of the duration of the decision horizon. Also, these values are closer to the

Table 3. Rolling stocks preventive components characteristics.

Type	Number	Mean Mileage Between Maintenance(mi)	CR_l
T_E	1	31 250	100
T_F	3	15 625	100

Table 4. The Values of the Numeric Application.

Name	Significance	Value
SH	Simulation Horizon	300 Days
M	The Size of the Rolling Stock Fleet	18
P	Number of Daily Missions	15
K	Number of Predictive Components	13
L	Number of Preventive Components	4
ML_{Equip}	Capacity of Maintenance in Vehicles per Day	2
ML_{Comp}	Capacity of Maintenance in Components per Day	4
LP	Cost Penalty on a lost mile	2 u.m
C_{lost}	Cost of Missing a Mission	10 ku.m
C_{cor}	Cost of Failure During a Mission	100 ku.m

optimal solution. These heuristics propose a compromise between the execution time to find the results and the solution quality.

In addition to the cumulative total cost over the simulation horizon, these methods are compared in terms of the total number of missed missions over the simulation horizon. In Figure 6, we presented the mean number of missed missions from several simulations of the used methods (GA, H2_v1, and H2_v2). We can note that the curves of the missed missions are quite similar to the curves of the total cost. We can also notice that the genetic algorithm presents the minimal value of lost missions. However, when the decision horizon gets too small or too larger the genetic algorithm start loosing its performance.

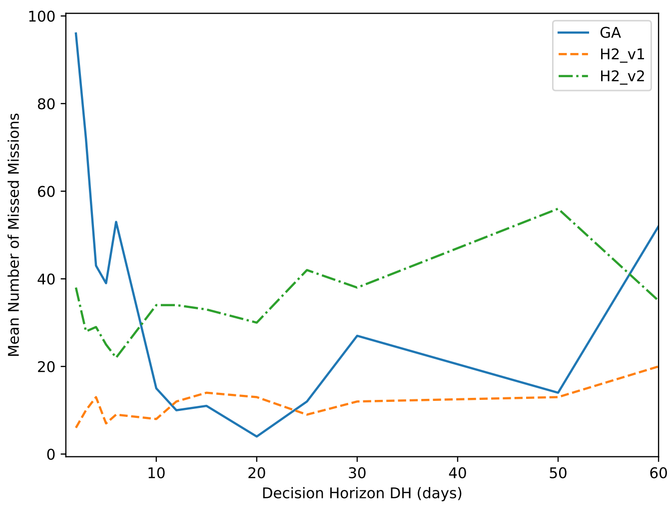

Figure 6. Mean missed missions of the different methods.

In Figure 7, we present the mean value of the lost distance before maintenance. This performance is measured as the mean value for all components and rolling units. We can note that the mean lost distance is almost stable for the both heuristics with a small difference between them. However, it is clear that the genetic algorithm manages better the maintenance of the rolling stocks. The mean value of lost distance when using GA is minimal for small decision horizon.

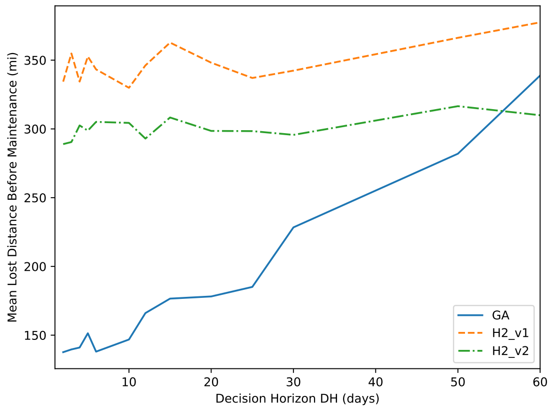

Figure 7. Mean lost mileage of the different methods.

Figure 8 presents the execution time spent by the proposed methods to solve the problem over the simulation horizon for a specific decision horizon value. We easily note that the genetic algorithm takes much more time to find improved solutions.

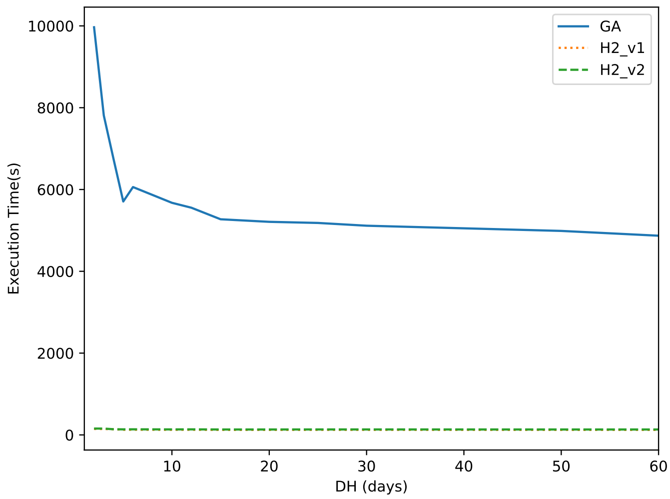

Figure 8. Execution time of the different methods.

5.3. Decision horizon duration study

In Figure 5, the cumulative total cost of the solution over the simulation horizon depends on the duration of the chosen DH. This influence can be seen through the fluctuation of the different curves. Almost all the methods present some fluctuations.

Based on this observation, a further investigation of this aspect is done by launching different simulations to see if these fluctuations are caused by the initial condition of the vehicles or the decision horizon. The obtained results are discussed per method in the following paragraphs.

5.3.1. Genetic algorithm

In the case of GA, the one-day decision horizon is ruled out of the study. The one-day decision horizon can provide locally optimized decisions for a certain number of periods by avoiding maintenance activities and using rolling units to their full degradation level. Then once all units are degraded, the algorithm can no longer schedule missions, thus all the missions are missed and no maintenance activities are done since it is less expensive to miss all missions without maintenance planned than to miss missions and plan maintenance interventions on the vehicles. This can be considered as a limit of the genetic algorithm performance. Figure 9 presents these results where the cumulative cost of a one-day decision horizon is compared to the ten-days decision horizon. It is important to note that in Figure 9, the time axis is limited to $80 - days$ for clarity purposes.

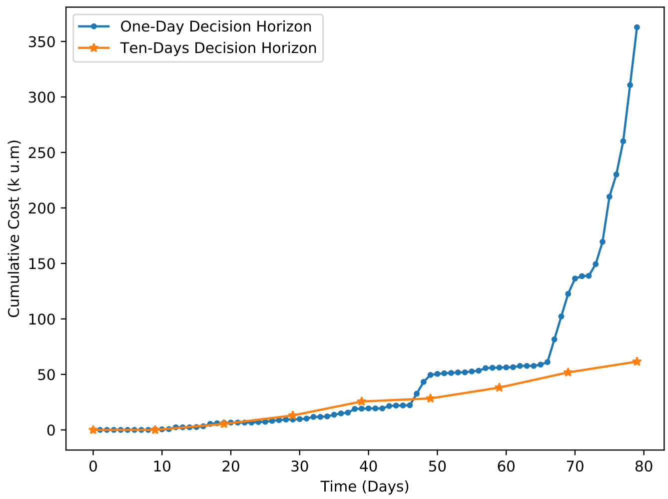

Figure 9. Cumulative cost of one-day and ten-days decision horizon.

When solving the same problem with different rolling vehicles' initial conditions, the obtained results of the cumulative total cost in function of the decision horizon have almost the same shape of fluctuation. These results can be seen in Figure 10. The fact that the evolution of the total cost has the same shape for different initial conditions, proves that the initial conditions of the vehicles have no big influence on the decision horizon value in the case of using the genetic algorithm as a decision-making method.

The results presented in Figure 10, are used to generate a box plot to display the variation of the total cost for each value of the decision horizon. For clarity purposes, the x-axis values, still, represents the decision horizon duration, but the scale is no longer applied.

Figure 10 and the box plot of Figure 11 show that the cumulative total cost over the simulation horizon, starts high, decreases to a minimal value, then increases once more. The minimal value of the cumulative total cost is obtained for a decision horizon

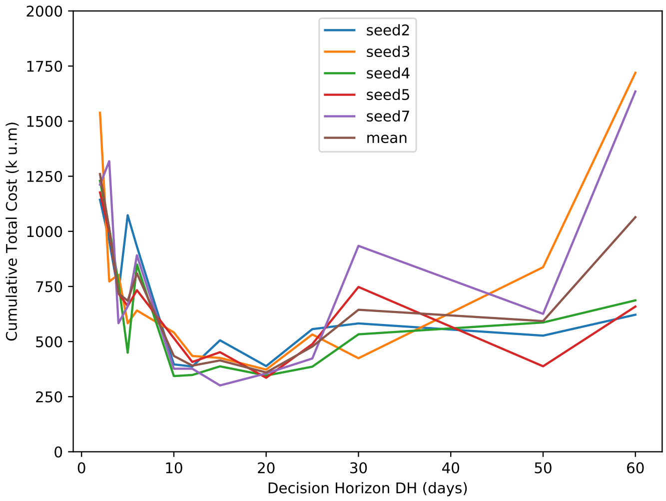

Figure 10. Total cost evolution for different initial conditions.

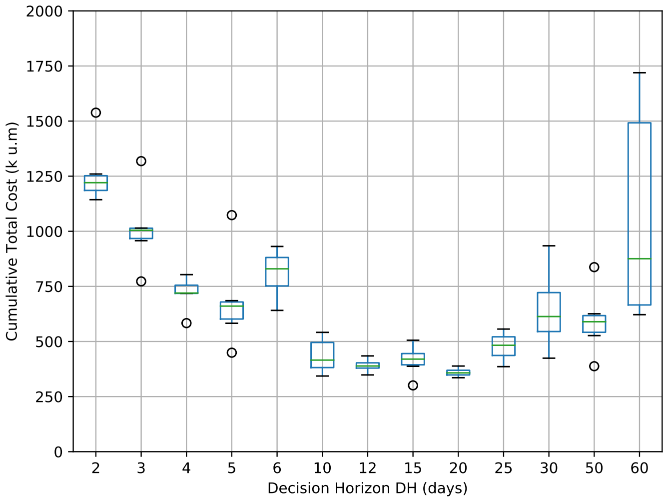

Figure 11. Box plot of total cost variation for the GA.

duration around 10-days to 25-days. For these same values of the decision horizon, the variation of the cumulative total cost is minimal from one execution to another. Thus, we conclude that for a task assignment and maintenance planning of rolling stocks as described in this problem and using a genetic algorithm, it is better to choose a rolling horizon of a duration that can vary from 10 to 25 days.

5.3.2. Heuristics H2_v1 and H2_v2

The variation of the cumulative total cost per decision horizon duration is captured in the case of the proposed heuristics H2_v1 and H2_v2. The obtained results confirm the observation of Figure 5. The heuristics are almost stable for all different decision horizon duration. Although there are some variations, as shown by the box plots in Figs. 12 and 13 that represents results from heuristic H2_v1 and H2_v2 respectively.

Despite the small variation in the cumulative total cost for different decision horizon

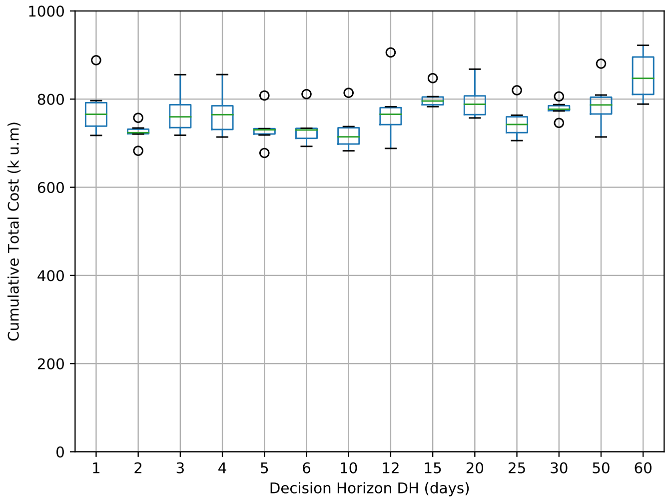

Figure 12. Box plot of total cost variation for heuristic H2_v1.

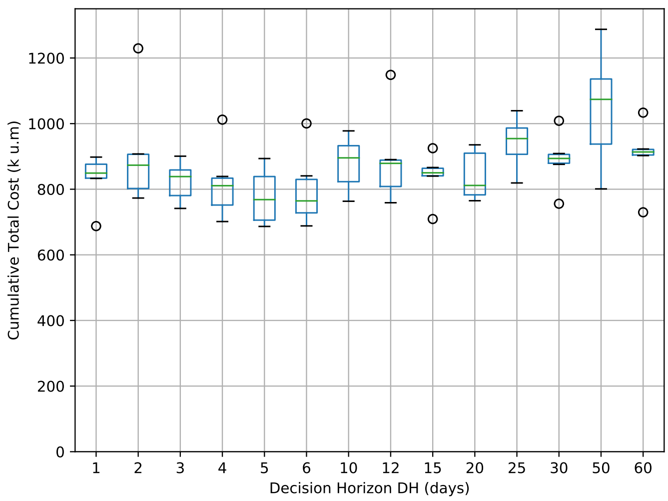

Figure 13. Box plot of total cost variation for heuristic H2_v2.

duration, minimal fluctuation and minimal values are obtained for a decision horizon of 5-days to 10-days in the case of heuristic H2_v1. As for heuristic H2_v2, the decision horizon of 4-days to 6-days presents the minimal variance of total cost and a globally minimal cost compared to other decision horizons.

5.3.3. Synthesis

The obtained values of decision horizon duration are different from one method to another. This proves that the decision horizon duration is a characteristic of the decision problem and depends also on the resolution method. The best decision horizon duration is presented in Table 5 per method. In the case of railway planning, it is more efficient to choose longer decision horizons. Therefore the genetic algorithm is more suited to solve this problem plus it provides the lowest total cost for decision horizons below 60-days.

Table 5. Best decision horizon per method.

Method	Decision Horizon Duration
GA	$DH \in \{10, 12, 15, 20\}$
H2_v1	$DH \in \{5, 6, 10\}$
H2_v2	$DH \in \{4, 5, 6\}$

6. Conclusion

In this paper, the decision horizon duration is studied for a the joint problem of railways vehicle scheduling and maintenance planning based on their health states. This study used several methods (i.e., genetic algorithm and three heuristics) to solve the joint problem for different values of decision horizon.

Results show that the decision horizon duration has an influence on the performance of the problem solving method. Whatever method used to solve the problem the duration of the decision horizon changes its performance. Also, we noticed that the obtained values of decision horizon duration that optimizes the performances (i.e. the cumulative total cost over the simulation horizon) are different from one method to another.

It was proven that the decision horizon is a characteristic of the considered problem and highly depends on the resolution method. Also, the initial conditions of the vehicles' components have a slight influence on the results, but they do not influence the choice of the duration of the decision horizon. To summarize, we propose a method to study the suitable decision horizon duration that minimizes the cumulative total cost.

In this work, we proved that the choice of the decision horizon duration requires a study of the used method. As future work, we suggest studying what factors of the problem definition influence the choice of the decision horizon. In this purpose, we propose to study the sensitivity of the decision horizon duration to the number of components in each railways vehicle, to the characteristics of the components degradation (e.g. speed of deterioration), and to the number of vehicles/missions.

The used methods to solve the joint problem, in this paper, are sequential and centralized (heuristics and evolutionary algorithms). However, this joint problem requires the involvement of two different disciplines, i.e., maintenance and operations. Each of these disciplines has its own objectives and treats the problem from its point of view. Another promising future work is to study the influence of the decision horizon on decentralized resolution methods. One of these methods is multi-agent systems. As future work, we suggest solving the joint problem of vehicle scheduling and maintenance planning while considering prognostic information using multi-agent systems, then, study the decision horizon influence on such method.

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