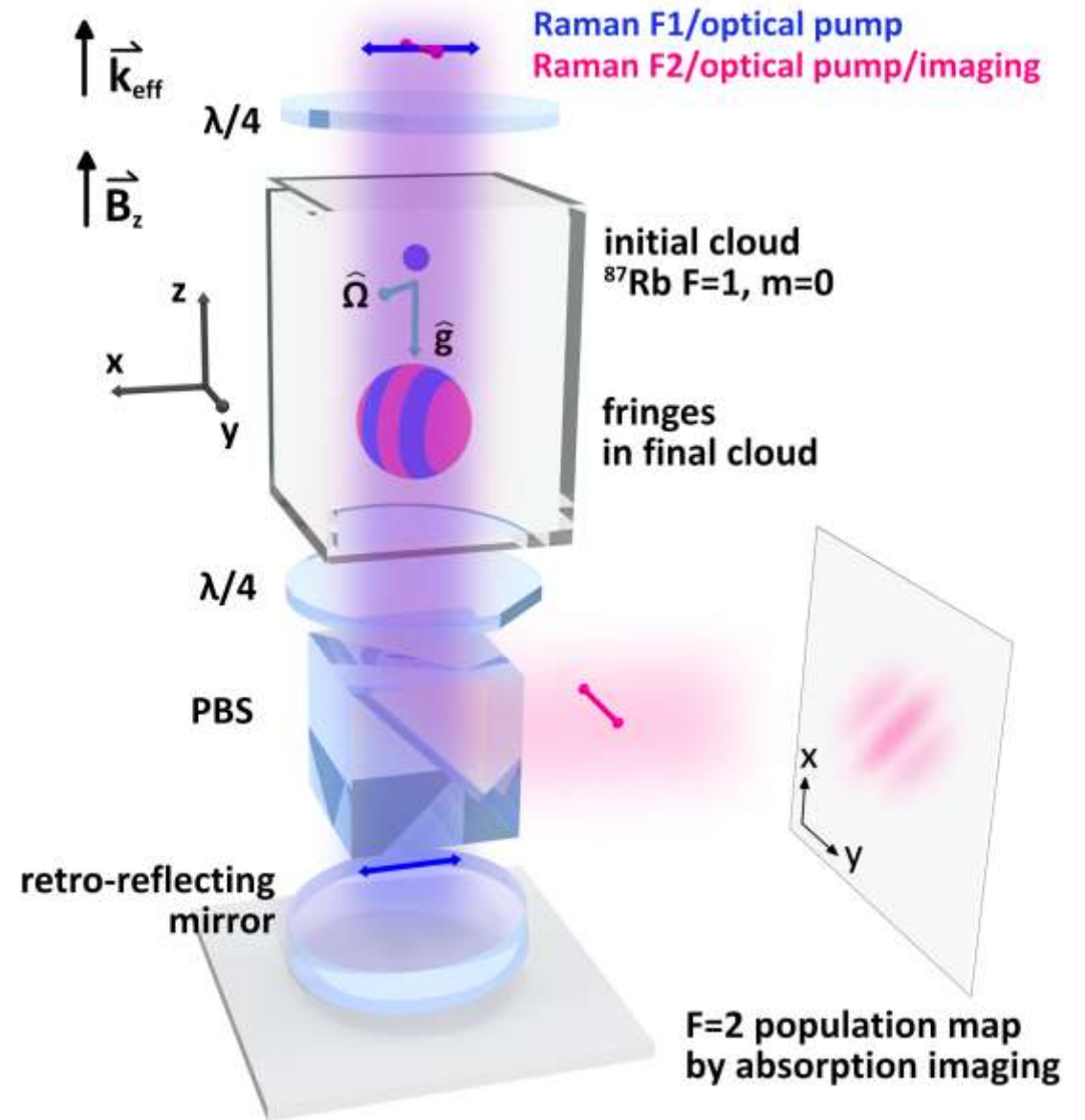


# Inertial sensing with point-source atom interferometry for interferograms with less than one fringe

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John Kitching<sup>1</sup>, and Elizabeth A. Donley<sup>1</sup>

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# Rotation measurement with interferometry

"The Feynman path integral approach to atomic interferometry. A tutorial," Pippa Storey and Claude Cohen-Tannoudji, Journal de Physique II, EDP Sciences, 1994, 4 (11), pp.1999-2027.

## Sagnac phase shift

$$\delta\varphi_{\text{photon}} = \frac{2\omega_0}{c^2} A\Omega \quad \text{Light wave}$$

$$\delta\varphi_{\text{atom}} = \frac{2M}{\hbar} A\Omega \quad \text{Matter wave}$$

A: Sagnac area

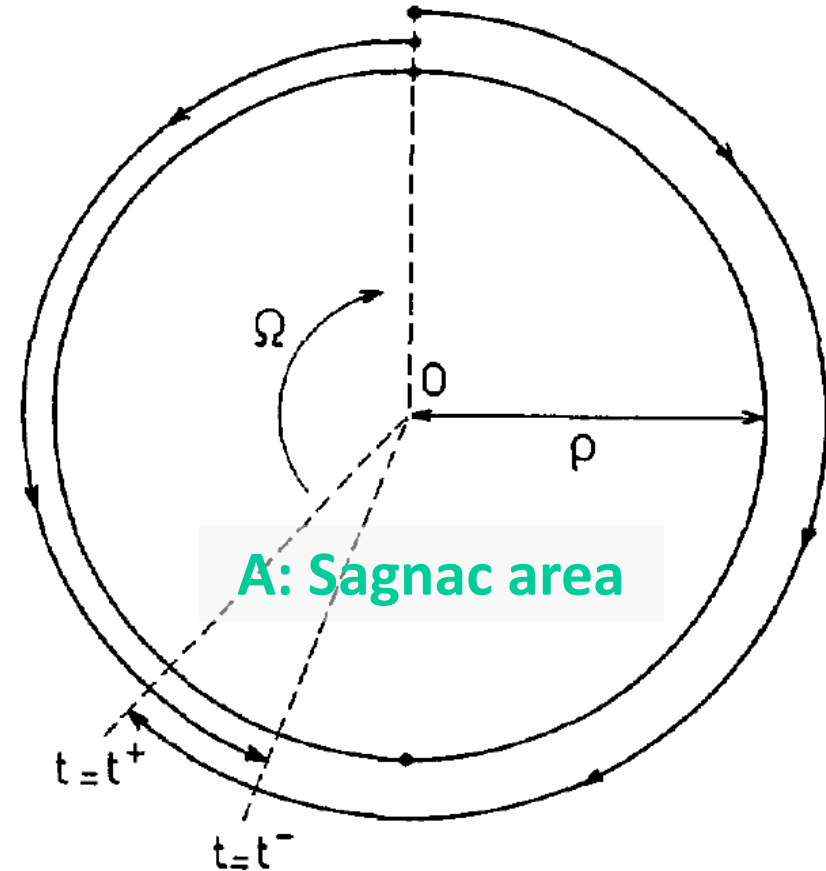
$\Omega$ : rotation rate

M: mass of atom

$\omega_0$ : angular frequency of light

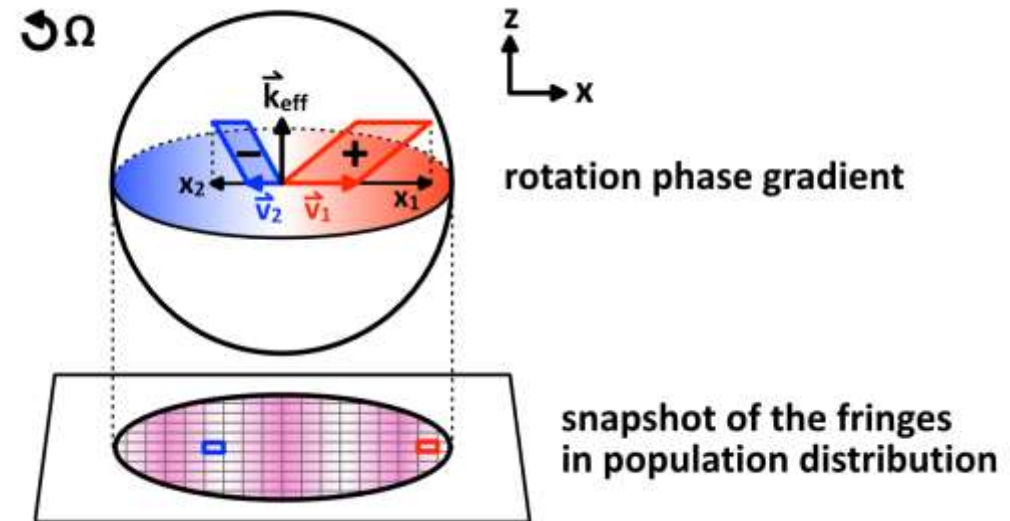
c: speed of light

$\hbar$ : reduced Plank constant



## Point source atom interferometry (PSI)

1. PSI is a parallel operation of many different Sagnac interferometers.
2. PSI enables direct rotation measurement without ambiguity.
3. PSI resolves a rotation vector in a plane with high dynamic range.



$$\varphi_{\Omega} = \frac{2m}{\hbar} \vec{\Omega} \cdot \vec{A} \quad \text{rotation phase}$$

A: Sagnac area (depends on atom velocity)

## Acknowledgement

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Liz Donley

Azure Hansen

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Eugene Ivanov

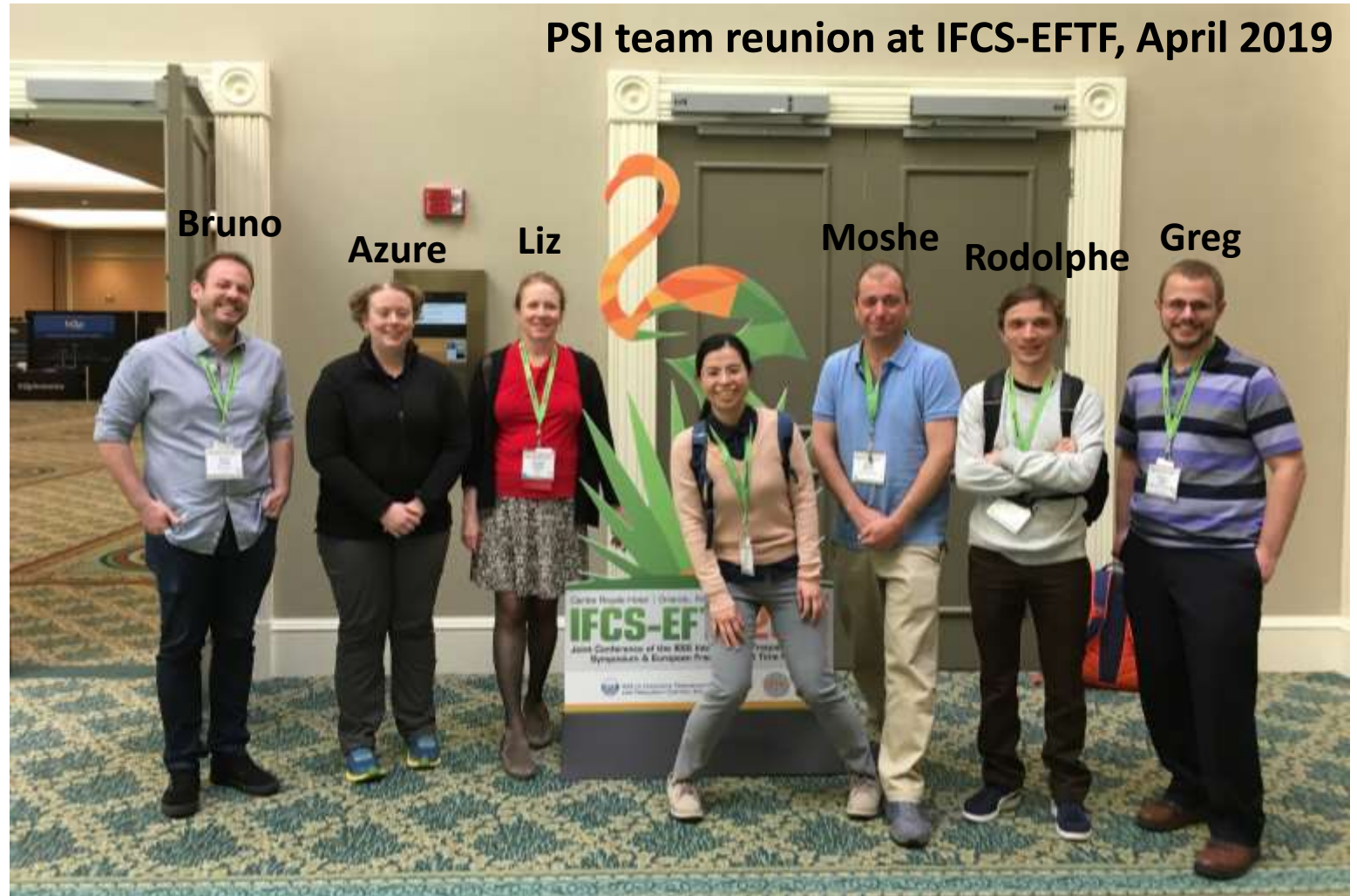
John Kitching

William McGehee

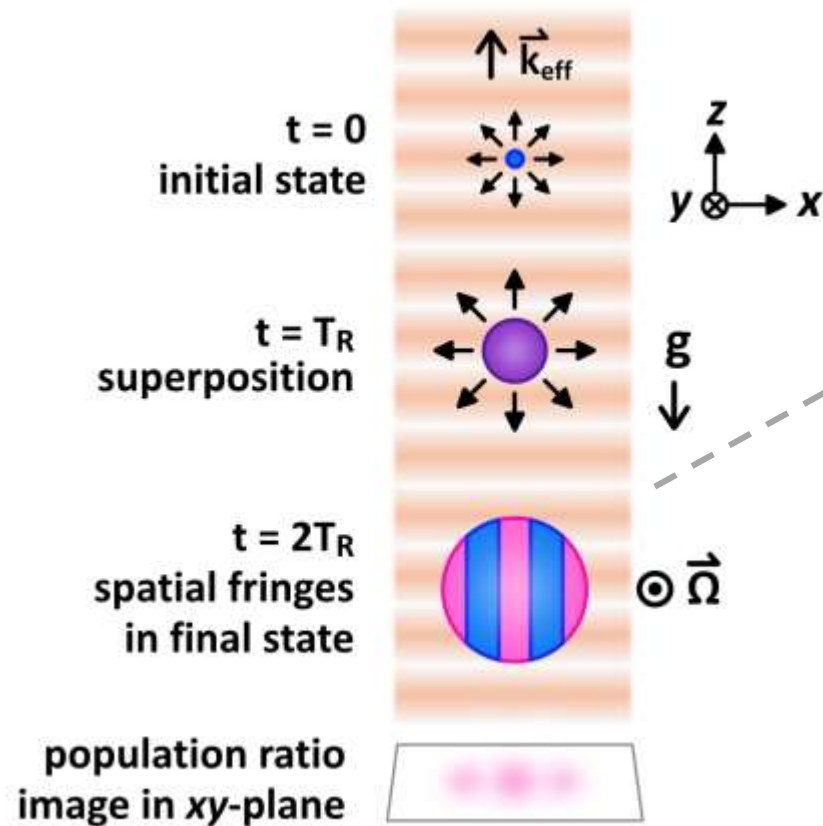
Bruno Pelle

Stefan Riedl

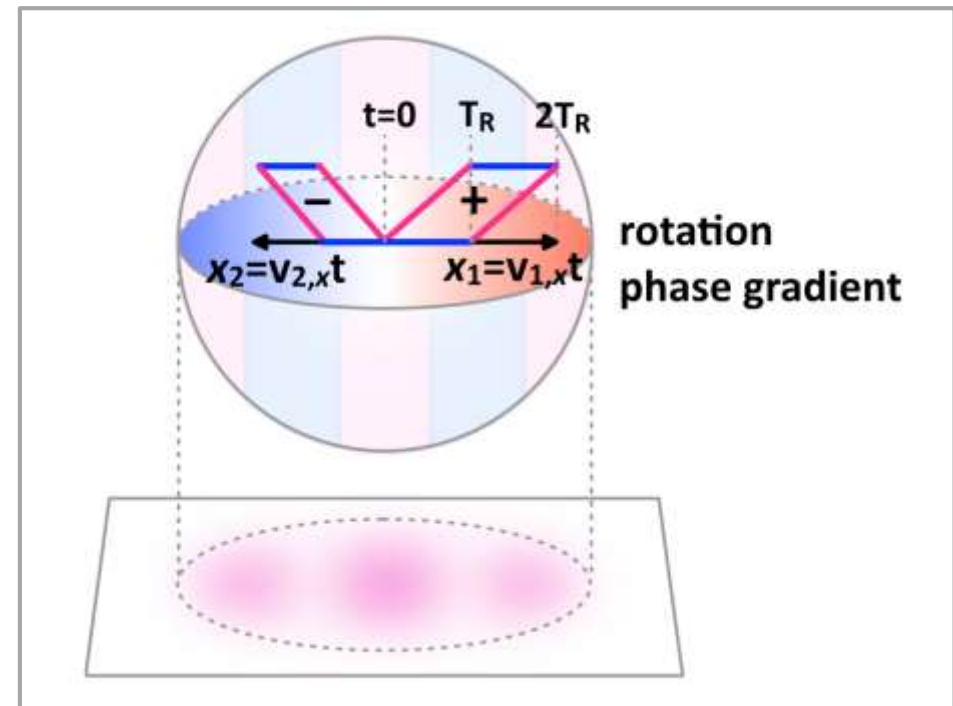
Moshe Shuker



# Expanding point-like atomic source



Different velocity classes are mapped to different locations in the image plane with the position-velocity correlation of a point source.





# Rotation phase gradient

Interferometer phase shift of the Raman  $\pi/2$ - $\pi$ - $\pi/2$  pulse sequence:

$$\varphi = \varphi_a + \varphi_0 + \varphi_\Omega$$

$$\varphi_a = \vec{k}_{\text{eff}} \cdot \vec{a} T_R^2 \text{ acceleration}$$

$$\varphi_\Omega = 2\vec{k}_{\text{eff}} \cdot (\vec{\Omega} \times \vec{v}) T_R^2 \quad \text{velocity dependent!}$$

$$\varphi_0 = \text{laser phase}$$

$T_R$  : time between Raman laser pulses

Position-velocity correlation of an **ideal** expanding point source:  $\vec{v} = \frac{\vec{r}}{T_{\text{ex}}}$   
 $T_{\text{ex}}$  : cloud expansion time



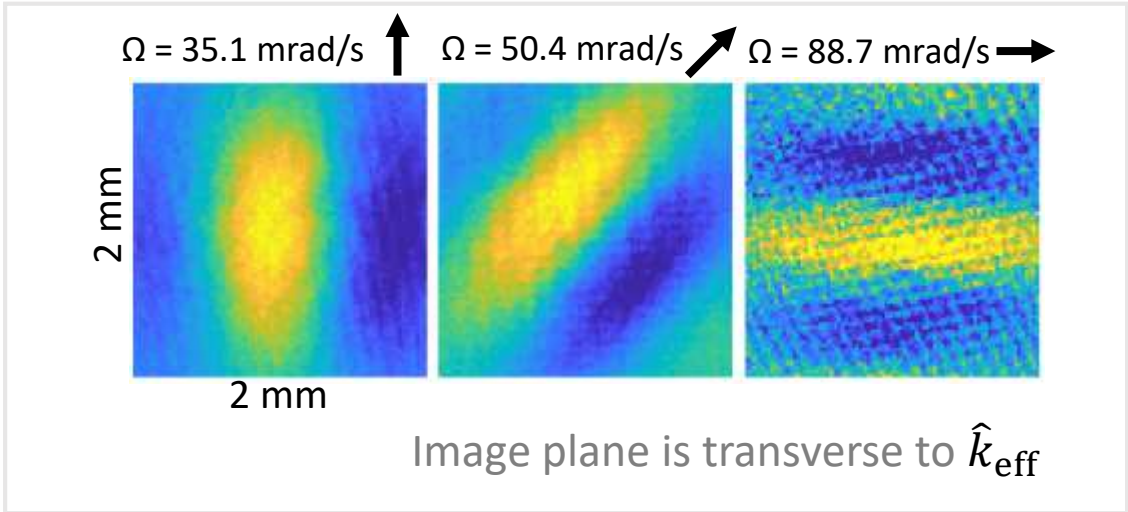
$\varphi_\Omega(\vec{r}) = \vec{k}_\Omega \cdot \vec{r}$  **rotation phase gradient**

$$\vec{k}_\Omega = \frac{2T_R^2}{T_{\text{ex}}} k_{\text{eff}} \Omega \hat{n}$$

$$\hat{n} = \hat{k}_{\text{eff}} \times \hat{\Omega}$$

$\varphi_a$  and  $\varphi_0$  : constant across cloud

- Fringe period : rotation rate
- Fringe direction : direction of the rotation vector
- Fringe phase : acceleration



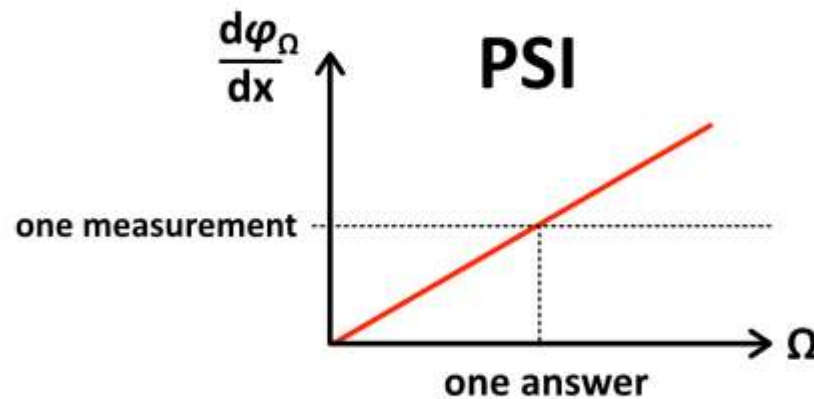
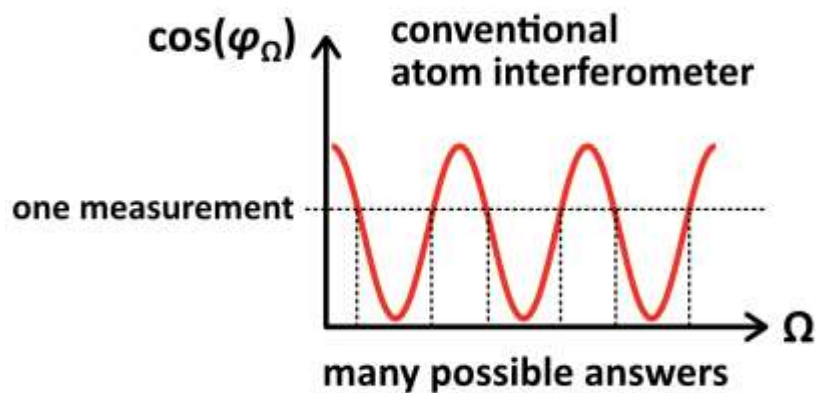
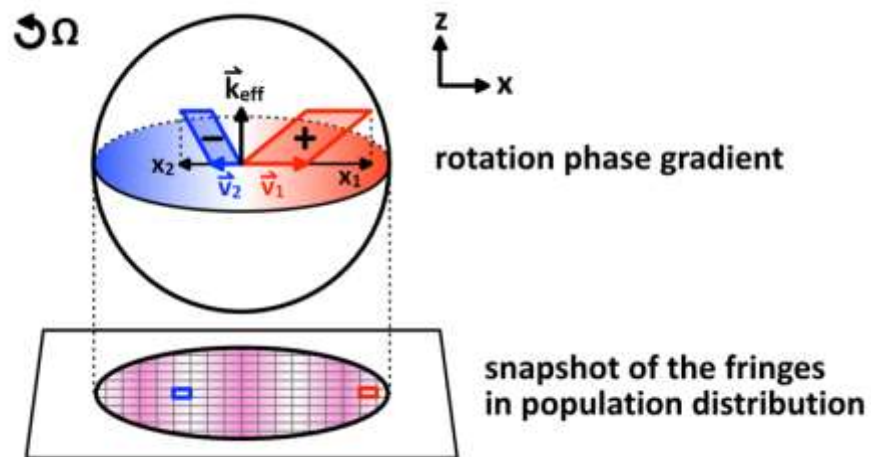
# Unambiguous rotation measurement

Conventional:

Population ratio  $\rightarrow$  phase  $\rightarrow$  rotation

PSI:

Fringe period  $\rightarrow$  phase gradient  $\rightarrow$  rotation



## PSI references

Multiaxis inertial sensing with long-time point source atom interferometry

[Enhanced Atom Interferometer Readout through the Application of Phase Shear](#)

Point source atom interferometry with a cloud of finite size

[Single-source multiaxis cold-atom interferometer in a centimeter-scale cell](#)

Concept study and preliminary design of a cold atom interferometer for space gravity gradiometry

[Rotation sensing with improved stability using point-source atom interferometry](#)

A Multi-Axis Atom Interferometer Gyroscope Based on a Grating Chip

[High sensitivity multi-axes rotation sensing using large momentum transfer point source atom interferometry](#)

Robust inertial sensing with point-source atom interferometry for interferograms spanning a partial period

## Authors

Susannah M. Dickerson, Jason M. Hogan, Alex Sugarbaker, David M. S. Johnson, and Mark A. Kasevich

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Gregory W. Hoth, Bruno Pelle, Stefan Riedl, John Kitching, and Elizabeth A. Donley

[Yun-Jih Chen, Azure Hansen, Gregory W. Hoth, Eugene Ivanov, Bruno Pelle, John Kitching, and Elizabeth A. Donley](#)

A. Trimeche, B. Battelier, D. Becker, A. Bertoldi, P. Bouyer, C. Braxmaier, E. Charron, R. Corgier, M. Cornelius, K. Douch, N. Gaaloul, S. Herrmann, J. Müller, E. Rasel, C. Schubert, H. Wu and F. Pereira dos Santos

[Chen Avinadav, Dmitry Yankelev, Moshe Shuker, Ofer Firstenberg, and Nir Davidson](#)

Xiaojie Li, Zhixin Meng, Peiqiang Yan, Jianwei Zhang, Yanying Feng

[Jinyang Li, Gregório R. M. da Silva, Wayne C. Huang, Mohamed Fouda, Timothy L. Kovachy, and Selim M. Shahriar](#)

Yun-Jih Chen, Azure Hansen, Moshe Shuker, Rodolphe Boudot, John Kitching, and Elizabeth A. Donley

## Link

<https://doi.org/10.1103/PhysRevLett.111.083001>

<https://doi.org/10.1103/PhysRevLett.111.113002>

<https://doi.org/10.1063/1.4961527>

<https://doi.org/10.1103/PhysRevApplied.12.014019>

<https://doi.org/10.1088/1361-6382/ab4548>

<https://doi.org/10.1103/PhysRevA.102.013326>

<https://doi.org/10.1109/INERTIAL48129.2020.9090092>

<https://arxiv.org/abs/2006.13442>

<https://doi.org/10.1364/OE.399988>

## Year

2013

2013

2016

2019

2019

2020

2020

2020

2020

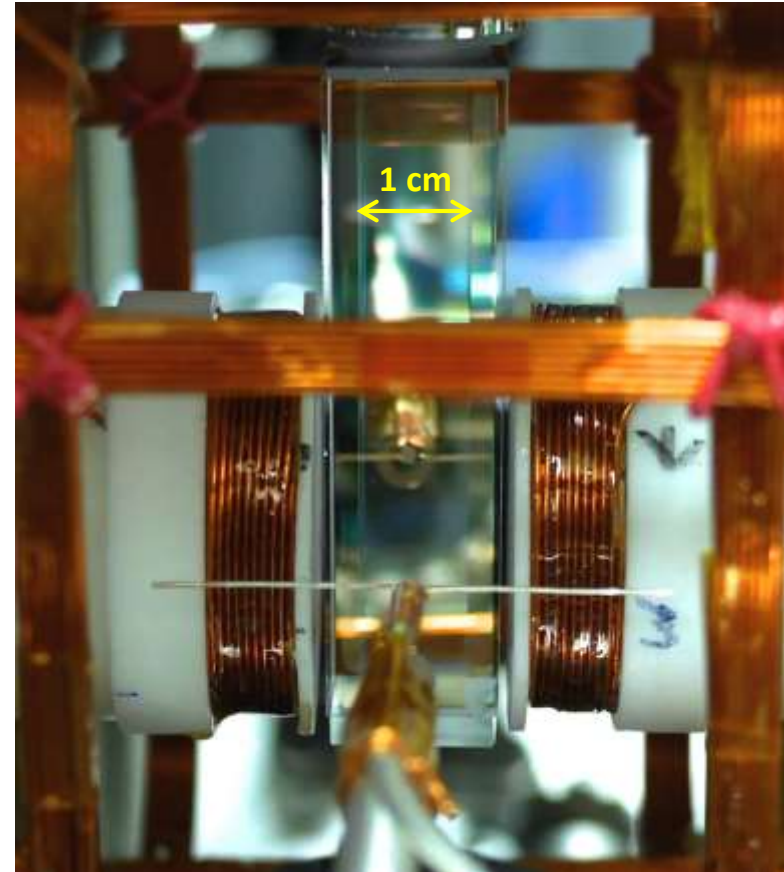


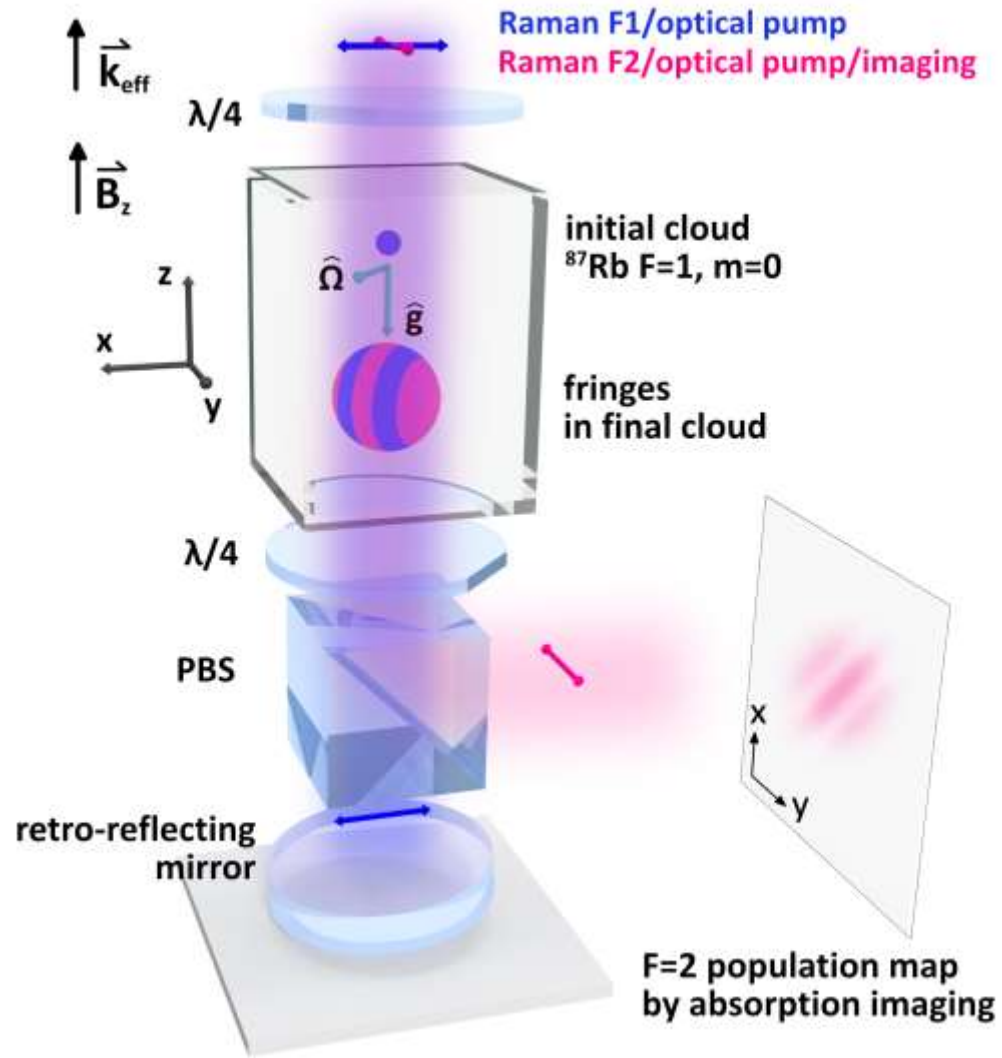
## NIST ADI group's PSI gyro

Experimental scheme:

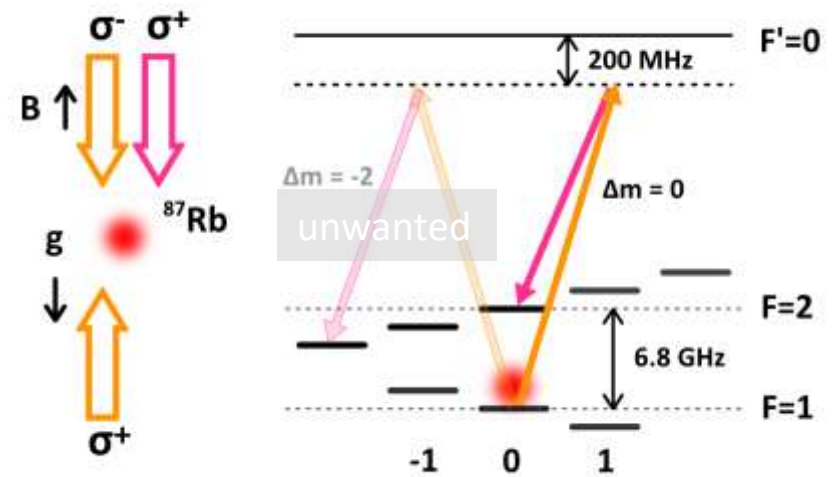
1. MOT, compressed MOT, and molasses
2. State preparation
3. Raman interrogations
4. State-selective absorption imaging

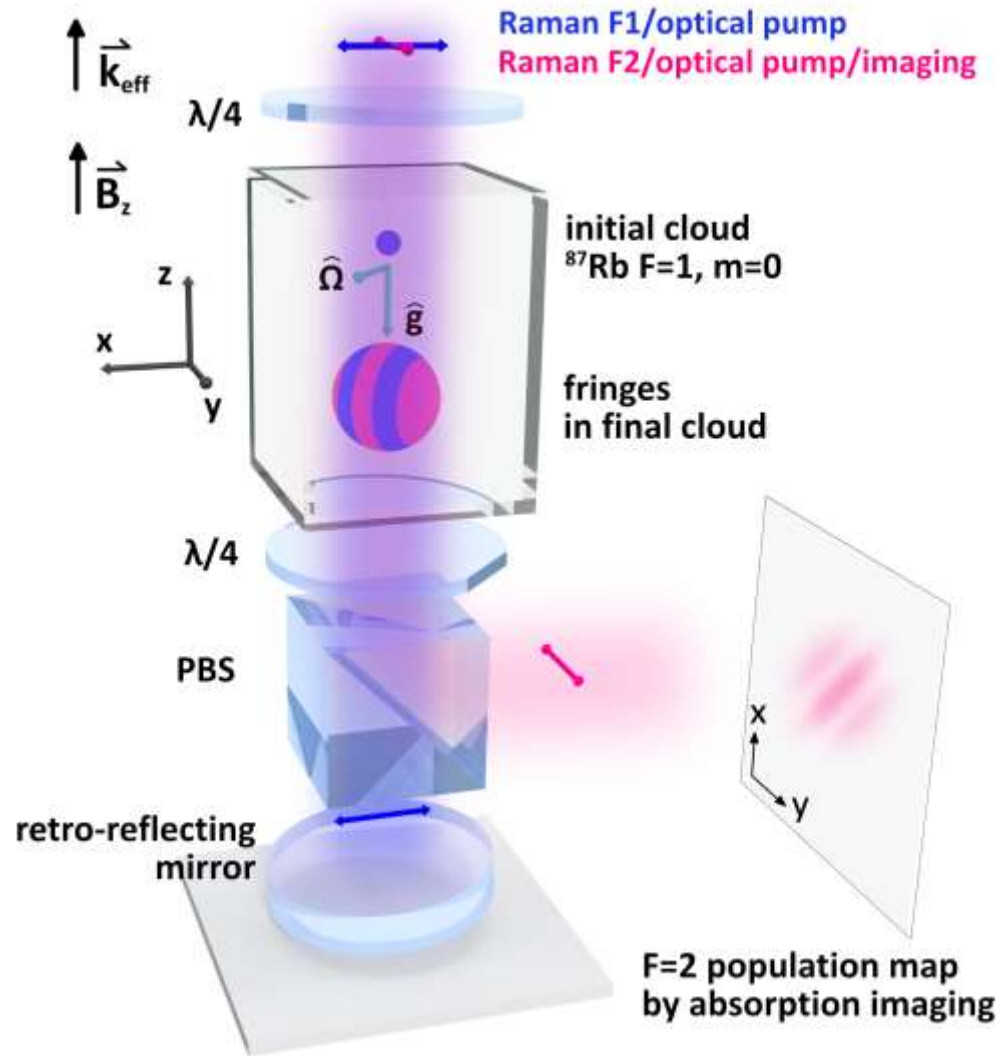
Repetition rate: 5 to 10 Hz





### two-photon Raman transition



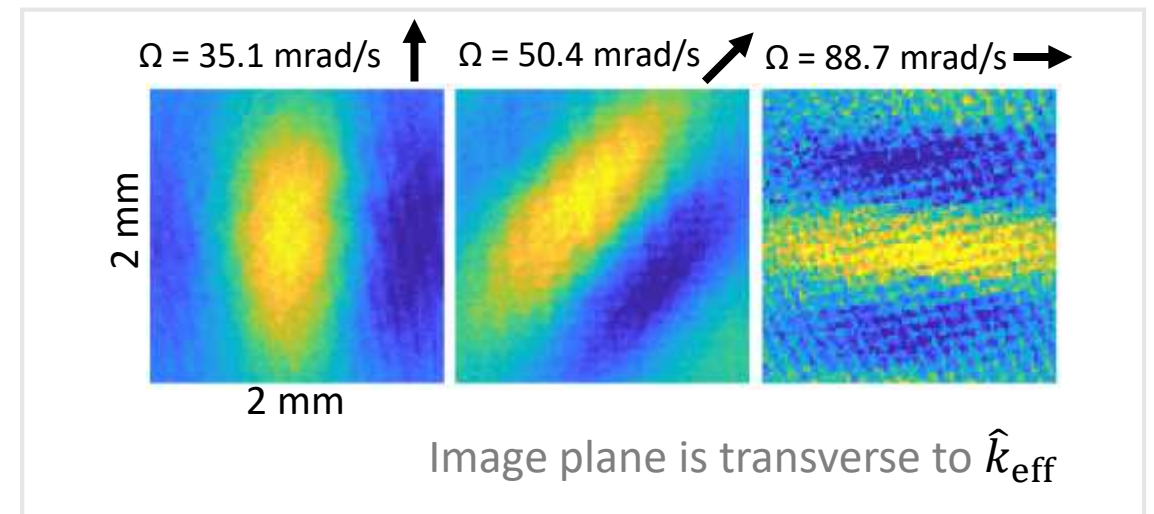


## Multi-axis sensitivity

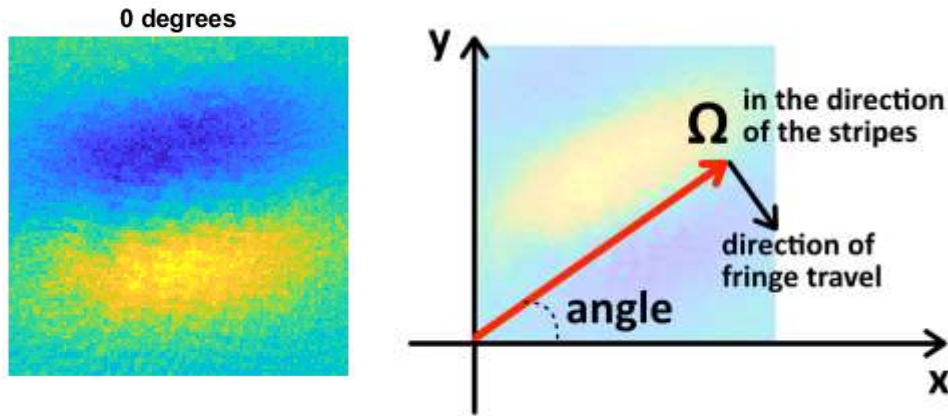
The effective wave vector  $\vec{k}_{\text{eff}}$  is in +z direction. Cloud is imaged in xy-plane.

PSI measures:

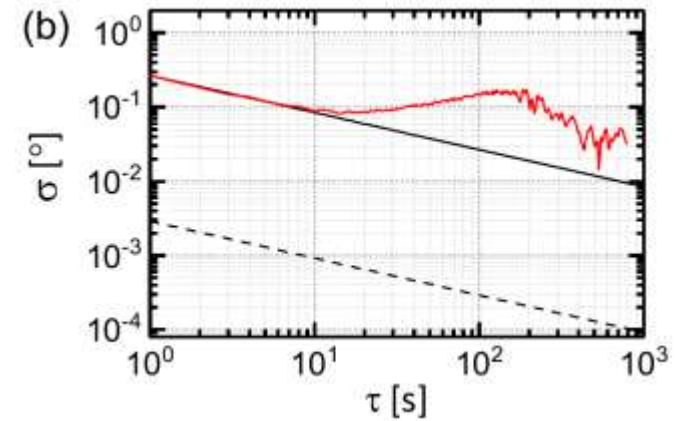
1. Acceleration in the z-axis
2. Rotation projected onto the xy-plane



# Experimental demonstration of the two-dimensional rotation measurement with point source atom interferometry



- Fringe period : rotation rate
- Fringe direction : direction of the rotation vector
- Fringe phase : acceleration



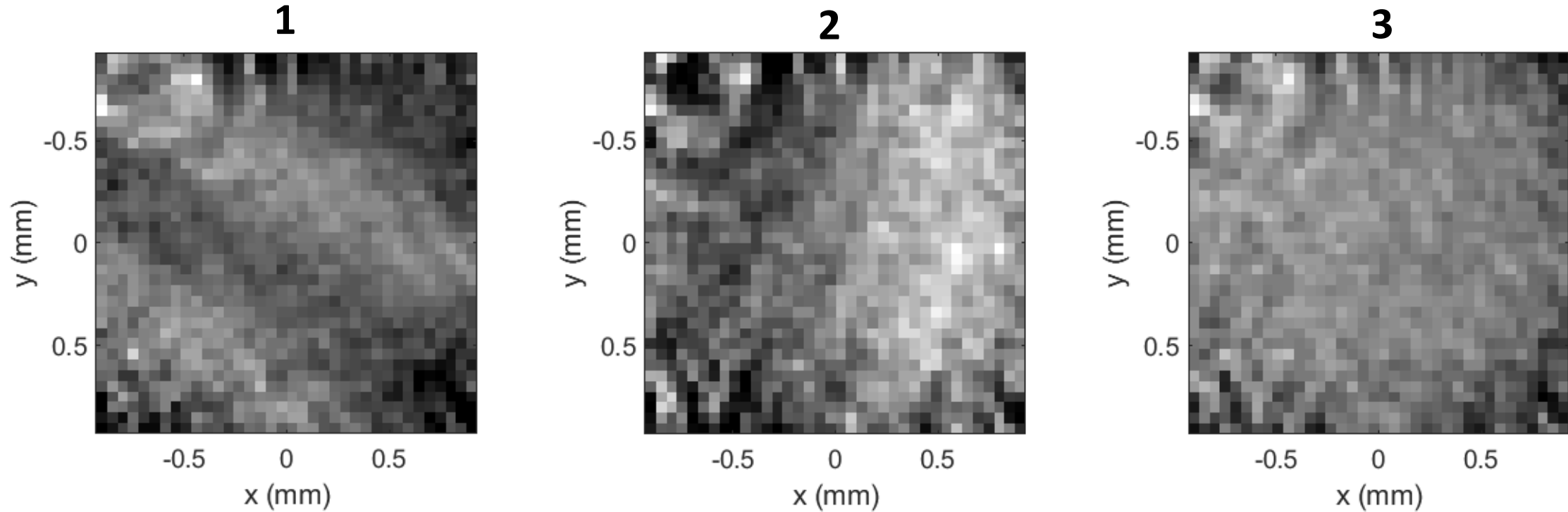
**Direction of the rotation vector in a plane,  $0.27^\circ$  at  $\tau = 1$  s**

[doi.org/10.1103/PhysRevApplied.12.014019](https://doi.org/10.1103/PhysRevApplied.12.014019)

Application ideas:

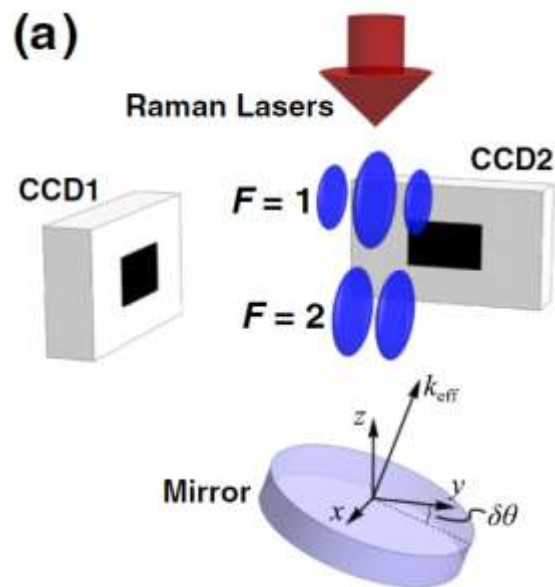
1. navigation, gyrocompassing
2. Fundamental physics, relativistic precession measurement

## Measurement of small rotation rates

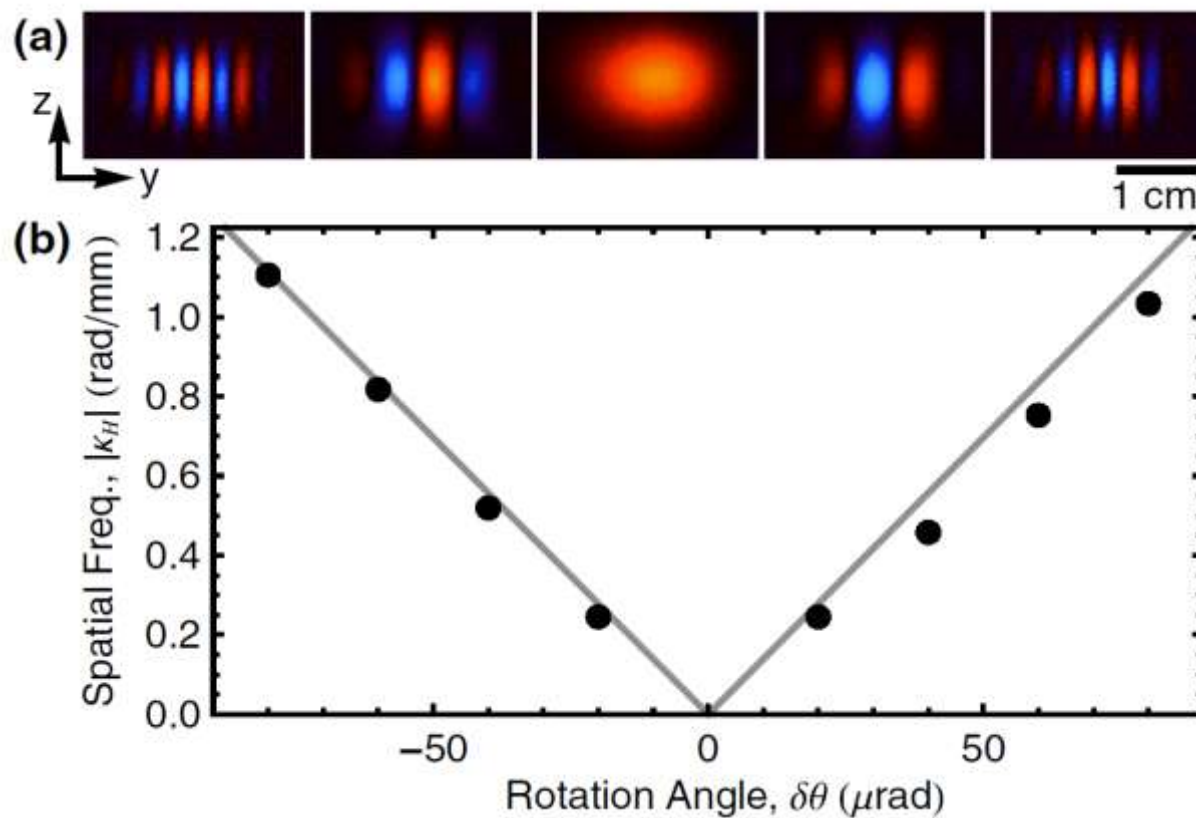


**How to measure rotation in case 3?**

## Method in literature: phase shear



“Enhanced Atom Interferometer Readout through the Application of Phase Shear,” A. Sugarbaker, S. M. Dickerson, J. M. Hogan, D. M. S. Johnson, and M. A. Kasevich, PRL **111**, 113002 (2013).





# Method in literature: ellipse fitting

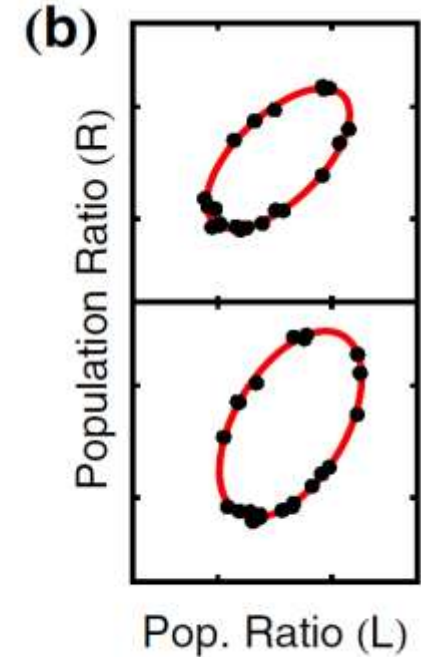
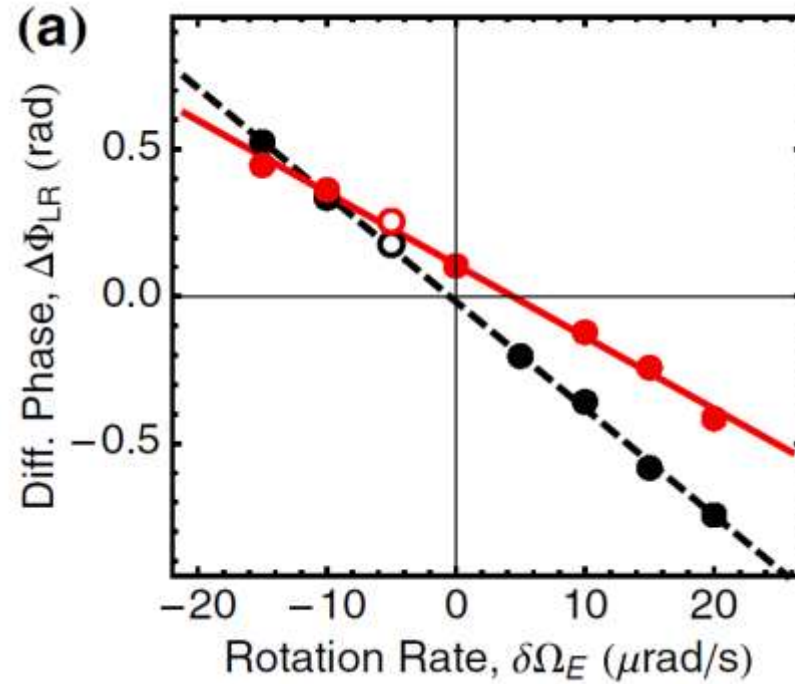
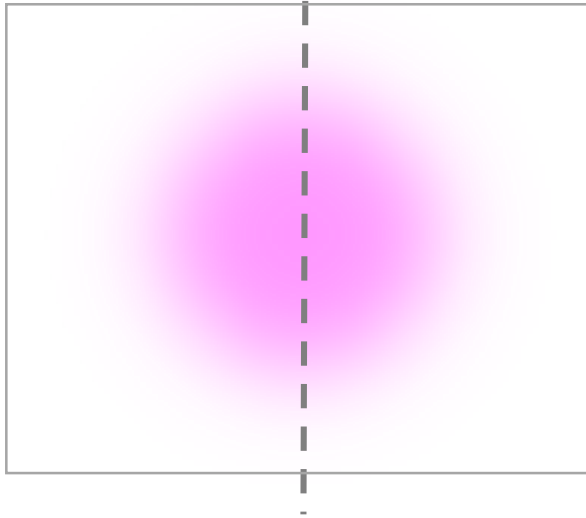
Ellipse equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Phase difference:

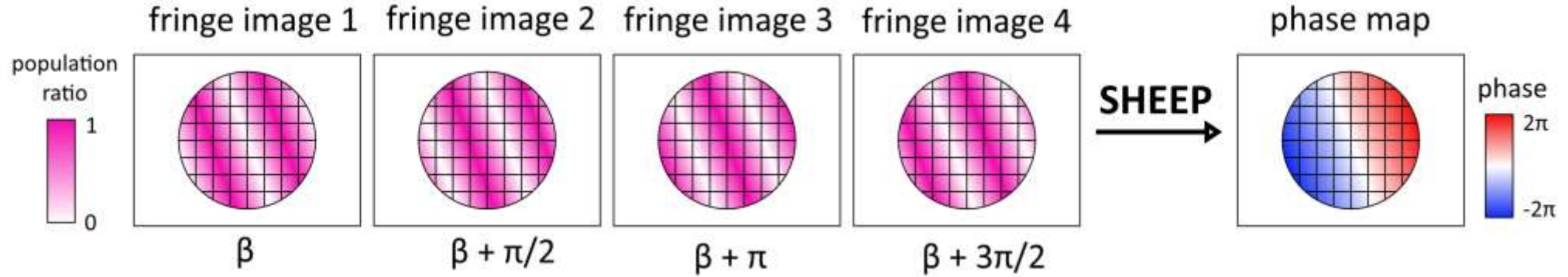
$$\Delta\Phi_{LR} = \cos^{-1}(-B/2\sqrt{AC})$$

Left population ratio = x | Right population ratio = y



“Multiaxis Inertial Sensing with Long-Time Point Source Atom Interferometry,” S. M. Dickerson, J. M. Hogan, A. Sugarbaker, D. M. S. Johnson, M. A. Kasevich, PRL **111**, 083001 (2013).

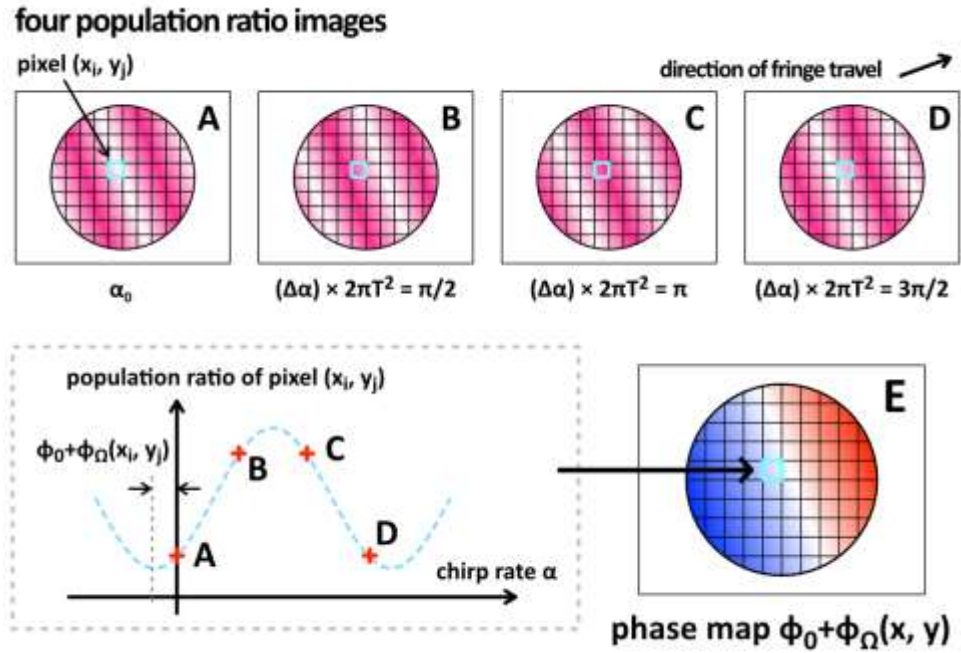
# Our approach



SHEEP—**S**imple, **H**igh dynamic range, and **E**fficient **E**xtraction of **P**hase map

<https://doi.org/10.1364/OE.399988>

# Phase shifting interferometry



## INPUT

Population ratio maps:

$$A(x_i, y_j) = \frac{1}{2} \{1 - c \cos[\phi_0 + \phi_\Omega(x_i, y_j)]\}$$

$$B(x_i, y_j) = \frac{1}{2} \{1 + c \sin[\phi_0 + \phi_\Omega(x_i, y_j)]\}$$

$$C(x_i, y_j) = \frac{1}{2} \{1 + c \cos[\phi_0 + \phi_\Omega(x_i, y_j)]\}$$

$$D(x_i, y_j) = \frac{1}{2} \{1 - c \sin[\phi_0 + \phi_\Omega(x_i, y_j)]\}$$

## OUTPUT

Phase map

$$E(x_i, y_j) = \tan^{-1} \frac{B(x_i, y_j) - D(x_i, y_j)}{C(x_i, y_j) - A(x_i, y_j)} + m\pi$$

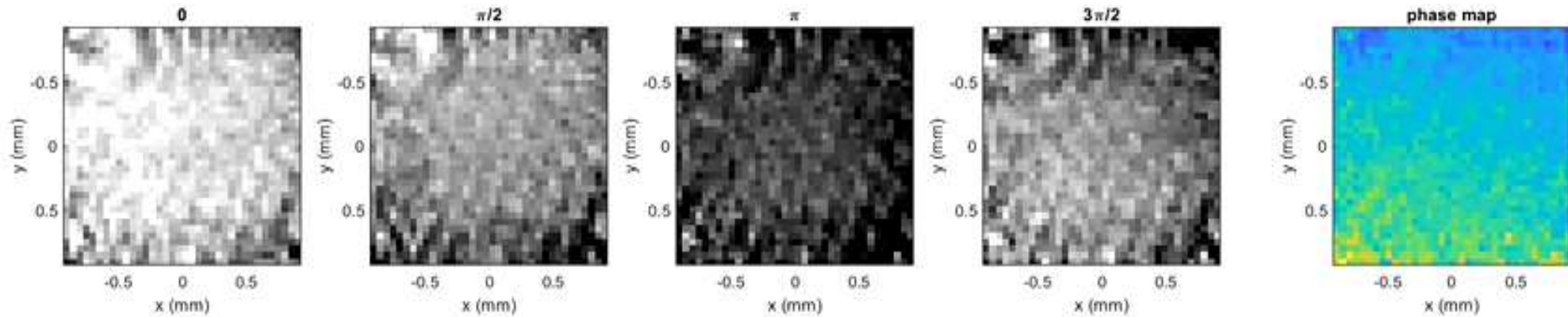
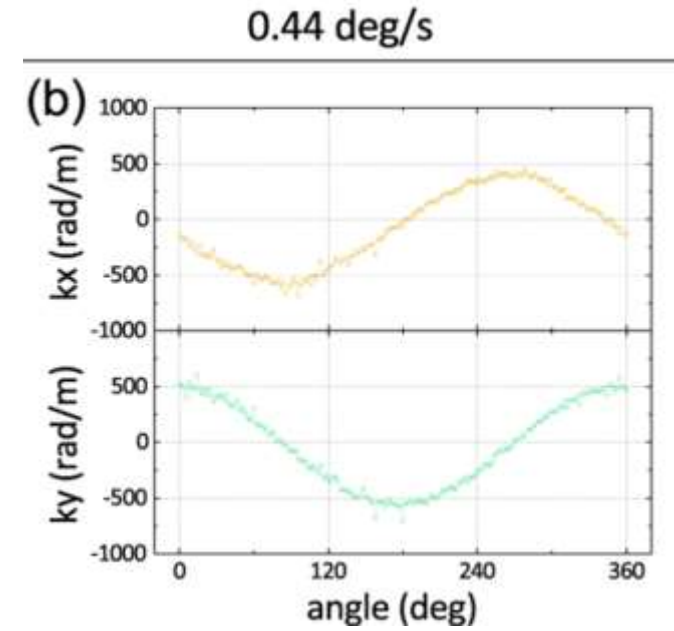
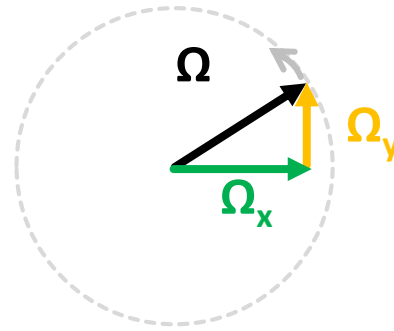
$$= \phi_0 + \phi_\Omega(x_i, y_j)$$

↑  
Stitching the  
tan<sup>-1</sup> output

- Rotation phase gradient : average of the differences in pixel values of adjacent pixels
- Acceleration phase : average of the entire phase map

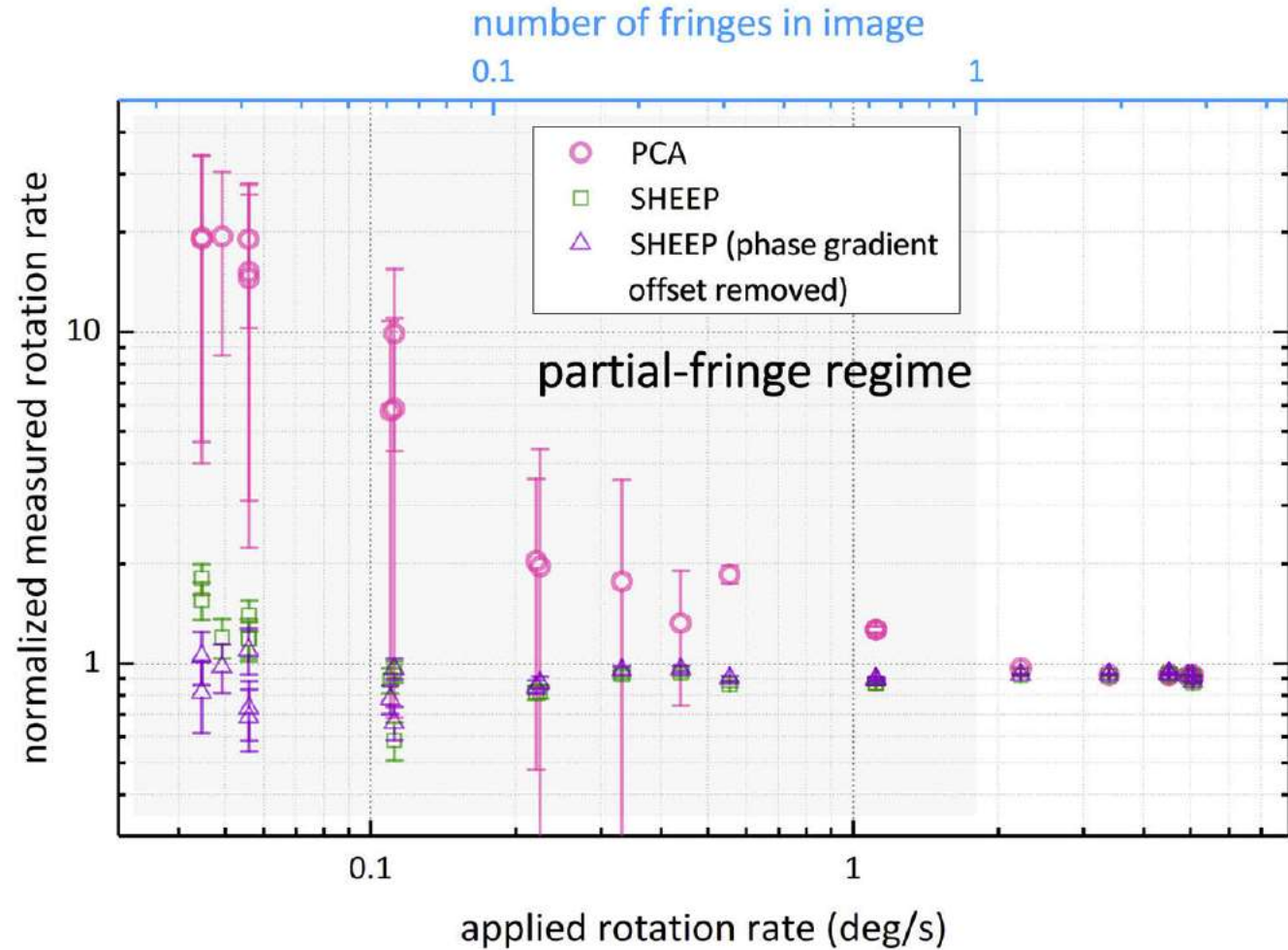
# Case 3 revisited with SHEEP method

1. The direction of the rotation traces a 360° range in 2° steps.
2. X- and y- components of the rotation vector vary sinusoidally.



# Dynamic range

- Robust and accurate
- Performs well over a wide range of rotation rates
- Lowest rotation rate  $0.045^\circ/\text{s}$  at  $T = 7.8\text{ ms}$ , corresponding to 0.025 fringes in the image.

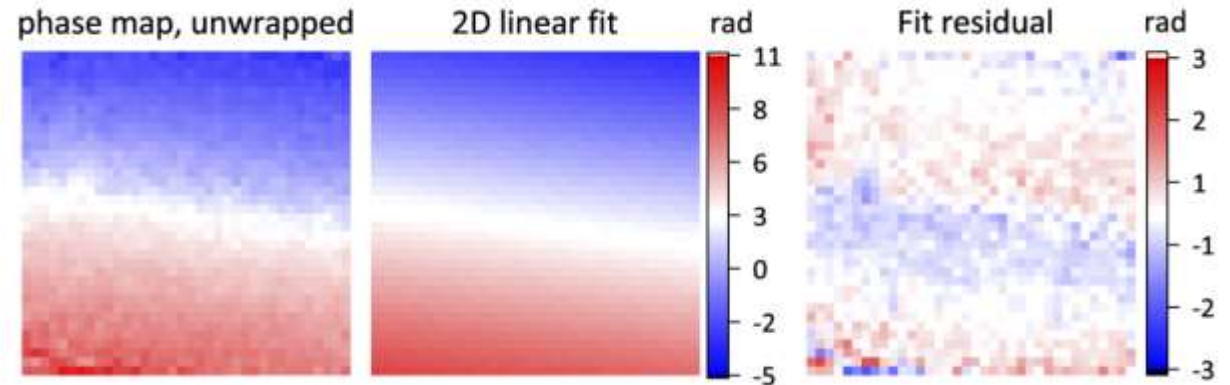




# Discussion

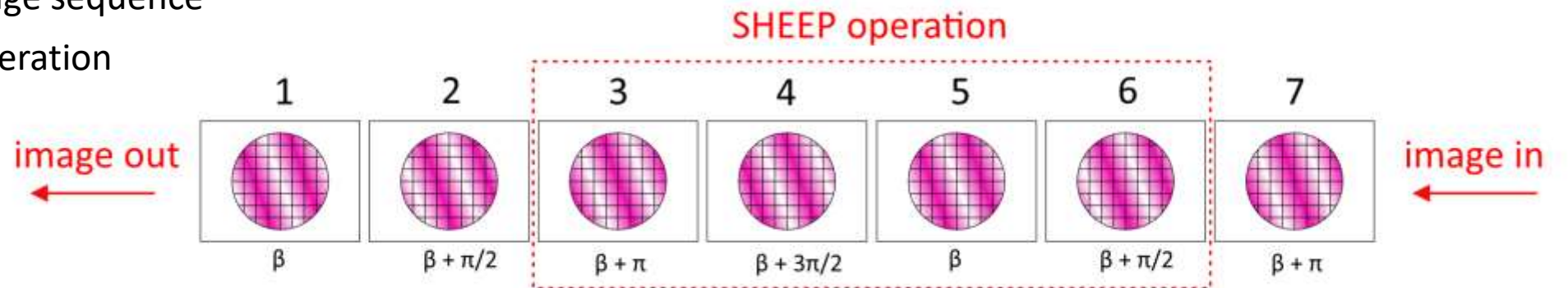
## 1. Vibrations

- Spurious phase between images
- Sensitivities:
  - Acceleration  $\propto T^2$
  - Rotation  $\propto T$
- Closed-loop operation



## 2. Bandwidth

- Three-image sequence
- Queue operation





## Conclusions:

1. PSI is simple compared to other atom interferometer techniques.
2. PSI measures two rotation components and one acceleration component at the same time.
3. PSI enables rotation measurement without ambiguity.
4. PSI has a high dynamic range.
5. The SHEEP method returns a phase map with rotation and acceleration information.
6. The SHEEP method does not require a contrast calibration, and it is applicable from the multiple-fringe regime well into the partial-fringe regime.

