Automated detection of contextuality proofs with intermediate numbers of observables

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Contextual geometries

$X \otimes I - I \otimes X - X \otimes X$

$I \otimes Y - Y \otimes I - Y \otimes Y$

$X \otimes Y - Y \otimes X - Z \otimes Z$

Finite geometries $G = (\mathcal{O}, L)$ parametrized by observables $O \in \mathcal{O}$ with $\forall l \in L, l \neq \varnothing, l \subset \mathcal{O}$ such that

$$\forall l \in L, \left(\forall \{O_1, O_2\} \subset l, O_1O_2 = O_2O_1\right) \wedge \left(\prod_{O \in l} O = \pm I\right).$$

They are contextual if

$$\nexists f : \mathcal{O} \to \{-1, 1\} \ / \ \forall l \in L, \prod_{O \in l} f(O) I = \prod_{O \in l} O. \qquad (1)$$

A linear problem

$$(\{-1, 1\}, \times) \to (\mathbb{Z}_2, +)$$

- Let A be the incidence matrix of G.
- Let e be the evaluation vector with $|L|$ entries such that, for the entry e_l corresponding to $l \in L$, $e_l I = \Pi_{O \in l} O$

Eq.(1) can be rewritten as $\nexists x \in (\mathbb{Z}_2)^n \ / \ Ax = e$

Here x is in in relation with f as follows: the entry corresponding to $l \in L$ would be $x_l = \Pi_{O \in l} f(O)$.

Symplectic space

A space encoding the Pauli group

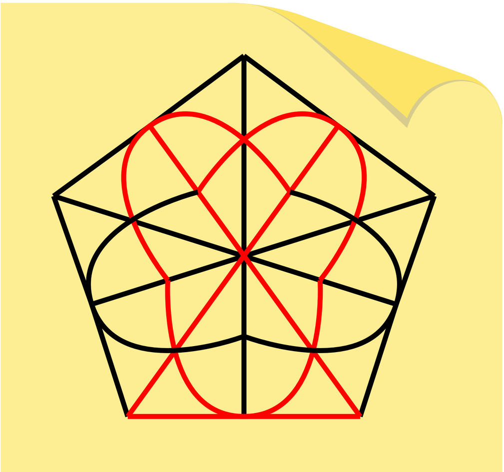
$$(0, 1, 1, 1) \leftrightarrow X \otimes Y$$

$((0, 0) \leftrightarrow I, (0, 1) \leftrightarrow X, (1, 0) \leftrightarrow Z, (1, 1) \leftrightarrow Y)$

With an inner product encoding commutativity

$$\left.\begin{matrix} p_1 \leftrightarrow O_1 \\ p_2 \leftrightarrow O_2 \end{matrix}\right\} \Rightarrow \left(\langle p_1 | p_2 \rangle = 0 \Leftrightarrow O_1O_2 = O_2O_1\right)$$

We chose the lines of the geometry G to be subspaces of the symplectic space.



Results

A Magma program [**dB21**] generates all elements of five families of geometries and automatically establishes their contextuality:

Geometries	$n = 2$	$n = 3$	$n = 4$	$n = 5$
Lines	C(1)	C(1)	C(1)	C(1)
Generators	C(1)	**N(1)**	**N(1)**	**N(1)**
Hyperbolics	C(10)	**C(36)**	**C(136)**	**C(528)**
Elliptics	N/A (6)	**C(28)**	**C(120)**	**C(496)**
Perpsets	N(15)	**N(63)**	**N(255)**	**N(1023)**

n: Number of qubits of the system
N: Non-contextual
C: Contextual
(k): There are k instances in this family
N/A: Not applicable
bold: New results

References

[**dB21**] H. de Boutray Magma-contextuality: Automated detection of contextuality proofs with intermediate numbers of observables. *QuantCert*, `https://quantcert.github.io/Magma-contextuality/`.

[**dHGM21**] H. de Boutray, F. Holweck, A. Giorgetti, and P.-A. Masson, Automated synthesis of contextuality proofs from subspaces of symplectic polar spaces. *arXiv:2105.13798 [math-ph, physics:quant-ph]*, May 2021.

[Hol21] F. Holweck. Testing Quantum Contextuality of Binary Symplectic Polar Spaces on a Noisy Intermediate Scale Quantum Computer. *arXiv:2101.03812 [math-ph, physics:quant-ph]*, January 2021.

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