HIGHER-GRADE ELASTIC BEAMS TO PROBE SCALING EFFECTS IN SOLIDS

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Abstract

The mechanical behavior of micrometer-sized structures differs from that of usual macroscopic objects. Their surface plays a key role in the overall behavior of the structure. Higher-grade elasticity, and especially second-strain gradient elasticity seems particularly suited to describe the observed size effects, and in particular the strong surface couplings at stake. These frameworks however involve a large number of constitutive parameters whose experimental identification seems today far from attainable. This contribution thus targets the development of robust beam equations for materials featuring higher-grade elasticity, which solve the deficiencies of the available theories, provides solutions involving on a limited number of parameters, and paves the way to the experimental identification of higher-grade elasticity parameters.

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1. INTRODUCTION

Because of their high surface over volume ratio, the mechanical behavior of micrometer-sized structures differs from that of usual macroscopic objects. Two main families of mechanical size effects are observed for solids :

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- a material parameter (for instance a stiffness) depends on the specimen size [1] ;

- strong surface couplings are observed. Their surface plays a key role, and this property has been proposed to devise micromechanical sensors of environmental changes [2]. In particular, a significant effort has been put on the development of biological sensors, based on cantilevers operated in static mode (see Fig. 1), and the published results are highlighting the need for a more basic understanding of coupled surface phenomena [3].

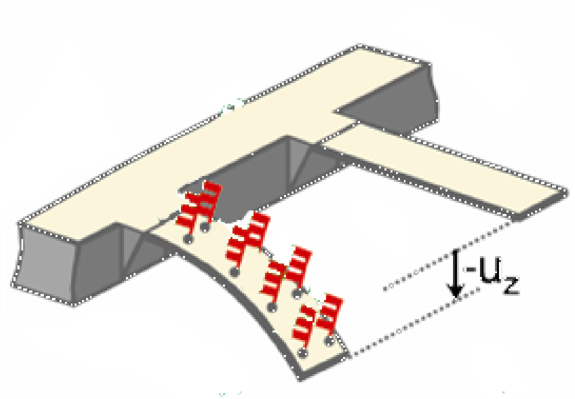

Figure 1: Principle of a cantilever-based sensor : a surface chemical modification induces a mechanical deformation (adapted from [2]).

It has been shown that the use of second strain gradient elasticity may provide a framework describing both these size effects, at the cost of additional modeling parameters which are yet to be identified [4]. Focusing on beams allows to simultaneously encompass most of the reported experimental results and to lower the complexity. The available higher-order beam theories are however only based on kinematic assumptions, so that they cannot render Poisson effect for instance [5]. The correct beam stiffness for vanishing higher-order elasticity parameters is sometimes recovered in a somehow arbitrary way, so that the consistency with the usual Cauchy material-based beam theory is questionable [6]. Refined higher-grade beam theories are therefore desirable, keeping their complexity as low as possible.

2. HIGHER-GRADE BEAM EQUATIONS

A method is proposed herein to build beam equations for materials featuring higher-grade elasticity. The proposed approach is based on the minimization of the constitutive equation gap in order to simultaneously satisfy kinematic and static conditions, including higher-order static admissibility conditions. It is shown that the resulting beam equations are consistent with the usual ones obtained with Cauchy materials when the dimensions are large enough and are yet tractable. The (few) driving parameters are obtained as closed-form expressions of the parameters of the three-dimensional constitutive law.

For second-strain gradient elasticity, the resulting beam equations are found to be non-local : the non-local kernel results from the stationarity condition, and the involved length parameter is again expressed as a simple function of the parameters of the three-dimensional law. The differences with first-strain gradient theories and the usual (kinematic) second-strain gradient approach are highlighted for several representative load cases, so that the scale range such refined theory is required is identified. The obtained solutions can also be compared to those obtained from asymptotic analysis [7] or from a Gurtin-Murdoch like approach [8], and the obtained results are

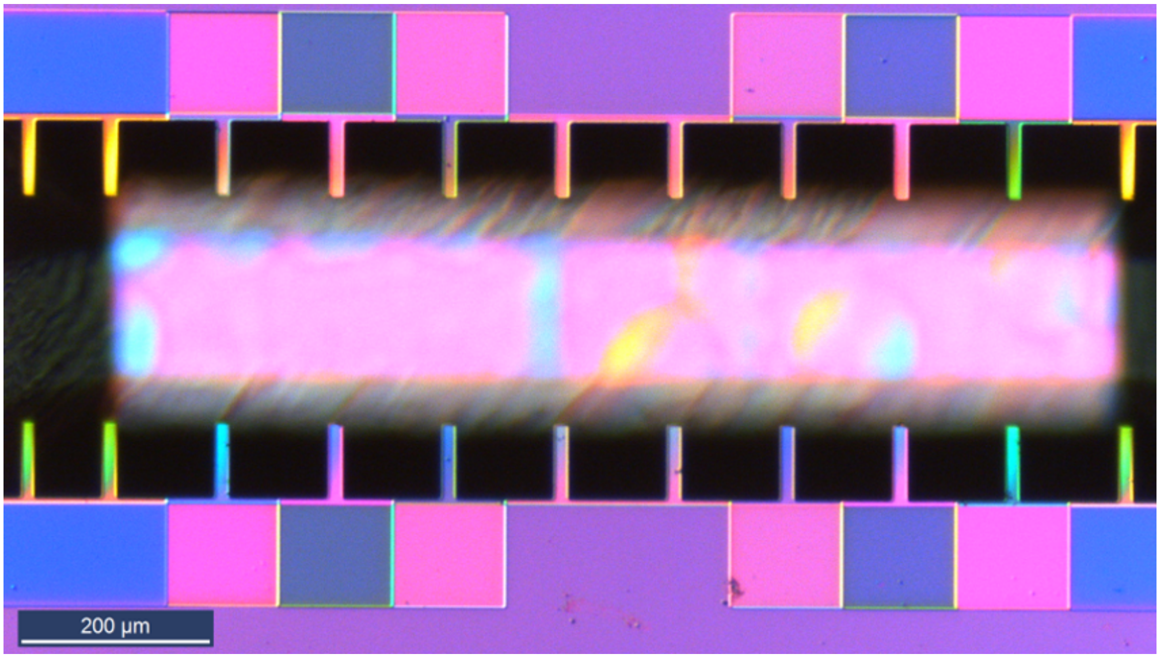

Figure 2: Optical view of a silicon nitride cantilevers array featuring different thicknesses (ranging from 230 to 550 nm).

used to define bulk-driven and surface-driven (ecto-elastic) elasticity regimes.

Additionally spanning the range of thermodynamically admissible materials, the role of Mindlin's cohesion modulus is exemplified and it is shown that the predicted behaviors cluster in few, rather different, families, depending on the higher-grade elastic parameters. This could trigger the development of innovative MEMS devices and paves the way for the experimental approach of these higher-grade materials.

3. EXPERIMENTAL IDENTIFICATION

Based on the above-described framework and making use of arrays of silicon nitride cantilevers featuring different thicknesses (see Fig. 2), indications for the robust experimental identification of the involved higher-grade elastic parameters will be given.

References

[1] Cuenot S., Frétigny C., Demoustier-Champagne S., Nysten B. Surface tension effect on the mechanical properties of nanomaterials measured by atomic force microscopy. *Phys. Rev. B*, **69**: 165410, 2004.

[2] Fritz J., Baller M.K., Lang H.P., Rothuizen H., Vettiger P., Meyer E., Güntherodt H., Gerber C., Gimzewski J.K. Translating biomolecular recognition into nanomechanics. *Science*, **288**: 316-318, 2000.

[3] Boisen A., Dohn S., Keller S.S., Schmid S., Tenje M. Cantilever-like micromechanical sensors. *Reports on Progress in Physics* **74(3)**: 036101, 2011.

[4] Mindlin R.D. Second gradient of strain and surface tension in linear elasticity. *Int. Journal Solids Struct.* **1**: 417-438, 1965.

[5] Dehrouyeh-Semnani A.M., Nikkah-Bahrami M. A discussion on incorporating the Poisson effect in microbeam models based on modified couple stress theory. *Int. J. Engineering Science* **86**: 20-25, 2015.

[6] Amiot F. An Euler-Bernoulli second-strain gradient beam theory for cantilever sensors. *Phil Mag Letters* **93(4)**: 204-212, 2013.

[7] Amiot F. A model for chemically-induced mechanical loading on MEMS. *J. Mechanics of Materials and Structures* **2(9)**: 1787-1803, 2007.

[8] Begley M. R., Utz M. Multiscale modeling of adsorbed molecules on freestanding microfabricated structures. *Journal of Applied Mechanics-Transactions of the ASME* **75(2)**: 021008, 2008.