Two-level global sensitivity analysis of the excitation contributions leading to acoustic noise in an electric motor for the purpose of robust optimization

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Abstract: This paper presents a sensitivity analysis methodology used for electric motor design. This innovative approach evaluates both global effects of parameter variations in their design range and of parameter deviations in their tolerance intervals on design objectives. For the purpose of robust optimization, this method helps to select the most influent design parameters and uncertain parameters, which are not necessarily the same. Suitable for any design approach, this method is particularly useful in dealing with objectives defined by non-linear and non-regular functions. In this paper, the method is applied to the sensitivity evaluation of an electric motor's acoustic criteria. The objective functions are the levels of electromagnetic tangential excitations responsible for acoustic emissions. The output mean torque is another objective. The sensitivity analysis shows that acoustic criteria appear generally more sensitive to parameter deviations than mean torque. Parameter deviations can be even more influent on acoustic criteria than larger parameter variations in their design range. As can be expected from the sensitivity results, the paper eventually shows that the acoustic optimization of the electric motor faces robustness issues.

1. Introduction

The design of electric motors dedicated to the transport industry is a process which requires that many performance criteria be simultaneously brought to a satisfactory level. Typical performance criteria such as torque density, power density and efficiency should be maximized, while torque ripple, manufacturing complexity and material costs should be minimized [1]. Moreover, it is important to reduce electric motor noise and vibrations by minimizing Noise, Vibration and Harshness (NVH) criteria.

The manufacturing and assembly of electric motor components leads to geometrical and material dispersions [2]. Moreover, some electric motor control parameter deviations can be observed in operation, e.g. because of position sensor errors [3]. These discrepancies between the parameters of a perfectly manufactured and perfectly operated motor and those in reality can affect the motor performance criteria, so the discrepancies must be taken into consideration in the design process.

The effect of parameter deviations on a motor's electro-technical performance criteria is explored in literature, particularly on cogging torque [4], [5], [6] or on mean torque and torque ripple [7], [8]. The impact of manufacturing dispersions on NVH criteria is more rarely studied. The effects of an oval stator shape and retracted teeth on the harmonic orders of radial force densities and the sound power level (SWL) of a claw-pole alternator were studied by Tan-Kim et al. [9]. Kolb et al. performed an exhaustive global sensitivity analysis of the manufacturing tolerances in a Permanent Magnet Synchronous Machine (PMSM) on the radial Maxwell stresses [2]. These analyses all study the effect of parameter dispersions for a given design. Consequently, they enable the designers to identify the tolerances which should be tightened, and those which can be widened, without modifying the design. However, tightening some tolerances often causes additional cost. Instead, the present approach studies the effect of parameter dispersions for an entire set of possible designs. It is therefore intended for approaches aiming at finding a design making motor performance criteria robust to tolerances. The need to use such robust methods for electric motor design is pointed out in [10] and some examples of applications can be found in literature; using the Taguchi robust design method [11], [12], or robust optimization methods based on a probabilistic definition of robustness (six-sigma method) [13], [14], or on a worst-case definition of robustness [13], [15], [16]. In a classical optimization approach, some design parameters must be chosen, and to reduce the size of the design space and the resulting optimization time, sensitivity analysis methods are usually used to select only the design parameters having the largest effects on the criteria of interest (i.e. the objective and constraint functions). Reducing the design space is also useful to shorten the computation time of robust design and robust optimization methods. In addition, robust design and robust optimization methods require considering the deviations of some parameters, which are not necessarily the optimization parameters, in order to calculate their specific robustness indicators (e.g. the signal-to-noise ratio for the Taguchi method, or the mean value and standard deviation of objectives for the six-sigma method). Whatever robust approach is used, the parameter deviations to consider should also be limited only to those which have a significant effect on the criteria of interest, in order to reduce the time necessary to determine the robustness estimators for a given design. A sensitivity analysis must therefore be run before the robust optimization, in order to identify on the one hand the most influent design parameters, and on the other hand the most influent parameter deviations. The global sensitivity analyses which are usually performed before a design optimization only determine the most influent design parameters [17], [18], by evaluating the effect of parametric variations which are large compared with the size of the tolerance intervals. While the sensitivity of a smooth function to parameter deviations in their tolerance intervals can reasonably be approximated by such methods, this is particularly improper for non-linear and non-regular functions. On the other hand, the sensitivity analyses of manufacturing tolerance, such as those performed by Kolb et al. in [2], usually only evaluate their effect for a fixed design, even though a global overview of the effect of parameter deviations on the criteria of interest for the entire design space is preferable before running a robust design or optimization approach.

This article presents a novel global sensitivity analysis method, which evaluates the global sensitivity of the criteria of interest for electric motor design, on the one hand to variations of the nominal values of the parameters, to select the most influent design parameters, and on the other hand to their deviations, to select the most influent deviations which must be simulated during the robust design process. It is particularly useful to handle non-linear and non-regular electric motor NVH criteria and was developed for this purpose, but it is also of interest when dealing with any non-linear and non-regular criterion which is potentially sensitive to small parametric variations.

The sensitivity analysis method is applied to study the mean torque produced by a PMSM dedicated to automotive traction, and also the most significant electromagnetic contributions responsible for its noise emissions at low speed. Therefore, this paper supplements the rare studies of electric motor NVH sensitivity to dispersions performed in [2] and [9], with new results showing the effect of several geometric and control parameter variations in their design space and in their tolerance intervals on the prevailing NVH criteria of the considered motor. These results show that specific attention should be paid to robustness when dealing with the design of silent electric motors, because their NVH indicators can be very sensitive to slight parameter dispersions.

The main characteristics of the considered PMSM and its NVH issues are presented in section 2. The sensitivity analysis method is then described in section 3, and results are presented and discussed in section 4. Finally, in section 5, some conclusions of the sensitivity analysis are illustrated by performing a multi-objective motor optimization which does not account for the parameter deviations, and by evaluating the robustness of an optimized design.

2. Motor characteristics and NVH diagnosis

2.1. Motor characteristics

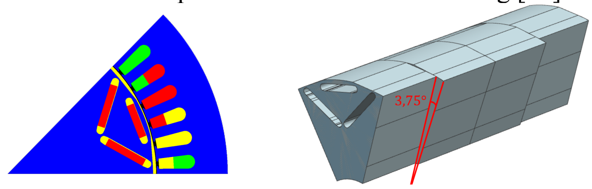
The 2D section of the considered motor is depicted in Fig. 1.(a). Its main characteristics are specified in Table 1.

Table 1 Motor characteristics

Maximum output power	75 kW
Maximum torque	180 N.m
RMS value of the current limit	240 A

The three rotor permanent magnets (PM) are grade N35UH Neodymium-Iron Boron magnets, and the active magnetic parts are made of standard silicon steel laminations which are 0.35 mm thick and have a core loss value of 2.5 W/kg at 50 Hz and 1.7 T.

A particularity of the motor is that the rotor is step-skewed: the rotor is made of three parts, and the central part is shifted by 3.75° with respect to the external parts, as depicted in Fig. 1.(b). Skewing is generally an efficient solution to reduce torque ripple, and rotor step-skew can be selected for its more convenient manufacturability and more attractive cost compared with continuous skewing [19].



(a) (b)

Fig. 1. *Motor geometry.*
(a) 2D section of the studied motor for one pole, **(b)** Rotor step-skewing

2.2. Simulation workflow

To evaluate the motor's performance, many criteria, including common electro-technical criteria and noise and vibration levels, can be calculated from a common transient electromagnetic simulation. Indeed, the electromagnetic surface force densities which are applied to both the rotor and the stator, and which are called Maxwell stresses, can be calculated from the radial and tangential airgap flux density outputs of the electromagnetic simulation, using the formulas:

$$\sigma_r = \frac{1}{2\mu_0}(B_r^2 - B_t^2), (1)$$

and

$$\sigma_t = \frac{1}{\mu_0}(B_r B_t), (2)$$

where σ is the surface force density, B is the magnetic flux density, μ_0 is the magnetic permeability of vacuum, and r and t denote radial and tangential components.

Torque can be computed by integrating the tangential components of the electromagnetic surface force densities along the airgap circumference. The radial and tangential surface force densities vary with time, i.e. with the rotation of the rotor, but they also have a distribution depending on space. Consequently, they dynamically excite the motor structure and cause vibrations, whose levels depend on the excitation's harmonic and spatial content as well as on the structure's properties. This step can be simulated using a structural model, which must be representative of the electric motor's structural properties and in particular those of the laminated stator. Some modelling guidelines for the structural design of electric motor stators are given in [20] and [21]. Finally, an acoustic model can be used to compute the acoustic power level radiated by the motor. This multiphysical simulation workflow is detailed in [22], and an example of validation is given in [23].

2.3. NVH diagnosis of the motor

The interest of analyzing and trying to minimize the acoustic emissions of the considered PMSM was demonstrated by prior experimental investigations on the complete powertrain including the PMSM, the gearbox and the power electronics unit. They are not detailed in this article. They show that high noise levels are reached at low speeds. This is particularly critical because at low motor speeds, electromagnetic noise prevails over aerodynamic and rolling noise in electric vehicles.

Therefore, to identify the causes of the noise emissions, the motor is simulated in the low-speed range from 500 rpm to 3000 rpm, using the simulation workflow detailed in2.2. Moreover, the torque is set to 40% of its maximum torque (72.7 N.m) for the simulations, which is a torque which is often required in electric vehicles.

The step-skewing of the rotor is an additional difficulty regarding the simulation process. Indeed, the hypothesis that the electromagnetic behavior in 2-D sections taken at different axial positions is the same is often made to reduce the electromagnetic model to a 2-D model. This hypothesis is no longer valid for step-skewed rotors. Consequently, two options exist to model step-skewed motors using finite element analysis (FEA): multi-slice 2-D or 3-D FEA [19]. The 3D FEA requires a high computation time which is prohibitive for optimization, so a multi-slice 2-D model is used, under the assumption that the electromagnetic behavior is the same in all 2-D sections belonging to a common rotor part. As the two external parts of the rotor are identical, only two electromagnetic simulations are required. After projecting the excitations produced by each rotor part on a structural finite-element model, the phenomenon causing the high noise levels is identified: the noise and vibrations are due to a resonance of an overall powertrain bending mode caused by the tangential excitations around this mode's natural frequency, i.e. around 1000 Hz. The shape of this overall bending mode, depicted in Fig. 2, is prone to be excited by the dynamic excitation of the tangential Maxwell surface force densities. Moreover, this mode shape involves deflections of the electric power unit and gearbox cover plates which have large radiating areas. In order to simplify the simulation workflow, it is decided to perform the sensitivity analysis of the spatial resultant of the tangential excitations causing the overall bending mode resonance, i.e. the engine orders 24, 48 and 96 of the torque. The engine order n of the torque corresponds to the sinusoidal wave of the instantaneous torque having a frequency which is n times larger than the rotation frequency. At the selected engine speeds, the torque engine orders of interest reach the natural frequency of the powertrain's overall bending mode (1000 Hz, i.e. 2500 rpm for engine order 24, 1250 rpm for engine order 48 and 625 rpm for engine order 96).

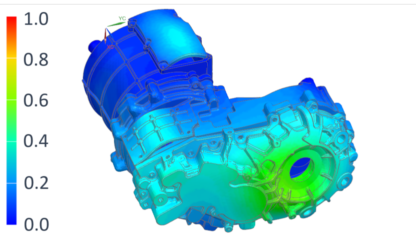


Fig. 2. *Representation of the powertrain overall bending mode shape (1026 Hz) with unit maximum amplitude color scale.*

These criteria of interest are expressed in decibels (dB):

$$O_i = 20\log_{10}\left(T_{h_i}\right), (3)$$

where O_i is the ith criterion and T_{h_i} is the torque harmonic contribution of engine order i.

As this overall bending mode resonance prevails over the other phenomena between 500 and 3000 rpm, expressing the variations of each torque engine order in dB gives an accurate estimation of the SWL variations to be expected.

Moreover, when modifying an electric motor design to improve its vibroacoustic behavior, attention must be paid to keeping the mean torque at an acceptable level. Therefore, this criterion is also investigated in the sensitivity analysis.

3. Description of the methodology

3.1. Presentation of the sensitivity analysis method

The Morris method [24] is used to evaluate the effect of the parameters on the criteria of interest. Unlike local sensitivity methods, the sensitivity indicators of the Morris method are calculated by evaluating the effect of parameter variations on the criteria of interest for several designs in the design space. Moreover, it is a One-Factor-At-a-Time method, which requires much fewer evaluations of the criteria of interest than stochastic approaches [2]. The differences are that the results obtained using the Morris method are qualitative while those of stochastic approaches are quantitative, and that the effects of non-linearity and interaction between parameters cannot be distinguished using the Morris method. Still, it is well suited to optimization parameter selection, as it provides a classification of the parameters' effects that relies on the entire design space.

The principle of the Morris method is described precisely in [24]. To understand the results presented in this article, a brief description of the method is necessary. One can consider the case where the sensitivity of a function denoted by f to N parameters p_1 to p_N must be estimated. For this purpose, a range of admissible values $[p_{j_{min}}, p_{j_{max}}]$ must be given for each parameter p_j. Several elementary effects are then calculated for each parameter p_j, by evaluating the variation of f when the parameter p_j is subject to a variation of Δ_j and the other parameters remain unchanged:

$$EE_{p_j} = \frac{f(p_1, \ldots, p_j, \ldots, p_n) - f(p_1, \ldots, p_j + \Delta_j, \ldots, p_n)}{\Delta_{j_{norm}}}, (4)$$

where $\Delta_{j_{norm}} = \frac{\Delta_j}{p_{j_{max}} - p_{j_{min}}}$ is the normalized value of the variation. The Morris method guarantees that $\Delta_{j_{norm}} > 0.5$, i.e. that the increment always exceeds 50% of the size of the admissible range, ensuring the global nature of the sensitivity measure.

The different elementary effects of each parameter p_j are calculated for different values of the other parameters and of p_j itself. Consequently, unless f is a linear function of the parameters p_1 to p_N, the elementary effects EE_{p_j} of a parameter p_j have different values.

Two sensitivity indicators are then calculated:

- $\mu^*_{EE_{p_j}}$ is the mean value of the absolute elementary effects of the parameter p_j. It can be read as the mean normalized variation of the function f when the parameter p_j is incremented by $\pm\Delta p_j$.
- $\sigma_{EE_{p_j}}$ is the standard deviation of the elementary effects. It is an indicator of the variability of the parameter effect within all the designs used to calculate the elementary effects.

This method can be directly applied to perform a sensitivity analysis aiming at identifying the most influent design parameters. In this case, the boundaries of the admissible ranges must be chosen for each parameter as their minimum and maximum admissible values in the design space. These parameters are named design parameters and their admissible ranges are named design ranges. If the sensitivity analysis is made to prepare an optimization, the criteria of interest are the objective and constraint functions of the optimization, and the parameters having the highest absolute mean elementary effect $\mu^*_{EE_{p_j}}$ on these criteria of interest should be selected as optimization parameters.

Moreover, to perform robustness analyses and robust optimization, the effect of the parameter deviations must be taken into consideration. For this purpose, the sensitivity of the criteria of interest to variations of the parameters in their entire design space is distinguished from their sensitivity to generally much smaller variations of parameters in their tolerance interval. Indeed, if the sensitivity analysis is performed to prepare a robust optimization, the impact of the variations of a given parameter in its design space may be too low to select it as an optimization parameter, but its deviations may imply effects on the criteria of interest which may be large enough to be considered. On the contrary, a given parameter may have a significant effect when subject to large variations in its design space, but local variations may cause no significant impact on the criteria of interest. This is particularly the case for NVH criteria which are non-regular and non-linear with respect to geometric and control parameters.

For this purpose, a parameter p_j is considered as the sum of its nominal value, denoted by p_j^{NOM}, and a deviation parameter p_j^{DEV}:

$$p_j = p_j^{NOM} + p_j^{DEV}. \quad (5)$$

In this way, the effect of variations of the parameters' nominal values in the design range, and of their deviations in the tolerance interval, can be estimated separately. The difference between the components p_j^{NOM} and p_j^{DEV} is only their admissible range. A significant advantage of this method, apart from the reduced number of samples in comparison with stochastic approaches, is that the effect of parameter uncertainties is also evaluated globally. It is valuable for the selection of the parameter deviations, which should be taken into consideration in the search for an optimum robust design when dealing with non-linear and non-regular NVH criteria.

3.2. Definition of design spaces and tolerance intervals

To perform the motor's sensitivity analysis, its parameters must be defined, as well as the limits of their design space and those of their tolerance intervals.

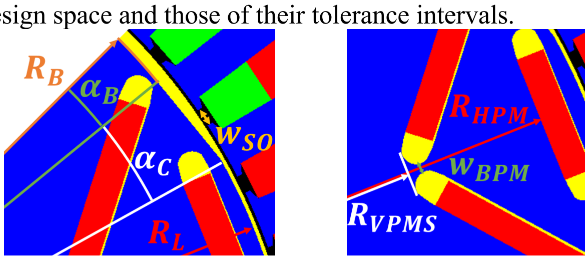


Fig. 3. *Geometric parameters of the PMSM.*

The 8 geometric parameters which are considered are depicted in Fig. 3 and described in Table 2. For geometric parameters, the effect of variations in both nominal values and deviations on the criteria of interest, i.e. the torque harmonics 24, 48 and 96 (the objective functions), and the mean torque (the constrained function), are investigated. In addition, the control parameter Δ_{RS} defines the rotor's mechanical angular position with respect to the stator's rotating field, so it directly defines the phase current shift angle. The nominal value of this parameter is directly defined by the control system so as to provide the maximum torque, and it is therefore considered as a fixed parameter which could not be modified during an optimization. However the effect of its deviation due to control system inaccuracies is studied.

The design ranges are defined to be as wide as possible and to provide a large design space offering good possibilities for the reduction of the objective functions. Their spans are specified in Table 2. These spans remain limited because the rotor geometry with three magnets provides limited design modification possibilities.

Table 2 Characteristics of design ranges and tolerance intervals

Param. symbol	Description	Design interval span	Standard deviation of 5 samples (σ_j)
R_L	Rotor lobe radius	0,5 mm	0.007 mm
R_B	Rotor flux barrier radius	0,5 mm	0.006 mm
α_C	Angle spanned by rotor chamfers	5°	0.5°
α_B	Angle spanned by rotor flux barriers	4°	0.28°
w_{SO}	Slot opening width	0,5 mm	Not measured
R_{VPMS}	Radial position of V-shaped PMs	0,5 mm	0.011 mm
w_{BPM}	Bridge between V-shaped PMs	0,5 mm	0.014 mm
R_{HPM}	Radial position of the horizontal PM	0,5 mm	0.035 mm

The tolerance intervals can be defined from the measurements made on five sheets of a manufactured motor. The standard deviation of the parameters can be calculated using the corrected sample formula:

$$\sigma_j = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(p_{j_k} - \bar{p}_j\right)^2}, (6)$$

where n is the number of available measures (5 in this case), p_{j_k} the value of the parameter p_j for the sample k, and $\bar{p}_j$ the mean value of p_j over the n measurements.

These standard deviations are also presented in Table 2. The tolerance intervals are assumed to have the limits $[p_j^{NOM} - 3\sigma_j, p_j^{NOM} + 3\sigma_j]$. Under the hypothesis that the parameter deviations follow a normal distribution, this assumption means that 99.7 % of the deviations of each parameter are inside their tolerance intervals. However, it should be noted that the sensitivity analysis method does not assume any particular parameter deviation distribution.

No experimental data is available for the deviations of the parameter w_{SO}. Its standard deviation is therefore fixed to the value of 0.07 mm, which is a large value in comparison with the standard deviations of other parameters given in

Table 2 (twice as high as R_{HPM}, which is the largest among the distance parameters).

The standard deviation of the control parameter Δ_{RS} is assumed to be 0.166°, and its tolerance interval [-0.5°,0.5°]. This corresponds to electrical shift angle deviations lying between -2° and 2° in 99.7% of the cases.

4. Results

The Morris sensitivity analysis following the described methodology is performed in Python using the SALib library [25]. The parameter names are followed by the exponent ^{NOM} when the effect of the variation of their nominal value in their design range is considered, and by the exponent ^{DEV} when the effect of their deviations in their tolerance interval is considered. In this study, the average absolute effect of each parameter and its standard deviation are calculated using 9 elementary effect values. Each criterion must therefore be evaluated for 162 different sets of parameters. Evaluating the criteria of interest for one design requires building and solving the corresponding finite element electromagnetic model (Altair Flux is used), and post-processing the torque. The computational time for one design is approximately 34 minutes on the workstation (128 GB RAM – 2 Intel Xeon CPU E5-2670 v2 processors), so the total computational time is around 93 hours. However, as the design evaluations are performed using 5 parallel threads, the computational time is reduced to approximately 19 hours.

4.1. Results interpretation

The sensitivity results regarding the output torque are depicted in Fig. 4.

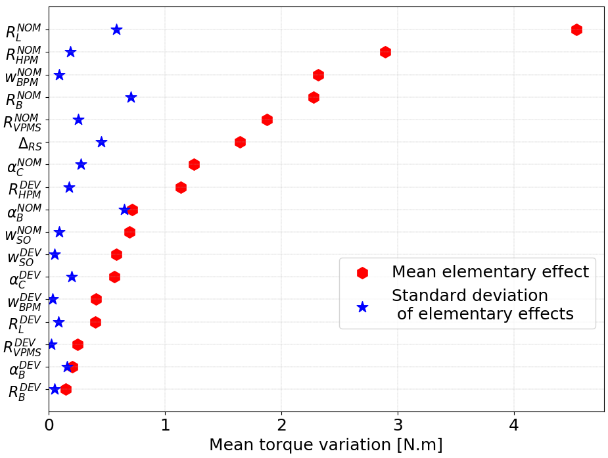


Fig. 4. *Mean torque sensitivity to parameter variations.*

For each parameter, the average value of absolute elementary effects is given at each line by the red point, while the blue star corresponds to the standard deviation of elementary effects. It appears that for the most influent parameter (the nominal value R_L^{NOM} of the rotor lobe radius R_L), the mean elementary effect reaches 4.5 N.m (for an average mean torque value of 64.7 N.m among all the tested configurations). Therefore, the possibilities of significantly improving the motor's torque performance by varying the geometric parameters' nominal values are not very significant. Fig. 4 also shows that the standard deviations are significantly lower than the average elementary effect values for almost all parameters. This means that the elementary effects of each parameter have similar values for the different trajectories, i.e. for different designs. This can be explained by the fact that mean torque is an overall performance criterion which is not affected much by parameter interactions; most parameter variations impact mean torque independently of each other (e.g. increasing the nominal value of the rotor lobe radius parameter R_L almost always results in an increase of mean torque, regardless of the value of the other parameters, because it reduces airgap size).

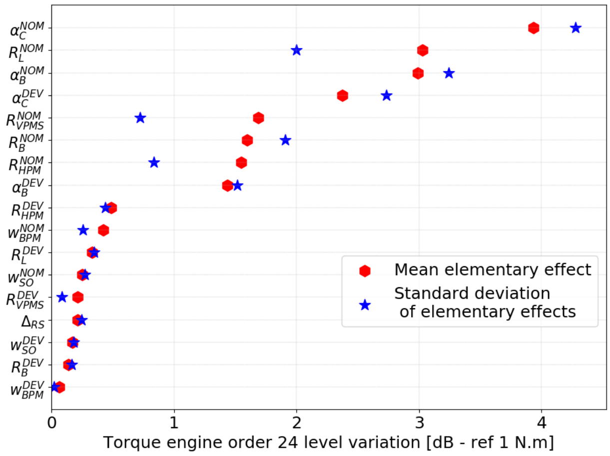


Fig. 5. *Torque engine order 24 sensitivity to parameter variations.*

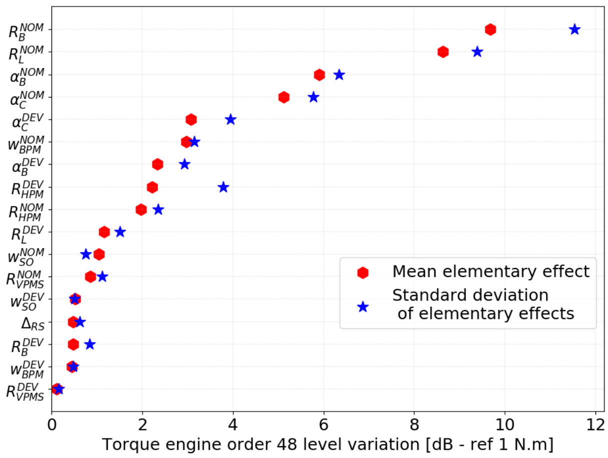


Fig. 6. *Torque engine order 48 sensitivity to parameter variations.*

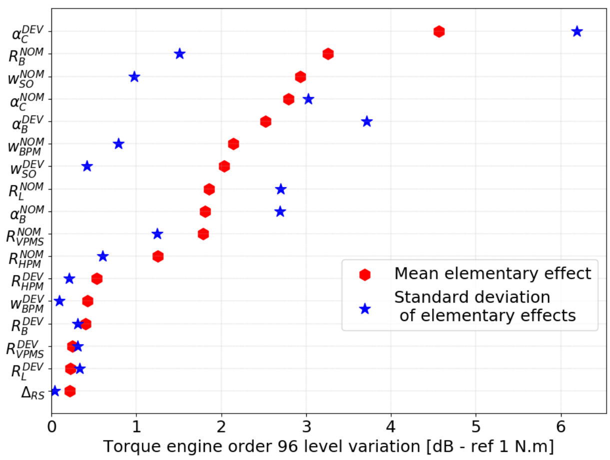


Fig. 7. *Torque engine order 96 sensitivity to parameter variations.*

The sensitivity results regarding the NVH criteria to be minimized, i.e. the levels expressed in dB of the torque engine orders 24, 48 and 96, are respectively presented in Fig. 5, Fig. 6 and Fig. 7.

For these criteria, a noticeable result is that the standard deviations of elementary effects are in the same order of magnitude as the average values of absolute elementary effects. This shows that the evolution of these NVH criteria is much more difficult to predict than that of mean torque, because the NVH criteria depend on the local distribution of the Maxwell stresses, which is highly dependent on the interaction between parameters. As a result, while a given parameter variation will generally have a similar effect on mean torque for any design, its effect on the NVH criteria will depend on the values of the other parameters, i.e. on the considered design.

The criterion which is the most sensitive to the variation of parameters in their design range is the level of the torque engine order 48: in average, the effect of the variation of the nominal value of the flux barrier radius R_B^{NOM} is a normalized change of almost 10 dB of the torque engine order 48. Three other parameters affecting the rotor's outer shape have an average normalized effect of more than 5 dB. The standard deviations of these effects are also particularly high. This is due to the fact that the step-skewing angle of 3.75° was chosen in order to minimize the torque engine order 48; for certain designs, simulation results show that this engine order can almost be eliminated, so the associated variations of the torque engine order 48 level, expressed in dB, are very high. However, some parameter deviations, such as those of the angles spanned by the rotor flux barriers α_B^{DEV} and the rotor chamfers α_C^{DEV} , also have a significant effect on this criterion, so they could potentially cause robustness issues.

The effect of the parameters on the two other NVH criteria is less significant. The results show that there are fewer design possibilities to reduce these criteria because the average effect of nominal value variations is lower, and also that the criteria are probably a bit less subject to robustness issues because the average effects of parameter deviations are also slightly lower.

Table 3 Elementary effects of the parameter R_{HPM}^{DEV} on the torque engine order 48 level

Design	Δ_{norm}	$f(p)$ (dB)	$f(p+\Delta)$ (dB)	Elementary effect (dB)
1	-0.6	-11.31	-10.26	-1.75
2	-0.6	-3.48	-3.07	-0.68
3	+0.6	-0.89	-1.05	-0.27
4	+0.6	-18.66	-25.32	-11.10
5	-0.6	-7.26	-7.44	0.30
6	+0.6	-4.54	-5.3	-1.27
7	-0.6	-7.41	-6.26	-1.92
8	+0.6	-10.18	-8.91	2.12
9	+0.6	1.76	2.09	0.55

A noteworthy exception can be observed in Fig. 7: the level of the torque engine order 96 is most sensitive to α_C^{DEV}, the deviations of the angle spanned by the rotor chamfers in their tolerance intervals. It is surprising to observe that the sensitivity to variations of a parameter in its tolerance interval is higher than the sensitivity to its variations in its design range, which is more extended. This phenomenon also happens for the effect of the parameter R_{HPM}, which defines the radial position of the horizontal magnet, on the level of the torque engine order 48. To investigate these results, the nine elementary effects of R_{HPM}^{DEV} on torque engine order 48 levels and of α_C^{DEV} on torque engine order 96 levels are examined.

Table 3 presents the values of the torque engine order 48 levels, denoted by f, for the 9 designs used to calculate the elementary effects of the parameter R_{HPM}^{DEV}. For that, the torque engine order 48 level is calculated for the sets of parameters p corresponding to the different designs 1 to 9, and for the sets of parameters $p+\Delta$ where Δ contains only zeros except for the value of R_{HPM}^{DEV}. The average value of the absolute elementary effects of this parameter reaches 2.21 dB. This is mainly due to the fourth elementary effect, which is much larger than the others. As a consequence, the R_{HPM}^{DEV} parameter's elementary effect greatly depends on the considered design. This explains that the standard deviation of the elementary effects reaches the large value of 3.79 dB. The reason for this high fourth elementary effect is illustrated in Fig. 8. To build this curve, all parameters except R_{HPM}^{NOM} and R_{HPM}^{DEV} are set to the values corresponding to the design 4 used to evaluate the fourth elementary effect of the parameter R_{HPM}^{DEV}, equal to -11.10 dB. The possible values of R_{HPM} are then swept in its range defined by the design range, which covers 0.5 mm, and the tolerance interval, which is equal to $[-3\sigma_j, 3\sigma_j]$ i.e. [-0.105 mm, 0.105 mm], and the value of the torque engine order 48 is calculated for each R_{HPM} value. **Erreur ! Source du renvoi introuvable.** shows that for this design the simulated engine order 48 can almost be eliminated, so the torque engine order 48 level is subject to very large variations when the parameter R_{HPM} varies, even for a variation of 0.126 mm (corresponding to 60% of the tolerance interval of R_{HPM}) applied to calculate the elementary effect of R_{HPM}^{DEV}.

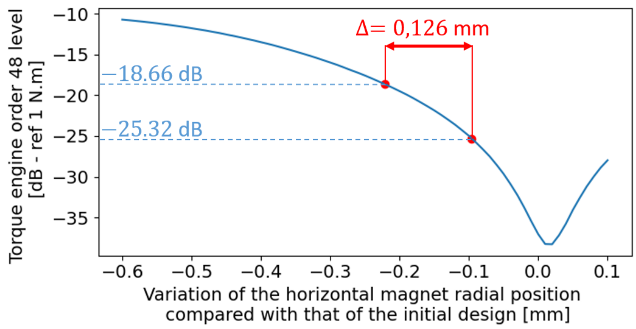


Fig. 8. *Effect of the parameter* $\boldsymbol{R_{HPM}}$ *on the torque engine order 48 level, for the design used to calculate its largest elementary effect due to deviations.*

A similar case can be observed for the effect of the parameter α_C^{DEV} on torque engine order 96. The large average value of elementary effects (4.56 dB) and the high standard deviation (6.19 dB) are mainly due the sixth elementary effect of α_C^{DEV}. For the sake of conciseness, the table containing all elementary effects of α_C^{DEV} on torque engine order 96 is not presented. Following the same methodology as for Fig. 8, the effect of α_C for the mentioned sixth design is depicted in Fig. 9.

It appears that for this particular design, the parameter deviations within the tolerance interval can be more influent than larger variations in the design range. Indeed, it can be seen in Fig. 9 that this design is a local minimum, so the function variability is very high in the local minimum's close neighborhood, and the function becomes more regular for designs which are more distant from the local minimum.

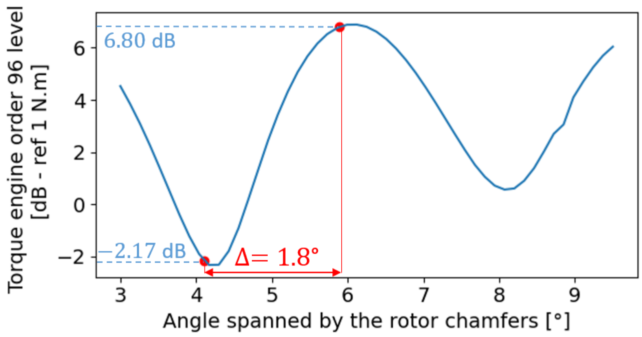


Fig. 9. *Effect of the parameter* $\boldsymbol{\alpha_C}$ *on the torque engine order 96 level, for the design used to calculate its largest elementary effect due to deviations.*

The high average elementary effects of α_C^{DEV} on torque engine order 96 and R_{HPM}^{DEV} on torque engine order 48 are due to the elementary effects calculated close to the local minima depicted in **Erreur ! Source du renvoi introuvable.** and Fig. 9. These high average elementary effects are not only because of the non-regular nature of torque engine orders 48 and 96, but also because of the Morris method's sampling including these local minima. Consequently, the obtained results are valid for the randomly generated trajectories of this analysis, but they cannot be generalized quantitatively to the entire design space. Still, they provide useful qualitative information about the global effect of parameter variations in design ranges and tolerance intervals. In addition, the more trajectories are used, the more generalizable the sensitivity analysis conclusions are.

4.2. Parameter selection for robust optimization

In addition to the overview of the parameter variations' effects on the criteria of interest and the resulting physical interpretations, the role of this new sensitivity analysis method is to select the most influent design parameters, and the parameters whose deviations must be taken into consideration in a robust design or robust optimization approach.

The chosen optimization parameters are the parameters for which the variation of the nominal value have the most effect on the criteria of interest. In the present case, these parameters are R_L, R_B, α_B and α_C. Some better optimization results may be obtained by adding other optimization parameters, but only at the cost of an increased optimization time.

The parameter deviations which should be taken into consideration for the robustness analyses and for robust optimization are those which have the most effect on the criteria of interest. Reducing the number of parameters whose deviations are considered reduces the number of simulations which must be run to evaluate the robustness of a design, regardless of the computed robustness indicator.

To evaluate the robustness of the torque engine order 48, shown to be the most likely to be subject to robustness issues, it is decided to consider the deviations of α_C, α_B and R_{HPM}. This choice would also be adapted for the evaluation of the robustness of the torque engine orders 24 and 96, which are particularly sensitive to the deviations of α_C and α_B. Finally, to study mean torque robustness, the deviations of Δ_{RS} and R_{HPM} could be considered.

5. Verification of the main sensitivity analysis results

It is difficult to validate the presented sensitivity analysis method itself, because this method involves simulating many different designs (162 in the example of section 4) to provide the sensitivity indicators. Consequently, it would not make sense to verify the effect of each parameter variation individually. A more meaningful approach is to confirm the sensitivity analysis results. Just as any sensitivity analysis method, the presented method identifies the parameters whose variations in their design range are the most influent. Moreover, two novel types of results are obtained using this two-level sensitivity analysis method, and should be verified:

- The first concerns the criteria minimization possibilities and their likeliness to face robustness issues if the parameters' tolerances are not taken into account during the design or optimization process. It appears that the torque engine order 48 has the best minimization possibilities, but that it is also the most subject to robustness issues. For the torque engine orders 24 and 96, fewer reduction opportunities are expected. Although more limited, some robustness issues can also be expected for these two engine orders if no robust design or optimization approach is adopted. These results are confirmed in this section by conducting a classical optimization minimizing the three criteria of interest, and analyzing the Pareto front as well as the robustness of a Pareto-optimal design.
- The second one concerns the parameters whose tolerances must be taken into consideration for a robust optimization. To evaluate the robustness of the torque engine orders 24, 48 and 96, it is decided to only consider the deviations of α_C, α_B and R_{HPM}. To confirm this conclusion, a possible approach is to conduct a robust optimization accounting for the three selected parameters' deviations, and verify that the optimized design robustness is satisfying when the all parameter deviations are considered. However, robust optimization methods are very time-consuming, even though the authors are investigating simplifying assumptions to reduce computational time. The parameter deviation selection based on the present sensitivity analysis method belongs to the computational time reduction process. Robust optimization results including this verification procedure should be presented in the future.

5.1. Classical multi-objective optimization results

To confirm the first results of the sensitivity analysis study, a multi-objective optimization aiming at minimizing the torque engine orders 24, 48 and 96 without reducing the mean torque by more than one percent is performed. Following the sensitivity analysis results, the chosen optimization parameters are R_L, R_B, α_B and α_C which can vary within the spans given in Table 2. To solve this multi-objective optimization problem, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) algorithm is used [26]. It is a genetic algorithm which very commonly serves to handle multi-objective problems because of its fast convergence towards sets of solutions which are close to the Pareto fronts of these problems and well spread over them. For this purpose, the python library *pymoo* is used [27]. An initial population of 40 individuals is first sampled randomly. After that, 50 generations of offspring are generated by performing random crossovers between parent designs and random mutations. In all, 540 design evaluations are performed. The same workstation is used as for the sensitivity analysis, and 5 parallel threads are also used. With this setup, the computational time is approximately 62 hours.

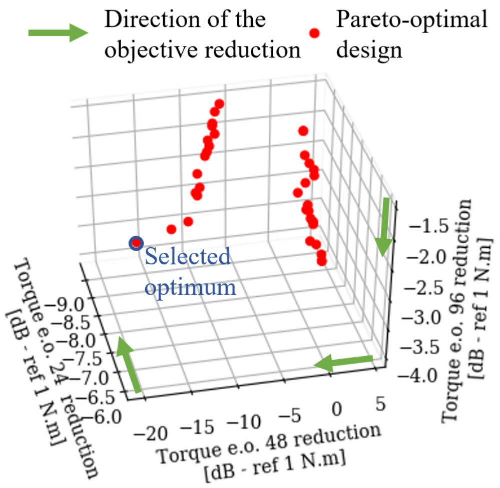


Fig. 10. *Values of the objectives of the designs belonging to the estimated Pareto front for the classical multiobjective optimization aiming at minimizing the torque engine orders 24, 48 and 96 – first overview*

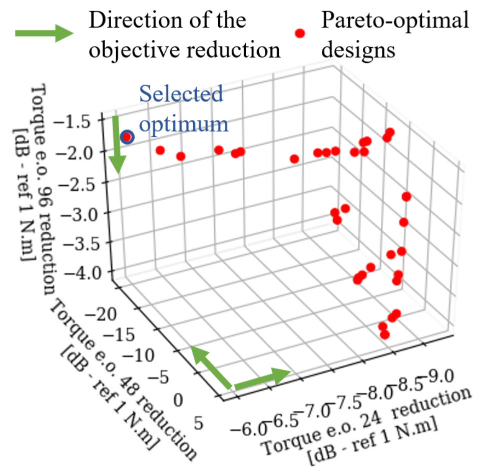


Fig. 11. *Values of the objectives of the designs belonging to the estimated Pareto front for the classical multiobjective optimization aiming at minimizing the torque engine orders 24, 48 and 96 – second overview*

Fig. 10 and Fig. 11 display the estimated Pareto front in the 3-dimensional objective space. The axes represent the variation of each objective with respect to the initial design, so that the initial design corresponds to the coordinates (0,0,0). Both figures depict the same Pareto front: they provide two overviews which are shifted by a 90° angle around the vertical axis.

The largest objective reductions are achieved on the torque engine order 48 level, as expected when looking at the sensitivity analysis results. In addition, the torque engine orders 24 and 96 levels can be reduced to a lesser but still significant extent by varying the four optimization parameters. The high dependency of the torque engine order 48 level on design can also be seen in the span of reductions covered by the Pareto-optimal designs regarding this objective, from a 20.0 dB reduction to a 4.3 dB increase, while the reductions of the torque engine order 24 and 96 levels expressed in dB respectively cover the much more restricted ranges [-9.3,-6.0] and [-4.0,-1.6].

5.2. Robustness analysis of the design minimizing the torque engine order 48

The sensitivity analysis results leads to the prediction that the torque engine order 48 level could be significantly reduced by using an optimization algorithm, but that the resulting designs might be non-robust if, as it is the case in the multi-objective optimization presented in section 5.1, the parameters' tolerances are not taken into consideration in the optimization process. To verify this assumption, in the Pareto front depicted in Fig. 10 and Fig. 11, the design which features the largest reduction of the torque engine order 48 level, denoted by "selected optimum", is subjected to a robustness analysis. The values of each objective for this design are depicted in Table 4.

Table 4 Results of a deterministic optimization for the nominal design

	Torque engine order 24 (dB)	Torque engine order 48 (dB)	Torque engine order 96 (dB)	Mean torque (N.m)
Initial design	3.99	-4.29	8.39	72.74
Selected optimum	-2.03	-24.32	6.68	72.24
Variation	- 6.02	-20.03	-1.71	-0.6 %

To evaluate the robustness of this design, its design parameters are fixed, and a probability distribution is assigned to the considered parameter deviations. The results of the sensitivity analysis presented in section 4.2 are used, so only the deviations of α_C, α_B and R_{HPM} are taken into consideration. Normal distributions are widely used for the definition of parameter deviations [13], [14], and this choice is made for this article.

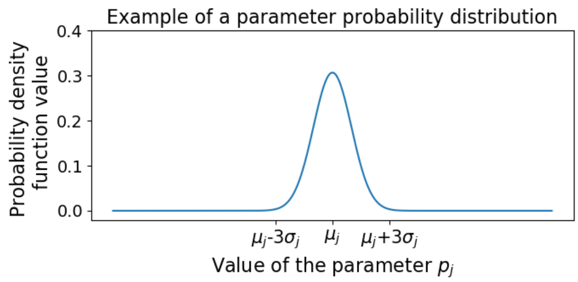


Fig. 12. *Example of the normal probability density functions for a parameter* $\boldsymbol{p_j}$.

The mean value of the deviation distribution is supposed to be zero: if it is not, the systematic deviation can be included in the design. Consequently, considering a parameter p_j, the mean value μ_j of its distribution is its nominal value, as depicted in Fig. 12. To define the standard deviation σ_j of the normal distribution of each parameter p_j, the experimental data on five rotor sheets depicted in Table 2 are reused.

A Monte-Carlo sampling is then performed to generate random sets of parameter deviation values, and the levels of torque engine orders 24, 48 and 96 are calculated for each of these random sets of parameters. The distribution of these three objectives are depicted in Fig. 13 for all the sets of parameter deviations. The distribution of this design's order 48 level has a very high variability, and confirms that the classical optimization method potentially leads to non-robust designs. However, robustness is criterion- and design-dependent: for this Pareto front design, the torque engine orders 24 and 96 levels are not significantly variable when taking parameter deviations into consideration.

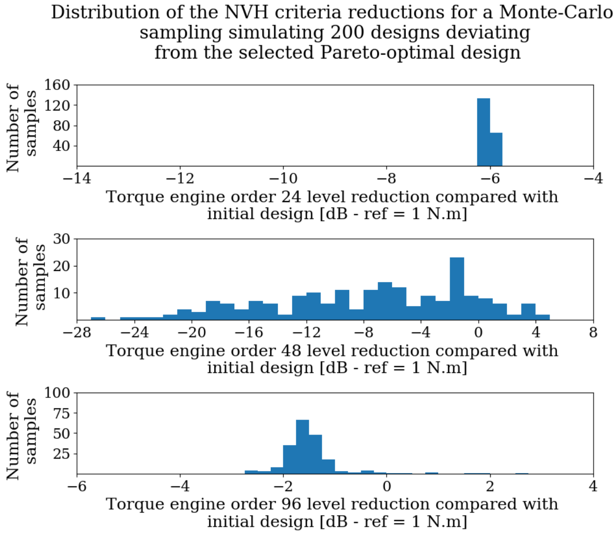


Fig. 13. *Distribution of the objectives when considering deviations of the parameters* $\boldsymbol{\alpha_C}$, $\boldsymbol{\alpha_B}$ *and* $\boldsymbol{R_{HPM}}$

Most of the designs in the Pareto front which feature reductions of the torque engine order 48 level are very close to the design whose robustness was investigated, and will therefore be subject to similar robustness issues. To achieve robust torque engine order 48 reductions, robust design or robust optimization methods should therefore be applied.

6. Conclusions

This article addresses the use of sensitivity analyses to prepare the robust vibroacoustic optimization of electric motors. Sensitivity analyses enabled the comparison of the global effect, i.e. in the entire design space, of parameter variations in their design range and tolerance intervals, on NVH criteria and on mean torque. The selection of pertinent NVH criteria is important: when the airborne motor noise is considered, the criteria must be good indicators of the acoustic power level variations. In this article, the specific excitation harmonic contributions which directly cause high motor noise levels at low rotating speeds were considered.

The definition of tolerance intervals also has an impact on the sensitivity results. Some values in this article were selected based on experimental results or on assumptions, but the described methodology can be applied to any other tolerance intervals.

The sensitivity analysis method presented is a novel and efficient tool to reduce the number of parameters considered in electric motor robust design or robust optimization processes when dealing with functions which are highly non-linear and non-regular. Indeed, for motor NVH criteria and a reasonable number of 162 design evaluations, an estimation of the most influent parameters for variations in their design space and for variations in their tolerance intervals was obtained. In the presented example, it resulted in the selection of four optimization parameters among eight possible candidates. This reduces the design space dimension, and consequently the number of design evaluations necessary for the optimization algorithm convergence towards the Pareto front. Moreover, three parameter dispersions were selected out of nine possible candidates. This reduces the number of simulations necessary to estimate the robustness indicators for a given design. For example, in a probabilistic robustness definition approach, if the objective functions are approximated by second-order Taylor expansions to estimate their expected value and variance [28], considering three parameter dispersions instead of nine decreases the number of required simulation from 55 to 10. For this problem, this complete sensitivity estimation could not have been obtained using existing sensitivity analysis methods. By employing methods from [2], [4]-[9], the effect of tolerances could have been estimated for a given design, but the dependence of this effect on the considered design would not have been brought to light, and some potential robustness issues could have been missed. Using global sensitivity analysis methods applying parameter variations which are large with respect to the tolerance interval spans, like those used in [17] and [18], the high sensitivity of some criteria to parameter variations in their tolerance intervals would not have been raised: a good example is the parameter R_{HPM}, which has a moderate effect on torque engine order 48 for large variations, but whose deviations can be very influent on this criterion for some designs and should therefore be considered in a robust design or optimization approach to avoid robustness issues.

Moreover, the presented results improve the knowledge of the effect of tolerances on electric motor noise and vibrations. In particular, the following conclusions regarding electric motor robustness can be made:

- For the studied motor, the NVH indicators are more sensitive to parameter variations than the mean output torque. The effect of a given parameter on the NVH criteria is generally more dependent on the design (i.e. the values of the other parameters) than its effect on mean torque. This means that the possibilities of optimizing NVH criteria are broader but are potentially subject to more robustness issues than the mean torque. Robust vibroacoustic design approaches are therefore particularly relevant, even in cases where robust motor optimization to improve its other electrotechnical performance criteria is not necessary.
- Depending on the prevailing phenomena in the noise generation process and on the motor topology, the different NVH criteria of the considered motor are more or less likely to have robustness issues. For some criteria, one could expect classical optimization methods to converge towards designs having acceptable robustness characteristics. For other criteria, the variations of some parameters in their tolerance intervals may have a significant influence, for some specific designs even more than larger parameter variations in their design range. In this case, robust optimization methods may be necessary.

Future work should include the combination of the presented sensitivity analysis method with a robust optimization method to achieve robust reductions of optimization objectives which cannot be robustly minimized using classical optimization methods. Robust optimization methods are very time-consuming; the effectivity and validity of the parameter deviation selection approach presented in this paper to reduce robustness evaluation time will be evaluated in the course of this future work.

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