Energy shaping plus Damping injection of Irreversible Port Hamiltonian Systems

Ignacio Villalobos * **Hector Ramrez** * **Yann Le Gorrec** **

* *Electronics Engineering Department, Universidad Tcnica Federico Santa Mara (UTFSM), Chile.* ignacio.villalobos.a@gmail.com, hector.ramireze@usm.cl

** *Department of Automation and Micro-Mechatronic Systems, FEMTO-ST UMR CNRS 6174, UBFC, 26 chemin de l'épitaphe, F-25030 Besançon, France.* legorrec@femto-st.fr

Abstract: Irreversible port Hamiltonian Systems (IPHS) are an extension of the classical Port Hamiltonian System (PHS) formulation for reversible systems. Irreversible port Hamiltonian systems not only expresses the conservation of the energy but also the irreversible entropy production as an structural property. These systems can be used to model and control a large class of thermodynamics systems such as RLC circuits with dissipation, heat ex-changers, continuous stirred tank reactor (CSTR) and in general multi-energy systems that have reversible and irreversible process. Control techniques such as IDA-PBC and Energy Shaping have been used to propose controllers for these systems using an energy based availability function to derive a Lyapunov stable closed loop energy function. This paper presents a systematic method using an energy based availability function with Energy Shaping and Damping Injection techniques to derive a controller for an IPHS system.

Keywords: Irreversible port Hamiltonian Systems, Irreversible thermodynamics, Energy Shaping, Damping Injection, RLC system, CSTR.

1. INTRODUCTION

The derivation of the control laws usually is based on an appropriate choice of a Lyapunov function which can be linked to the Hamiltonian of the system and derived through the resolution of a set of associated matching equations. Then synthesize an energy shaping controller to modify the equilibrium of the Lyapunov function with a damping injection input to converge to this point (van der Schaft and Jeltsema, 2014). In irreversible phenomena, the irreversible process of the system makes the energy function a non convex function. In Ramirez et al. (2013a) the framework of the IPHS has been combined with the framework of thermodynamic availability function (Alonso and Ydstie, 1996); (Alonso and Ydstie, 2001); (Jillson and Ydstie, 2007) to derive an asymptotically stability condition for irreversible thermodynamic systems.

In (Ramirez et al., 2013a) a class of quasi PHS, nameley irrversible port-Hamiltonian systems (IPHS), has been proposed to model a large class of thermodynamic systems. These systems express as a structural property the first and second principles of thermodynamics. IPHS have are a non-linear system with a physically meaningful structure and just as PHS systems, they are defined with respect to the total energy of the system which make it possible to interconnect them with other reversible or non-reversible systems (Ramirez et al., 2013b). Recently the IPHS structure have been employed to derive non-linear passivity based controllers (Ramrez et al., 2016).

In this paper we propose a systematic control method for IPHS using Casimir invariant function in order to shape the closed loop energy function with an energy shaping-damping injection control input in order to the energy function to converge to the desire minimum point and to be shaped. We use the framework of an availability function for the irreversible process of the systems and derive an energy based controller for an IPHS. The paper is organized as follows. Section 2 presents the basics on IPHS. In section 3 we derive an energy shaping plus damping injection controller for an IPHS and we get as an example a controller for a RLC system. Section 4 shows the IPHS representation of a CSTR system and we get an energy shaping plus damping injection controller for a particular case of a reaction of two species. In section 5 we present some conclusions of the work.

2. IRREVERSIBLE PORT HAMILTONIAN SYSTEMS

IPHS have been proposed in Ramirez et al. (2013b) as an extension of PHS. These system represent not only the energy balance but also the entropy balance associated with the irreversible process. Let us first define a Poisson bracket (Maschke et al., 1992) with respect to a constant skew symmetric matrix $J = -J^\top$ acting on any two smooth functions Z and G as

$$\{Z, G\}_J = \frac{\partial Z^T}{\partial x}(x) J \frac{\partial G}{\partial x}(x). \tag{1}$$

Definition 1. An IPHS is defined by the dynamical equation

$$\dot{x} = J_{ir}\left(x, \tfrac{\partial U}{\partial x}\right)\frac{\partial U}{\partial x} + g\left(x, \tfrac{\partial U}{\partial x}\right)u \tag{2}$$

$$y = g\left(x, \tfrac{\partial U}{\partial x}\right)^{\top}\frac{\partial U}{\partial x} \tag{3}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ the input, $y(t) \in \Re^m$ the output, the smooth function $U(x) : \Re^n \to \Re$ is the Hamiltonian and $g \in \Re^{n\times m}$ is the input map. The skew-symmetric structure matrix $J_{ir} \in \Re^{n\times n}$ is defined as

$$J_{ir}\left(x, \tfrac{\partial U}{\partial x}\right) = J_0(x) + R\left(x, \tfrac{\partial U}{\partial x}\right)J \tag{4}$$

with $J = -J^T$, $J_0 = -J_0^T$ and there exists a smooth entropy like function $S(x) : \Re^n \to \Re$ which is a Casimir function of J_0, i.e.,

$$\frac{\partial S}{\partial x}^{\top} J_0 = 0. \tag{5}$$

The non-linear modulating function R is defined as

$$R\left(x, \tfrac{\partial U}{\partial x}\right) = \gamma\left(x, \tfrac{\partial U}{\partial x}\right)\{S, U\}_J \tag{6}$$

where $\gamma\left(x, \frac{\partial U}{\partial x}\right) : \Re^n \to \Re$, $\gamma \geq 0$, a non linear positive function.

Definition 1 is the composition of a conservative and a non-conservative dynamics characterized respectively by the matrices J_0 and RJ. The balance equations of the entropy function $S(x)$ and the energy function $U(x)$ of the IPHS express the first and second principles of thermodynamics: the conservation of the energy and the irreversible creation of entropy. Taking the time derivative of the energy function gives

$$\begin{aligned}\frac{dU}{dt} &= \frac{dU^T}{dx}(J_0 + RJ)\frac{dU}{dx} + \frac{dU^T}{dx}gu\\ &= y^T u\end{aligned}$$

by skew-symmetry of J_{ir}, expressing that the IPHS is a lossless dissipative system with supply rate $y^T u$. If we take the time derivative of the entropy function it follows that

$$\begin{aligned}\frac{dS}{dt} &= \frac{dS^T}{dx}J_0\frac{dU}{dx} + R\frac{dS^T}{dx}J\frac{dU}{dx}\\ &= \{S, U\}_{J_0} + \gamma\left(x, \tfrac{\partial U}{\partial x}\right)\{S, U\}_J^2\\ &= \gamma\left(x, \tfrac{\partial U}{\partial x}\right)\{S, U\}_J^2 = \sigma \geq 0\end{aligned}$$

where the therm $\{S, U\}_{J_0} = 0$ because of (5) and where σ corresponds to the internal entropy production.

2.1 Example: RLC plus dissipation system

Consider a RLC system including the dynamics of the thermal effects of its electrical components. So we can consider that all electrical components are a function of the temperature. The IPHS formulation of the thermodynamic RLC circuit is (Ramirez et al., 2019)

$$\begin{bmatrix}\dot{Q}\\ \dot{\phi}\\ \dot{S}\end{bmatrix} = \left(\begin{bmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} + \frac{r}{T}\frac{\phi}{L}\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\right)\begin{bmatrix}\frac{Q}{C}\\ \frac{\phi}{L}\\ \frac{L}{T}\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\end{bmatrix}u \tag{7}$$

where the internal energy $U_e(Q, \phi, S)$ is the sum of the electrical energy plus an entropy like function

$$U_e(Q, \phi, S) = \frac{1}{2}\frac{Q^2}{C(S)} + \frac{1}{2}\frac{\phi^2}{L(S)} + U_s(S) \tag{8}$$

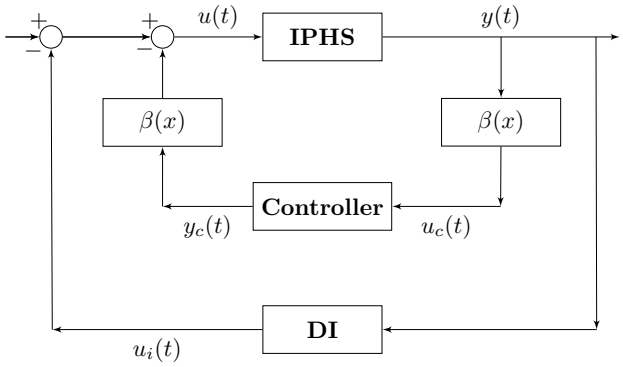

Fig. 1. Energy shaping plus damping injection control of an IPHS.

with time variation given by

$$\begin{aligned}\frac{dU_e}{dt} &= \frac{\partial U_e}{\partial Q}\dot{Q} + \frac{\partial U_e}{\partial \phi}\dot{\phi} + \frac{\partial U_e}{\partial S}\dot{S}\\ &= -r(S)\left(\tfrac{\phi}{L(S)}\right)^2 + \frac{\partial U_e}{\partial S}\frac{dS}{dt} + y_e^T u_e\end{aligned}$$

From Gibb's relation (Callen, 1985) it is known that $\frac{\partial U_e}{\partial S} = T(S)$. Taking $u_e = 0$ it follows that $\dot{U}_e = 0$ and it goes that

$$\frac{dS}{dt} = \frac{r(S)}{T(S)}\left(\frac{\phi}{L(S)}\right)^2 = \sigma_r \tag{9}$$

with σ_r the internal entropy production of the system. Notice that (7) has the structure of the Definition 1.

3. PASSIVITY BASED CONTROL OF IPHS

Passivity based control (PBC)techniques use the passivity properties of a system to achieve closed-loop stability. They take benefit of the existence of an open-loop storage function to drive the system to a desired equilibrium point (van der Schaft and Jeltsema, 2014). Among the PBC techniques there are two of particular interest: damping injection, which consists in adding damping to a system such that it is driven to the minimum of its energy function, and energy shaping which consists in changing the closed-loop energy function such that it has a minimum at the desired equilibrium. The main difference between a conservative or reversible system and an irreversible thermodynamic system, is that while the reversible energy function has its minimum at the dynamic equilibrium the internal energy has not. Hence the internal energy cannot be used as a candidate Lyapunov function for PBC design. In (Alonso and Ydstie, 2001) the *available storage function* is introduced as candidate Lyapunov function for PBC of irreversible thermodynamic processes and different definitions of this function have been used for Lyapunov based control of thermodynamic systems (Alonso and Ydstie, 1996, 2001; Ydstie, 2002; Hoang et al., 2011, 2012) and Ydstie (2002). Based on that work, let us introduce the definition of the energy based availability function (Ramrez et al., 2016).

Definition 2. The energy based availability function is defined as

$$A(x, x^*) = U(x) + U_a(x, x^*) \tag{10}$$

Where $U_a(x, x^*) = -U(x^*) - \frac{\partial U}{\partial x}(x^*)^T(x - x^*)$, with $U(x)$ being the internal energy of the system and x^* the

desired equilibrium point of the system. $A(x, x^*)$ is strictly positive as soon as one of the extensive variables is fixed.

The energy based availability function is hence constructed using the internal energy of the system and it will be strictly positive as long as one of the extensive variables is fixed (see (Ramrez et al., 2016; Alonso and Ydstie, 2001; Jillson and Ydstie, 2007) more details). On the other hand, control by interconnection by means of Casimir function has proven to be a powerful tool for energy shaping control of PHS (van der Schaft and Jeltsema, 2014). According to the Definition 1, an IPHS system can be seen as the interconnection of a purely conservative part and purely dissipative one. This property can be exploited for the design of energy shaping controllers by combining the use of Casimir functions and the availability function.

3.1 Energy Shaping for IPHS

Casimir functions, which are structural invariants of a system are instrumental to perform energy shaping of PHS (van der Schaft and Jeltsema, 2014). They are characterized by a set of partial differential equations (PDE) which depend on the structure matrices of a system. Consider a control system in IPHS form

$$\begin{aligned}\dot{\xi} &= \bar{R}\left(\xi, \tfrac{\partial U_c}{\partial \xi}\right)(J_c - R_c)\frac{\partial U_c}{\partial \xi}(\xi) + g_c\left(\xi, \tfrac{\partial U_c}{\partial \xi}\right)u_c(t)\\ y_c &= g_c^T\left(\xi, \tfrac{\partial U_c}{\partial \xi}\right)\frac{\partial U_c}{\partial \xi}(\xi)\end{aligned} \tag{11}$$

with $\xi \in \Re^l$ the state space vector, $u(t) \in \Re^m$ the input, $y(t) \in \Re^m$ the output, $g_c(\xi) \in \Re^{l\times m}$ the input map and Hamiltonian function $U_c(\xi)$. Define the state modulation power-preserving interconnection

$$\begin{pmatrix} u \\ u_c \end{pmatrix} = \begin{pmatrix} 0 & -\beta(x) \\ \beta(x) & 0 \end{pmatrix}\begin{pmatrix} y \\ y_c \end{pmatrix} \tag{12}$$

where $\beta(x) \in \Re$. The closed loop system then takes the form

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} J_{ir} & -g\beta g_c^T \\ g_c\beta g^T & \bar{R}(J_c - R_c) \end{pmatrix}\begin{pmatrix} \frac{\partial U_{cl}}{\partial x} \\ \frac{\partial U_{cl}}{\partial \xi} \end{pmatrix} \tag{13}$$

with closed-loop Hamiltonian function $U_{cl}(x, \xi) = U(x) + U_c(\xi)$. Defining

$$J_{cl} = \begin{pmatrix} J_{ir} & -g\beta g_c^T \\ g_c\beta g^T & \bar{R}(J_c - R_c) \end{pmatrix} \tag{14}$$

Let us explode the Casimir function potentials to control system design. We look for structural invariant functions of the form $C_i(x, \xi_i) = F_i(x) - \xi_i, i = 1, .., l$ with $F(x) = [F_1, ..., F_l] \in \Re^l$ some smooth well defined function of x, then on every invariant manifold defined by $\xi - F(x) = \kappa$ with $\kappa = [\kappa_1, ..., \kappa_l] \in \Re^l$ a constant that depends of the initial states of the plant and the controller. Then, the closed loop Hamiltonian energy function can be rewritten as $U_{cl}(x, \xi) = U(x) + U_c(F(x) + \kappa)$. As an invariant to the structure of the system, the follow relation $\frac{\partial C^T(x,\xi)}{\partial x}J_{cl} = 0$ holds for every pair (x, ξ). This condition leads to the set of partial differential equations

$$\begin{aligned}\frac{\partial F^T}{\partial x}(x)J_{ir} &= g_c\beta g^T\\ -\frac{\partial F^T}{\partial x}(x)g\beta g_c^T &= \bar{R}(J_c - R_c)\end{aligned} \tag{15}$$

Following the same procedure as in van der Schaft and Jeltsema (2014) we get the set of partial differential equations

$$\begin{aligned}\frac{\partial F^T}{\partial x}(x)J_{ir} &= g_c\beta g^T\\ R_c &= 0\\ \frac{\partial F^T}{\partial x}J_{ir}\frac{\partial F}{\partial x} &= \bar{R}J_c\end{aligned} \tag{16}$$

These are the matching equations for an IPHS system with an IPHS controller form, using a modulating function β between inputs and outputs. These matching equations can be interpreted in the framework of the matching equations of a PHS (van der Schaft and Jeltsema, 2014) but with an interconnection matrix J_{ir}, with a scalar function β and with a modulating function $\bar{R}$. Assuming that such smooth function $F(x)$ exists then the control law $u_e(x) = -\beta(x)g_c^T\frac{\partial U_c(F+\kappa)}{\partial \xi}$ shaped the closed loop Hamiltonian energy function $U_{cl}(x) = U(x)+U_c(F(x+\kappa))$ and the reduced IPHS may be written as

$$\dot{x} = J_{ir}\frac{\partial}{\partial x}(U + U_c \circ F) = J_{ir}\frac{\partial U_{cl}}{\partial x}$$

3.2 Damping Injection for IPHS

The energy shaping control input shaped the closed loop Lyapunov energy function with respect to a desire equilibrium point, but one have yet to stabilize the system to actually converge to this desire point. Lets suppose that the Hamiltonian function is shaped with respect to a point x^* with the control input $u_e = -\beta(x)\frac{\partial U_c oF(x)}{\partial x}$. Thus we have the closed loop system

$$\dot{x}(t) = J_{ir}\frac{\partial U_{cl}}{\partial x} + gu(t) \tag{17}$$

Lets close the system (17) with a damping injection input such that $u_i = -Kg^t\frac{\partial U_{cl}}{\partial x}$. The new system can be written as

$$\dot{x}(t) = (J_{ir} - gKg^T)\frac{\partial U_{cl}}{\partial x} \tag{18}$$

Then taking the time derivative of the closed loop energy function $U_{cl}(x)$ follows

$$\begin{aligned}\frac{dU_{cl}}{dt} &= \frac{dU_{cl}^T}{dx}\frac{dx}{dt}\\ &= \frac{dU_{cl}^T}{dx}(J_{ir} - gKg^T)\frac{\partial U_{cl}}{\partial x}\\ &= \{U_{cl}, U_{cl}\}_{J_{ir}} - \{U_{cl}, U_{cl}\}_M\end{aligned}$$

Where $\{U_{cl}, U_{cl}\}_{J_{ir}} = 0$ and then

$$\frac{dU_{cl}}{dt} = -\{U_{cl}, U_{cl}\}_M < 0$$

with $M = gKg^T$ a semipositive symmetric matrix; this implies that $K = K^T$ and such that $M \geq 0$. Using the Lasalle's theorem, the control law $u(t) = -\beta(x)\frac{\partial U_c oF(x)}{\partial x} - Kg^t\frac{\partial U_{cl}}{\partial x} = u_e(t)+u_i(t)$ stabilizes the system to the largest invariant set included in the set $\{x \in \Re \; s.t \; g^T\frac{\partial U_{cl}}{\partial x} = 0\}$.

3.3 Example: Control of the RLC system

Consider the IPHS model that it is shown in the section 2, Definition 1. We will apply the results of the subsections 3.1 and 3.2 to get an energy-shaping plus damping injection input control. We want to look for Casimir functions of the form $C(x,\xi) = F(x) - \xi$ such as $\xi = F(x) + \kappa$ for the system (7) with $F(x) = F(Q,\phi,S) : \Re^3 \to \Re$ a smooth function and with the IPHS controller

$$x_c = \bar{R} J_c \frac{\partial U_c}{\partial x_c} + g_c u_c$$

By applying the third equation of (16) it gets $J_c = 0$ and $g_c = 1$. The first equation of (16) results in the system

$$-\frac{\partial F}{\partial \phi} = 0, \qquad \frac{\partial F}{\partial Q} + \frac{r\phi}{TL}\frac{\partial F}{\partial S} = \beta, \qquad -\frac{r\phi}{TL}\frac{\partial F}{\partial \phi} = 0$$

This system has multiple solution. We take

$$\frac{\partial F}{\partial Q} = \alpha_1 \qquad \frac{\partial F}{\partial \phi} = 0 \qquad \frac{\partial F}{\partial S} = \alpha_2$$

with $\beta = \alpha_1 + \alpha_2 \frac{r\phi}{TL}$ where $\alpha_1, \alpha_2 \in \Re$. The smooth function F then takes the form

$$F = \alpha_1 Q + \alpha_2 S$$

The Casimir obtained allows to make control on the charge of the capacitor and on the entropy of the system. The desired Hamiltonian energy for the closed loop is

$$U_{cl}(Q,\phi,S) = \frac{1}{2}\frac{(Q-Q^*)^2}{C} + \frac{1}{2}\frac{\phi^2}{L} + A(S,S^*)$$

where $A(S,S^*) = U_s(S) - [U_s(S^*) + T^*(S-S^*)$ is the availability function (10) for the irreversible phenomena of the system. The interconnection between the controller and the system has a closed loop energy function given by

$$U_{cl}(Q,\phi,S) = U + U_c$$

where U_c is the energy of the controller and U is the internal energy of the system. The simplest choice for the energy of the controller is then

$$U_c = -\frac{QQ^*}{C} + \frac{1}{2}\frac{(Q^*)^2}{C} - U_s(S^*) - T(S^*)(S-S^*)$$

Taking $\alpha_1 = \frac{-Q^*}{C}$ and $\alpha_2 = -T^*$ with $\kappa = \frac{1}{2}\frac{(Q^*)^2}{C} - U_s(S^*) + T^*S^*$ it follows that $\beta = \frac{-Q^*}{C} - T^*\frac{r\phi}{TL}$. The control input is then given by

$$u_e = -\beta \frac{\partial U_c}{\partial \xi} = -\beta = \frac{Q^*}{C} + T^*\frac{r\phi}{TL}$$

We have the energy shaping control input so we have yet to design the damping injection input in order for the closed loop system to converge to x^*. Following the result presents in the subsection 3.2 we take the control input $u_i(t) = -Kg^t\frac{\partial U_{cl}}{\partial x}$ with K a positive constant. Taking $K = \alpha > 0$ as an adjustment parameter, the damping control input is then

$$u_i(t) = -\alpha\frac{\phi}{L}$$

The control law for the system takes the form

$$u(t) = \frac{Q^*}{C} + T^*\frac{r\phi}{TL} - \alpha\frac{\phi}{L}$$

which is a classic proportional plus integration control law. It is interesting to note that the control input of the energy shaping already has a damping input with the term $T^*\frac{r\phi}{TL}$ which it comes natural when we note that the IPHS is not entirely dissipative.

4. EXAMPLE: THE CSTR SYSTEM

In this section we will get an energy shaping plus damping injection controller for the CSTR System. We will show its IPHS model in subsection 1 and get a particular case of a reaction of two species. In subsection 2 we get the controller for this particular case.

4.1 IPHS Model

Let us consider a CSTR system with the following reversible reaction scheme:

$$\sum_{i=1}^{m} \xi_i A_i \overset{r}{\rightleftharpoons} \sum_{i=1}^{m} \eta_i A_i \tag{19}$$

with ξ_i, η_i being the constant stoichiometric coefficients for species A_i in the reaction. We will consider the following assumptions for the standard operation of the reactor (Aris, 1989); (Favache and Dochain, 2009):

1. The reactor operates in liquid phase.
2. The molar volume of each species are identical and the total volume V in the reactor is constant through the reaction.
3. The initial number of moles of a species in the reactor is equal to the number of moles of the inlet of the sames species.
4. For a given steady state temperature T and steady state input there is only one possible steady state for the mass. This mean that each steady state temperature is associated with a one unique steady state temperature.

Lets show now the IPHS model of the CSTR system which has been derived before in Ramrez et al. (2016). The system can be expressed as an IPHS system as

$$\dot{x}(t) = RJ\frac{\partial U}{\partial x}(x) + gu(t)$$

with the state vector $x = [\mathbf{n}\ S]^T$, where $\mathbf{n} = (n_1, ..., n_m)^T$ with n_i the number of moles of the species i inside the reactor; S the total entropy of the system and $U(x)$ the Hamiltonian energy function.

$$J = \begin{bmatrix} 0 & \cdots & 0 & \bar{\nu}_1 \\ & & & \\ 0 & \cdots & 0 & \vdots \\ 0 & \cdots & 0 & \bar{\nu}_m \\ -\bar{\nu}_1 & \cdots & -\bar{\nu}_m & 0 \end{bmatrix}, \frac{\partial U}{\partial x} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_m \\ T \end{bmatrix}$$

where J is a constant skew-symmetric matrix whose elements are the signed stoichiometric coefficients of the chemical reaction $\bar{\nu}_i = \xi_i - \eta_i$, a number which is positive or negative depending on whether the species i is a product or a reactant; $\frac{\partial U}{\partial x}$ the differential energy vector of intensive variables with T being the temperature in the reactor and μ_i the chemical potencial of the species i; R is the modulating function and is given by

$$R = \frac{rV}{T}$$

where $r = r(\mathbf{n}, T)$ is the reaction rate which depends on the temperature and on the reactant mole numbers vector $\mathbf{n}$. The input vector is $u = [u_1, u_2]^T$ with $u_1 = F/V$ the dilution rate, where F is the volumetric flow rate, and $u_2 = Q$ the heat flux from the cooling jacket; the input map g is given by

$$g = \begin{bmatrix} \bar{\mathbf{n}} & \mathbf{0} \\ \phi(x) & 1/T \end{bmatrix}$$

with $\bar{\mathbf{n}} = \mathbf{n}_e - \mathbf{n}$, where $\mathbf{n}_e = (n_{e1}, ..., n_{em})^T$ is the vector containing the numbers of moles of species i at the inlet and $\phi(x) = \sum_{i=1}^{m}(n_{ei}s_{ei} - n_i s_i) + \frac{n_{ei}}{T}(h_{ei} - Ts_{ei} - \mu_i)$, where s_{ei} is the inlet molar entropy,s_i is the molar entropy and h_{ei} is the inlet specific molar ethalpy of species i.

As a particular case, in order to control the system, we take $m = 2$ and the reaction is

$$\xi_1 A_1 + \xi_2 A_2 \overset{r}{\rightleftharpoons} \eta_1 A_1 + \eta_2 A_2 \tag{20}$$

This reaction then can be modeling with the constant skew-symmetric matrix, the differential energy vector and the input map, respectively

$$J = \begin{bmatrix} 0 & 0 & \bar{\nu}_1 \\ 0 & 0 & \bar{\nu}_2 \\ -\bar{\nu}_1 & -\bar{\nu}_2 & 0 \end{bmatrix} \quad \frac{\partial U}{\partial x} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ T \end{bmatrix} \quad g = \begin{bmatrix} \bar{\mathbf{n}}_1 & 0 \\ \bar{\mathbf{n}}_2 & 0 \\ \phi(x) & 1/T \end{bmatrix}$$

with $\phi(x) = \sum_{i=1}^{2}(n_{ei}s_{ei} - n_i s_i) + \frac{n_{ei}}{T}(h_{ei} - Ts_{ei} - \mu_i)$ and with state space vector $x(t) = [\mathbf{n}_1 \ \mathbf{n}_2 \ S]^T$; the input of the system is $u = (u_1, u_2)^T$ with an internal energy of the system U.

4.2 Passivity based control of the CSTR

In this section we shall get a controller for the CSTR for the particular case $m = 2$. The system has by states n_1, n_2, S so we will parametrize the design and look for Casimir functions of the form $C_1(n_1, \xi_1) = F_1(n_1) - \xi_1$, $C_2(n_2, \xi_2) = F_2(n_2) - \xi_2$ and $C_3(S, \xi_3) = F_3(S) - \xi_3$ such that $\xi_1 = F_1(n_1) + \kappa_1$, $\xi_2 = F_2(n_2) + \kappa_2$ and $\xi_3 = F_3(S) + \kappa_3$ with $F(\mathbf{n}_1, \mathbf{n}_2, S) = [F_1 \ F_2 \ F_3] : \Re^3 \to \Re^3$ a smooth function. Taking the IPHS controller

$$x_c = \bar{R} J_c \frac{\partial U_c}{\partial x_c} + g_c u_c$$

The state space vector of the controller is then $x_c = [\xi_1 \ \xi_2 \ \xi_3]^T$ with $J_c \in \Re^{3\times 3}$ a skew symmetric matrix; the input map is such that $g_c \in \Re^{3\times 2}$ and the modulating state space β a scalar function. For the design of the controller we take

$$g_c = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \end{bmatrix}$$

For the energy design using the availability function, we would like to get an energy close loop Hamiltonian function $U_{cl} = U(x) - [U(x^*) + \frac{\partial U}{\partial x}(x^*)^T(x - x^*)]$. The energy of the interconnection between the system and the controller is also $U_{cl} = U + U_c$ where U is the internal energy of the system and U_c is the energy of the controller. Thus the controller energy is chosen as

$$U_c = -[U(x^*) + \frac{\partial U}{\partial x}(x^*)^T(x - x^*)]$$

Energy which is equal to

$$U_c = (-\mu_1^* n_1 + \mu_1^* n_1^*) + (-\mu_2^* n_2 + \mu_2^* n_2^*) + (-T^* S + T^* S^*) - U(n_1^*, n_2^*, S^*)$$

With the election of the Casimir function and with the energy of the controller the vector of partial differential equations of the function F is

$$\frac{\partial F}{\partial x} = \begin{bmatrix} -\mu_1^* & 0 & 0 \\ 0 & -\mu_2^* & 0 \\ 0 & 0 & -T^* \end{bmatrix}$$

Lets us apply now the matching equations. From the first equation of (16) we get the system of equations

$$g_{11} = 0 \qquad g_{21} = 0 \qquad g_{31} = \frac{T^*\bar{\nu}_2}{\bar{n}_2}$$

$$g_{12} = -\mu_1^*\bar{\nu}_1 T^* \qquad g_{22} = -\mu_2^*\bar{\nu}_2 T^* \qquad g_{31} = \frac{T^*\bar{\nu}_1}{\bar{n}_1}$$

$$g_{32} = -g_{31}\phi T^* \qquad \beta = R$$

The system has a solution if

$$\frac{\bar{n}_1}{\bar{\nu}_1} = \frac{\bar{n}_2}{\bar{\nu}_2} \tag{21}$$

In Prigogine and Defay. (1954) for batch reactors the equality (21) is the expression of De Donder's extent reaction

$$\frac{n_{0i} - n_i}{\bar{\nu}_i} = \xi$$

where this property can be easily extended to the CSTR with assumption 3 and such that the numbers of moles of each specie is equal to the numbers of moles at the inlet: $\mathbf{n}(t = 0) = \mathbf{n}_0 = \mathbf{n}_e$ (Aris, 1989). The result in (21) is a particular case of the condition obtained in Ramrez et al. (2016) where an IDA-PBC light approach is taken. The third equation of (16) gives

$$J_c = T^* \begin{bmatrix} 0 & 0 & \mu_1^*\bar{\nu}_1 \\ 0 & 0 & \mu_2^*\bar{\nu}_2 \\ -\mu_1^*\bar{\nu}_1 & -\mu_2^*\bar{\nu}_2 & 0 \end{bmatrix}$$

with $\bar{R} = T^*$ a real constant. The controller then can be written as

$$x_c = T^* \begin{bmatrix} 0 & 0 & \mu_1^*\bar{\nu}_1 \\ 0 & 0 & \mu_2^*\bar{\nu}_2 \\ -\mu_1^*\bar{\nu}_1 & -\mu_2^*\bar{\nu}_2 & 0 \end{bmatrix} \begin{bmatrix} -\mu_1^* \\ -\mu_2^* \\ -T^* \end{bmatrix} + \begin{bmatrix} 0 & -\mu_1^*\bar{\nu}_1 T^* \\ 0 & -\mu_2^*\bar{\nu}_2 T^* \\ T^*\xi & -(T^*)^2\xi\phi \end{bmatrix} u_c$$

$$y_c = g_c^T \begin{bmatrix} -\mu_1^* \\ -\mu_2^* \\ -T^* \end{bmatrix}$$

The energy shaping control law $u_e = -\beta g_c^T \frac{\partial U_c}{\partial x_c}$ is then given by

$$u_e = -\frac{rV}{T} \begin{bmatrix} -(T^*)^2\xi \\ -(\mu_1^*)^2\bar{\nu}_1 T^* - (\mu_2^*)^2\bar{\nu}_2 T^* - (T^*)^3\xi\phi \end{bmatrix}$$

Recall that the damping injection input is given by $u_i = -Kg^T\frac{\partial U_{cl}}{\partial x}$. We design $K \in \Re^{3\times 3}$ such that $M = gKg^T \geq 0$ with $K = K^T$. A simple possible choice is to take $M = diag(0, 0, \alpha)$, with $\alpha > 0$. The election of K that allows this is $K = \alpha$ diag$(0, 0, T^2)$ and then the damping input takes the form

$$u_i = \alpha \begin{bmatrix} 0 \\ T(T - T^*) \end{bmatrix}$$

The closed loop system is then

$$\dot{x} = (-gKg^T + RJ)\frac{\partial U_{cl}}{\partial x}$$

with $\frac{\partial U_{cl}}{\partial x} = [\mu_1 - \mu_1^* \ \mu_2 - \mu_2^* \ T - T^*]^T$. We note that the closed loop system is also an IPHS with dissipation gKg^T. It has yet to prove that the closed loop system is asymptotically global stable. In effect, taking the time derivative of the Hamiltonian closed loop energy we get

$$\frac{dU_{cl}}{dt} = -\frac{\partial U_{cl}^T}{\partial x} M \frac{\partial U_{cl}}{\partial x} = -\alpha(T - T^*)^2$$

which is positive defined and vanishing only at $T = T^*$. By applying the La Salle's invariance theorem in a sufficient small region of $T = T^*$ and with assumption 4 then the proof follows.

5. CONCLUSION

This papers presents a systematic design method based on the IPHS extended formulation, using the thermodynamic availability function for irreversible part with an IPHS controller structure, and a modulated state function for the output and input of the system. An energy-shaping controller has been design using the framework of the well known Casimir functions to find structural invariants for the system. Casimir function has to satisfy the matching equations in order to be useful for the system. The controller modify the natural equilibrium point of the system but it doesn't guarantee the the system actually converge to this point. A damping injection input control has been design in order for the system to be asymptotically global stable. A controller for the PHS with thermal effects, which is a system with a reversible-irreversible dynamic, has been design using the energy shaping plus damping injection technique using the availability function for the irreversible process. A controller for a CSTR system, which is purely dissipative, with two reaction has also been design proposing an energy controller function and finding Casimir functions that satisfy the matching equations. Future work will deal with numerical implementations of the controller.

REFERENCES

Alonso, A.A. and Ydstie, B.E. (1996). Process systems, passivity and the second law of thermodynamics. *Computers & Chemical Engineering*, 20, S1119 – S1124.

Alonso, A.A. and Ydstie, B. (2001). Stabilization of distributed systems using irreversible thermodynamics. *Automatica*, 37(11), 1739 – 1755.

Aris, R. (1989). Elementary chemical reactor analysis. *Butterworths Series in Chemical Engineering*.

Callen, H. (1985). Thermodynamics and an introduction to thermostatistics. *Wiley, New-York*.

Favache, A. and Dochain, D. (2009). Thermodynamics and chemical systems stability: The cstr case study revisited. *Journal of Process Control*, 19(3), 371 – 379.

Hoang, H., Couenne, F., Jallut, C., and Gorrec, Y.L. (2011). The port Hamiltonian approach to modeling and control of continuous stirred tank reactors. *Journal of Process Control*, 21(10), 1449 – 1458.

Hoang, H., Couenne, F., Jallut, C., and Gorrec, Y.L. (2012). Lyapunov-based control of non isothermal continuous stirred tank reactors using irreversible thermodynamics. *Journal of Process Control*, 22(2), 412 – 422.

Jillson, K.R. and Ydstie, B.E. (2007). Process networks with decentralized inventory and flow control. *Journal of Process Control*, 17(5), 399 – 413.

Maschke, B., Schaft, A.V.D., and Breedveld, P. (1992). An intrinsic Hamiltonian formulation of network dynamics: non-standard poisson structures and gyrators. *Journal of the Franklin Institute*, 329(5), 923 – 966.

Prigogine, I. and Defay., R. (1954). Treatise on thermodynamics. *Chemical thermodynamics. London, Great Britain: Longmans Green and Co*, 1.

Ramirez, H., Maschke, B., and Sbarbaro, D. (2013a). Irreversible port-Hamiltonian systems: A general formulation of irreversible processes with application to the cstr. *Chemical Engineering Science*, 89, 223 – 234.

Ramirez, H., Maschke, B., and Sbarbaro, D. (2013b). Modelling and control of multi-energy systems: An irreversible port-Hamiltonian approach. *European Journal of Control*, 19(6), 513 – 520.

Ramirez, H., Sbarbaro, D., and Gorrec, Y.L. (2019). Irreversible port-Hamiltonian formulation of some non-isothermal electrochemical processes. *IFAC-PapersOnLine*, 52(7), 19 – 24.

Ramrez, H., Gorrec, Y.L., Maschke, B., and Couenne, F. (2016). On the passivity based control of irreversible processes: A port-Hamiltonian approach. *Automatica*, 64, 105 – 111.

van der Schaft, A. and Jeltsema, D. (2014). Port-Hamiltonian systems theory: An introductory overview. *Foundations and Trends in Systems and Control*, 1(2-3), 173–378.

Ydstie, B. (2002). Passivity based control via the second law. *Computers & Chemical Engineering*, 26(7), 1037 – 1048.