Efficient broadband vibration energy harvesting based on tuned nonlinearity and energy localization

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Abstract. In this letter, nonlinearity and energy localization are experimentally tuned in an electromagnetic vibration energy harvester in order to enhance its output performances. The nonlinear device consists of two moving magnets guided by elastic beams and coupled by a repulsive magnetic force. The mechanical nonlinearity is introduced by considering large displacements of the beams and the energy localization is achieved by mistuning the mass of one of the moving magnets. The critical load resistance is determined experimentally in order to tune the nonlinearity level and to drive the harvester beyond its critical amplitude. Consequently, the performances of the device, in terms of harvested power only from the perturbed dof oscillator and frequency bandwidth, are enhanced respectively up to 19.4% and 116% compared to the performances of the nonlinear periodic system.

Keywords: Vibration energy harvesting, magnetic coupling, energy localization, geometric nonlinearity, multimodal method, quasiperiodic structure.

1. Introduction

Over the last years, various techniques of energy harvesting from different sources have been proposed and deeply studied [1]. Energy harvesting from vibrations is one of the most studied concepts [2] which aims at converting the kinetic energy due to the vibrations of a system to electricity. Although the progress in energy harvesting field is continuous, energy harvesters still have limitations [3]. For instance, most existing vibration energy harvesting devices are effectively operating in a narrow bandwidth around their resonance frequency which limits their application in domains where energy prevails over a larger frequency bandwidth. To overcome this problem, several approaches have been proposed namely the adoption of multimodal configurations [4, 5, 6] and the introduction of the nonlinearity [7]. These methods have been widely studied and proved reliable results in terms of increasing the total harvested power or the frequency bandwidth [8]. Multiple researchers have adopted the multimodal approach which helps to cover a certain range of frequency to achieve a broader operating bandwidth [9]. Recent works investigate the benefits of the multimodal method with the functionalization of the energy localization phenomenon which is exploited to increase the amplitude of vibration and enhance the harvested power [10, 11, 12, 13]. This phenomenon known as Anderson localization occurs when a symmetry-breaking perturbation is applied to a periodic structure and is manifested by the confinement of the vibration energy in regions close to the perturbed region instead of being propagated in an uniform manner to all system's regions. Despite the fact that multimodal techniques guarantee the widing of the frequency bandwidth of harvesters, they require hard technological constraints and high costs to be implemented. Consequently, several researches have proposed the introduction of the nonlinearity as an alternative approach to overcome these limitations. In the field of energy harvesting, the nonlinearity of the device is presented in many works in different ways. It can be introduced by changing the design characteristics as done in [14], by imposing high displacements [15, 16] or via the interaction of the oscillator with the magnetic field [16, 17, 18, 19], etc. These works proved that the introduction of the nonlinearity enlarges significantly the device bandwidth.

While the effects of nonlinearity in a mode localized vibration energy harvester has been analytically studied [20], to the best of the authors knowledge, tuning simultaneously both the nonlinearity and the mistuning for quasiperiodic multimodal vibration energy harvesting has not been deeply addressed and experimentally demonstrated with respect to the critical amplitude and the localization robustness. In this letter, mode localization and nonlinearity are tuned in a multimodal electromagnetic vibration energy harvester, enabling the enhancement of the harvested power and the frequency bandwidth. Moreover, the results are described by an analytical model which is in good agreement qualitatively with the experiments and can serve as a design tool to drive the harvester displacement beyond its critical amplitude while ensuring robust energy localization. The energy localization phenomenon is achieved by mistuning the mass of one of the moving magnets, while the nonlinearity is introduced by driving the structure at large displacements. To overcome the issue of the high mechanical damping, the moving magnets are elastically guided by means of bi-clamped coupled beams.

2. Theoretical and experimental approaches

The energy localization phenomenon has been implemented in linear systems [11]. It has been shown that it allows an improvement of the harvested power and permits harvesting the vibration energy from the perturbed dof oscillator instead of harvesting from all dofs. This approach helps reducing the number of the harvesting circuits and thus their cost and enables the simplicity of the electronic part. As to the nonlinearity, it has already been one of the efficient techniques to enlarge the frequency bandwidth. The introduction of the nonlinearity should widen the bandwidth without destroying the recovered power and the localization phenomenon. For that, a theoretical approach is necessary to explain the physics phenomena and show the benefits of this combination. To do that, the harvester shown in Figure.1a is proposed. It is composed of two weakly coupled magnets guided by elastic beams (Neodymium magnet $N35$, diameter $6mm$, residual magnetic flux density B of 1.37 T). The center magnets are placed between the top and bottom fixed magnets. The magnetic poles are oriented in such a way that a repulsive force is created between each two successive magnets. These forces express a linear magnetic stiffness. Threaded rigid bars are used to tune the coupling by varying the gap and a copper coil is wrapped around each moving magnets. When the device is submitted to a basis harmonic imposed acceleration, the magnets oscillate around their equilibrium positions and a current is induced in coils when magnets oscillate (Lenz' law). The two moving magnets considered in the studied system are illustrated by the equivalent model of two degrees of freedom (dofs) in Figure.1b. The proposed harvesting

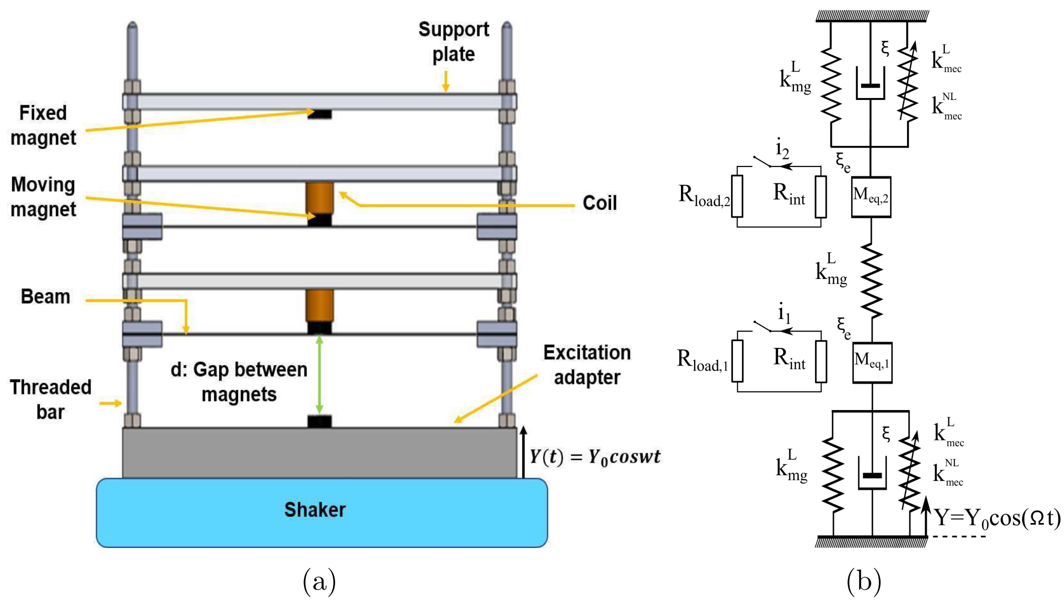

Figure 1: (a) The designed electromagnetic VEH (b) The mechanical and electrical equivalent model of the VEH

device is, then, modeled using two coupled Duffing equations. Dividing the equation of motion (I.11) detailed and solved in the supplementary material by M_{eq}, the following canonic equations of motion of the quasiperiodic system are obtained:

$$\begin{cases} \ddot{a}_1 + 2\,\xi\,\omega_0\,\dot{a}_1 + \omega_0^2\,[(1+2\beta)\,a_1 - \beta a_2] + f_{nl}\,a_1^3 = (1+p)\ddot{Y} \\ \alpha\ddot{a}_2 + 2\,\alpha\,\xi\,\omega_0\,\dot{a}_2 + \omega_0^2\,[(1+2\beta)\,a_2 - \beta a_1] + f_{nl}\,a_2^3 = (\alpha+p)\ddot{Y}, \end{cases} \tag{1}$$

Where a_j $(j = 1, 2)$ is the generalized coordinate of each dof, $\ddot{Y}$ is the acceleration of the basis excitation, $\xi = \xi_e + \xi_m$ is the damping factor of the system with ξ_e and ξ_m are respectively the electrical and mechanical damping. The damping factor is estimated experimentally by the half-power bandwidth method [22]. The estimated mechanical damping factor is of $\xi_m = 0.59\%$ which can be neglected compared to the electrical one leading to the approximation $\xi \approx \xi_e$. $\alpha = {}^{M_{eq,2}}/_{M_{eq,1}}$ is the mistuning

coefficient representing the mass ratio between the perturbed and the unperturbed magnets. In the case of a periodic system, $\alpha = 1$ and the equivalent masses are equal to $M_{eq,1} = M_{eq,2} = 9.8\ g$. $\beta = {}^{k^L_{mg}}/_{k^L_{mec}}$ is the coupling coefficient equal to 0.1% for a fixed gap $d = 50\ mm$ with k^L_{mg} and k^L_{mec} are respectively the linear magnetic and mechanical stiffness. $\omega_0 = \sqrt{{}^{k^L_{mec}}/_{M_{eq,1}}}$ is the eigenfrequency of the first decoupled dof. p is a mass ratio and $f_{nl} = {}^{k^{NL}_{mec}}/_{M_{eq,1}}$ is the mechanical nonlinear term while assuming that the magnetic nonlinearity k^{NL}_{mg} is neglected compared to the mechanical nonlinearity in the case of a weak coupling between the two magnets ${}^{k^{NL}_{mg}}/_{k^{NL}_{mec}} = 0,28\%$ [15].
The current flowing in the load resistance provides an electric power harvested from only the perturbed magnet and which can be calculated as $P = {}^{max(V_j)^2}/_{R_{load,j}}$; where V_j is the voltage generated by the coil j ($j = 1, 2$) and $R_{load,j}$ is its corresponding load resistance.
The energy localization is quantified by the rate τ, calculated by the following expression:

$$\tau(\%) = 100 \frac{\mid max(V_2) - max(V_1) \mid}{Sup(max(V_1), max(V_2))} \tag{2}$$

As illustrated in the Figure.2, the optimum of the energy localization rate in the linear case is very sensitive to the variation of the mistuning coefficient, whereas the optimum of this output response is robust towards the input parameters in the nonlinear case. This robustness is advantageous especially in the real applications because uncertainties and errors in measurements will not affect strongly the results.

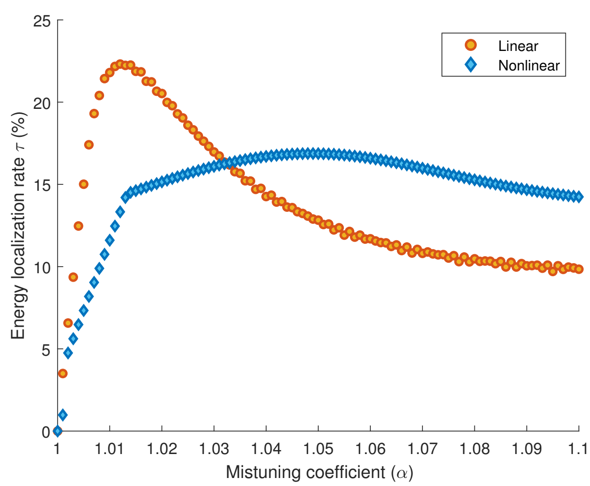

Figure 2: Variation of the energy localization rate with respect to the mistuning coefficient for an open-loop circuit (the coils are not mounted): highlighting the robustness of the optimum localization in the nonlinear case.

Moreover, the energy localization rate for the open-loop circuit illustrated in Figure 2 is higher than the closed-loop circuit in both linear and nonlinear cases shown in the figures of the supplementary material. This is due to the decreasing of the oscillation amplitude while introducing the electrical damping. As depicted in the figures of the

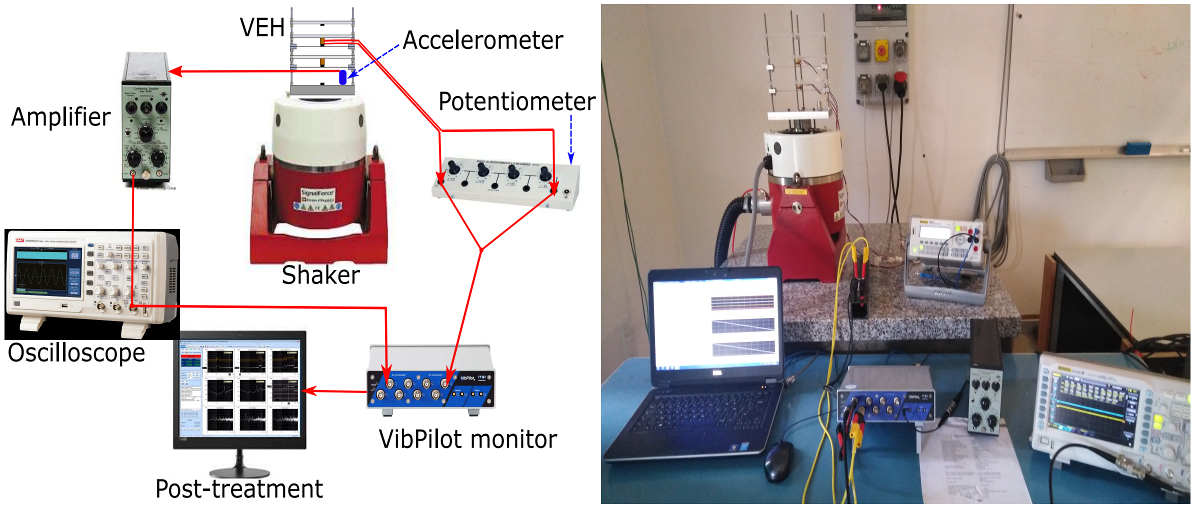

Figure 3: Test bench for a sinusoidal sweep excitation

supplementary material, the energy localization rate in the linear case is higher than the rate in the nonlinear one. Furthermore, the harvested powers from linear and nonlinear cases are roughly identical and the parameters giving the maximum of harvested powers in the two cases lead also to nearly identical localization energy rates. In conclusion, the introduction of the nonlinearity allows enlarging the frequency bandwidth while maintaining the same energy localization phenomenon rate and the harvested power compared to the linear case. In order to validate these benefits, the nonlinear term and the critical amplitude were determined experimentally for the proposed VEH. To do that, the device was fabricated and the experimental test bench, illustrated in Figure.3, was implemented. The designed device was mounted on the shaker which supplies mechanical vibration to the VEH. In order to generate signals as a response to the input acceleration to the shaker, an "m + p VibRunner" software has been used. During the experiments, the frequency was swept from 75 Hz to 98 Hz and the acceleration applied to the device was monitored by an accelerometer mounted on the VEH basis. For the open-loop circuit, the velocities of the dofs were measured by the laser doppler vibrometer while for the closed-loop circuit, the output voltages of the coils were stored through load resistances controlled by potentiometers. Throughout the experimental study, root mean square (rms) values were considered for the measured quantities.
Since the coupling is weak ($\beta = 0.1\%$), it is assumed that the closed-form expression of the critical amplitude already determined for a single dof in [21] remains valid for a 2-dofs system. The critical amplitude A_{cr} is the amplitude of transition from linear to nonlinear behavior. Its classical form for a Duffing oscillator with mechanical nonlinearity [21] is $A_{cr} = 1.687 \times {}^{h}/_{\sqrt{Q}}$ where $Q = 84.5$ is the quality factor and $h = 0.6\ mm$ is the thickness of the beam. Using this expression, $A_{cr} = 0.11\ mm$ which is in good agreement with the experimental results. Using the proposed model, the relation between A_{cr} and f_{nl} is giving by $A_{cr} = 2.335\, \omega_0 \sqrt{{}^{2\xi}/_{f_{nl}}}$. Thus, the identified f_{nl} is $1.28 \times 10^6 (m.s)^{-2}$.
Since the behavior of the oscillators is nonlinear, up and down sweeps were performed

for all the following experimental tests because of the significant difference in their peak magnitudes. In order to illustrate the definition of the bandwidth and to highlight the effect of nonlinearity, the frequency responses of the resonator for linear, critical and nonlinear behaviors are plotted in Figure.4a where it can be seen that the bandwidth BW_{nl} in the nonlinear case ($a_{rms} = 0.09\ g$) is remarkably enhanced up to 50% compared to the bandwidth BW_l of the linear one. Consequently, in order to take advantage of

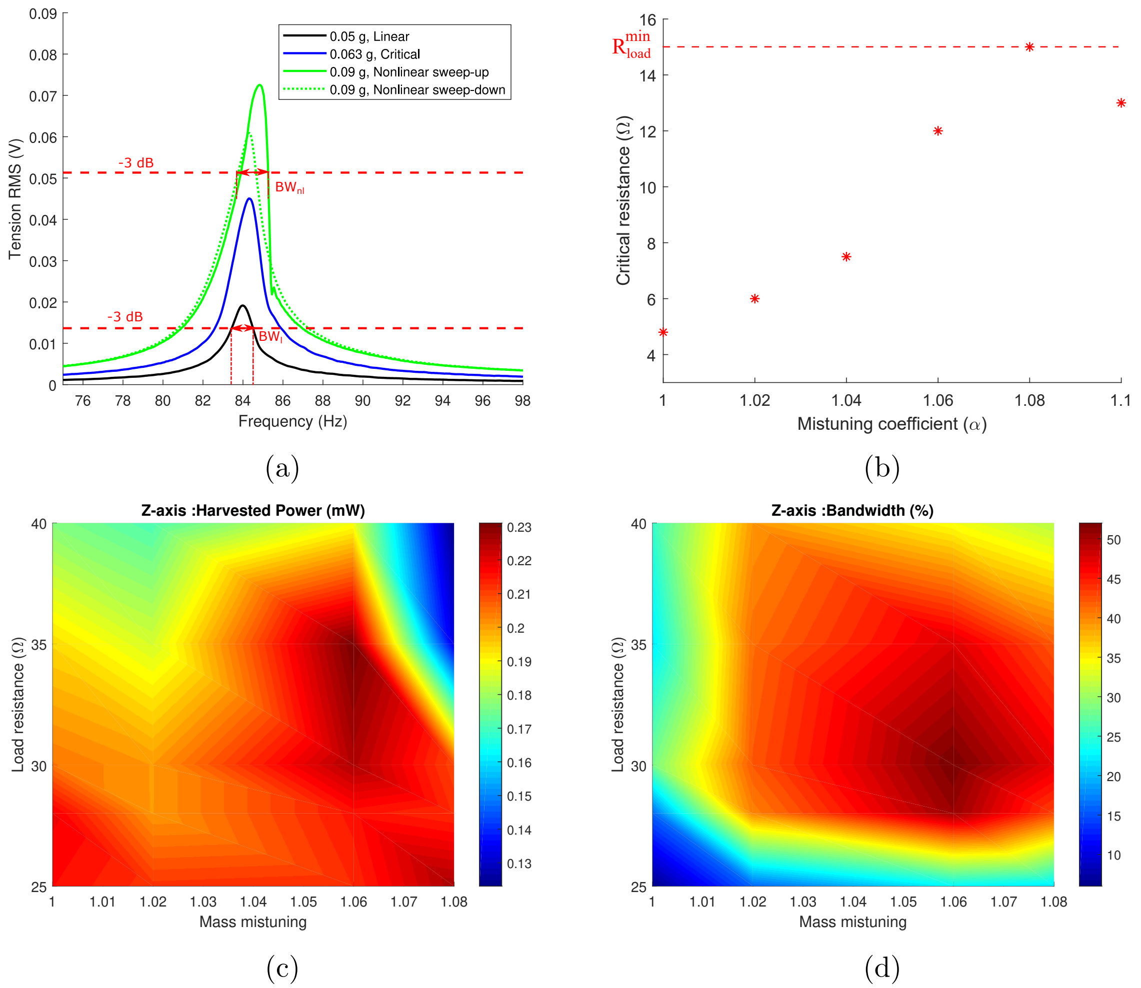

Figure 4: (a) Frequency responses of the moving magnets: linear, critical and nonlinear behaviors ($\alpha = 1; R_{load} = 4.8\Omega$) (b) Measured critical resistances with respect to mass mistuning coefficient ($a_{rms} = 0.063g$); Experimental results of (c) the harvested power from only the perturbed dof oscillator and (d) the frequency bandwidth with respect to load resistances and mass mistuning ($a_{rms} = 0.09g$).

the nonlinearity, the resonator should be driven beyond its critical amplitude. Then, the critical resistance, characterizing the lower bound limit of the harvester critical behavior, was determined for each mistuning coefficient in order to tune the level of nonlinearity. In fact, for a fixed mistuning coefficient, the load resistances were varied slightly and the harvester frequency response was visualized gradually while up and down sweeps were performed until capturing the resistance that ensures the transition from linear to nonlinear behavior. To do that, the measured critical resistances with respect to the measured mass mistuning coefficients were plotted in Figure.4b. As shown, while keeping the same base excitation level characterizing the critical behavior of the resonator in the periodic case ($a_{rms} = 0.063\ g$), the critical resistance is sensitive to the variation of the mass mistuning coefficient. According to this figure, the maximum value obtained is $R_{load}^{min} = 15\ \Omega$. In order to guarantee a nonlinear behavior of the oscillator, the load resistances should be higher than R_{load}^{min} ($R_{load} \geq 15\ \Omega$).

Aiming to obtain simultaneously the maximum of the harvested power and the frequency bandwidth that the resonator can reach, the load resistance and the mass mistuning coefficient are varied and the results obtained are depicted in Figures.4c and 4d. Figure.4c exibits the variation of the harvested power with load resistance and mass mistuning coefficient. The maximum scavenged power is obtained when $\alpha = 1.06$ and $R_{load} = 35\ \Omega$. As to the frequency bandwidth, it is shown in Figure.4d, that its maximum is reached when $\alpha = 1.06$ and $R_{load} = 30\ \Omega$.

Although the performance of the proposed harvester in terms of harvested power and bandwidth are increased, the maxima of these output performances are not given by the same parameters. Then, in order to obtain the optimal configuration that enhances the performances of the resonator, the mass perturbation and the load resistance should be tuned simultaneously.

Among a multitude of admissible solutions, the compromise solutions that suits for our problem are contained in the design space $\alpha^* \in [1.04\ ,1.06]$ and $R_{load}^* \in [28\ ,35\Omega]$ according to Figures.4c and 4d. These compromise solutions are obtained via a multiobjective optimization procedure. Recent studies related to this subject can be found in [23, 24]. This solution highlights the effects of the combination of the nonlinearity and the energy localization on the performances of the proposed device. In fact, the functionalization of these two phenomena simultaneously shows an improvement up to 19.4 % of the harvested power and 116 % of the bandwidth at $\alpha^* = 1.06$ and $R_{load}^* = 35\ \Omega$ as a chosen optimal configuration compared to the performances of the nonlinear periodic resonator. A quantitative comparison between theoretical and experimental results is achieved. The eigenfrequency calculated by the model is in good agreement with the experimental results ($Error < 1$ ‰). The model with the injected values α^* and R_{load}^* already found experimentally predicts the frequency bandwidth and the harvested power respectively with errors of 8% and 16%. This discrepency is mainly due to the manufacturing defects of the beams that affect the periodicity of the structure in term of mass mistuning. It is also due to the potentiometer accuracy while tuning the load resistance throughout the measurements which affects both the bandwidth and the harvested power.

3. Conclusion

In this work, a new concept combining the benefits of the nonlinearity and the energy localization phenomenon has been implemented in order to enhance the performances of an electromagnetic vibration energy harvester. The variation of the critical resistances with respect to the mass mistuning coefficients has been determined in order to tune the nonlinearity level. In fact, the mechanical nonlinearity enlarges the frequency bandwidth and offers a higher robustness of the optimum energy localization rate compared to the linear system. Concerning the energy localization controlled by the mass mistuning, it enhances the harvested power and its functionalization allows harvesting the vibration energy from one coil instead of two coils. Consequently, the technological constraints and the electric circuit cost can be reduced in the case of a device with multiple dofs oscillators. The optimal parameters that improve the device performances simultaneously have been also determined, which enables an enhancement up to 19.4 % in term of harvested power and up to 116 % in term of frequency bandwidth compared to the nonlinear periodic device. Finally, it is expected that the proposed concept can be generalized to a large-scale quasi-periodic system and the device can be miniaturized in order to improve more its output performances.

Please see supplementary material for more results about the theoretical approach.

The data that supports the findings of this study are available within the article and its supplementary material.

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