# Inverse Convolution Method for Periodic Media under Deterministic and Stochastic Condition


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***Abstract*—Wavenumber extraction has attracted widespread attention in periodic structures field to reveal many unique properties, such as complex band structures which give them potential to many applications such as vibration isolation, unable filters, wave guides and more. This paper optimizes a wavenumber extraction tool for estimating the wave propagation characteristics and energy propagation features of one-and two-dimensional periodic structures under deterministic and stochastic situations by dispersion curve and K-space analysis. This inversed method is referred as "INCOME". It uses the principle of the Prony series and the Bloch-Floquet theorem based on convolution framework. Compare to other methods proposed to obtain complex dispersion curve (1D) and dispersion surface (2D), the proposed method only requires the frequency response at a series of periodic measurements points as input parameters. The frequency response is easier to be obtained experimentally and numerically. On the other hand, the proposed method only relies on linear algebra instead of nonlinear optimization and allows a coherent estimate of the full K-Space in deterministic condition. More importantly, this proposed method is extended to the wavenumber extraction of stochastic condition in this paper, which greatly extends the practicability of INCOME.**



***Keywords- Periodic structure, Wavenumber Extraction, Inverse method, Deterministic and Stochastic condition, Wave propagation***


## I. INTRODUCTION

More recently, in order to overcome the numerical computational issue and extend applications of obtaining band structures to more complex structures, a number of inverse methods for wavenumber extraction are developed to provide the estimation of band structure from frequency response measured numerically or experimentally, leading to some significant practical values compared to traditional methods for wavenumber extractions [1,2,3]. One advantage of identifying wavenumbers from the inverse method is that this method does not require the detailed information of structure, such as the boundary conditions and material properties. On the other hand, in practice, the material parameters are complex so that it is difficult to make modeling, especially considering the damping factor which is hard to feature, thus using frequency response from experimental measurement directly is more useful. Generally, inverse methods contain two categories. The purpose of the first category is to solve the linear problem, involving Prony method and ESPRIT method [4]. These two methods are easy to manipulate but limited to 1D application and constrained to periodic measurements. Comparatively, the second category contains McDaniel's method, Inhomogeneous Wave Correlation (IWC) method [5] and Inverse Wave Decomposition (IWD). They are all not limited to periodic measurements and latter two methods can be extended to extract wavenumbers of 2D periodic structure in some special conditions but hard to calculate because of nonlinear algebra problem. Additionally, L. Junyi proposed wave superposition method (WSM) to analyze band structure but it is limited to measure a portion of the dispersion surface (2D) or volume (3D) [6]. Afterwards, as an extension of prony method, Boukadia proposed a novel inverse method referred to INCOME which can be applied to solve 1D and 2D problems of wavenumber extraction and only relies on linear algebra. However, periodic measurement is still a strong constraint that cannot be relaxed in this method. This paper optimizes INCOME to overcome the limitation of periodic measurements by considering different stochastic conditions.

## II. INCOME METHOD

### A. *Principle*

The extraction process can be summarized as two steps: Modelling and wavenumber identification. For modeling, through the execution of pronys' method, a recurrence relationship with uncertain coefficients is established. Then the convolution framework is used to simplify the expression of this recurrence relationship. Thus the problem of wavenumber extraction converts to that of obtaining solution of convolutional kernel which contains the coefficients of characteristic polynomial. For identification, the key of this process is to obtain the coefficients of characteristic polynomial. The least square method is used to obtain the coefficients of characteristic polynomial and then process wavenumber identification in deterministic situation.

### B. *Simulation examples*

The effectiveness of INCOME in deterministic condition is validated by the wavenumber extraction of two pairs of bending waves in a Timoshenko beam with resonators. Figure 1 shows a model of Timoshenko beam with resonators and the corresponding results in the positive direction are as follows:


Figure 1. The model of a Timoshenko beam with resonator

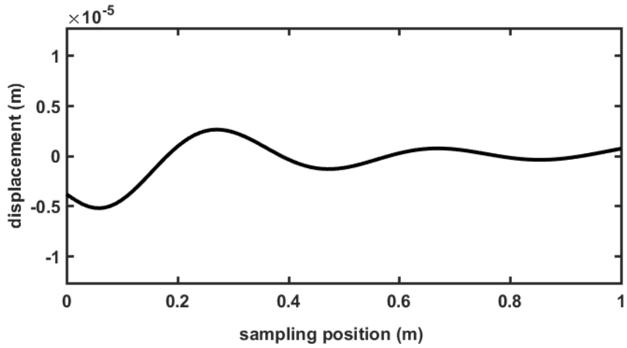


Figure 2. The frequency response curve at 500 Hz

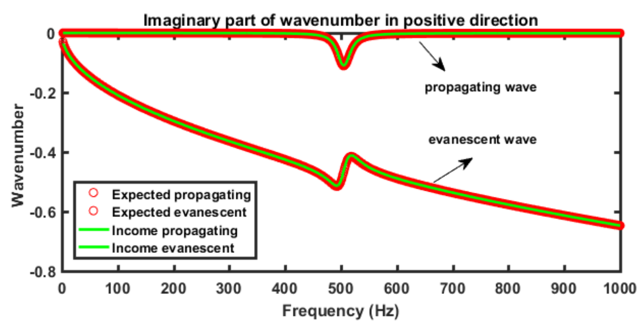


Figure 3. The real part of the dispersion curve

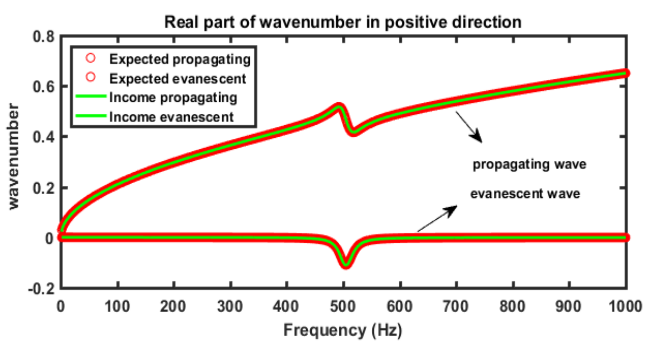


Figure 4. The imaginary part of the dispersion curve

The length L of beam is set to 1m and it is divided to 100 elements evenly. The weight and height are set to 2cm and 1cm respectively. The damping, Young's modulus, density and Poisson are set to 0.005, 210 $\times (1 + 0.005i)$ GPa, 7800 $kg/m^3$, 0.3 respectively. 30 resonators are distributed evenly through the Timoshenko beam. The mass of resonators is equal to 5% of the overall beam. The Natural frequency of resonator is 500Hz and the damping factor of resonator is 0.05. The Figure 2 shows the frequency response curve in 500Hz, which shows the amplitude of displacement is decaying with wave propagating. The Figure 3 and the Figure 4 present the complex dispersion curve of waves propagating in positive direction, which can illustrate that the INCOME is effective to extract wavenumbers. On the other hand, the dynamic properties of periodic structure can be explained. When the frequency reaches the natural frequency of the spring oscillator, resonance occurs. The energy is dissipated during the process of resonance, leading to the phenomenon of band gap existing near the natural frequency. In the band gap, the wave can't propagate. This principle is mainly reflected by the imaginary part of the wavenumber. For example, in the imaginary part of the dispersion curve, the wavenumber of the propagating wave changes to negative number near the natural frequency of spring oscillator, which leads to the exponential attenuation of displacement and the appearance of band gap.

## III. Perspectives and Conclusion

The theory of INCOME in deterministic condition is established and its effectiveness is validated by different cases in 1D. It only relies on linear algebra instead of non-linear optimization and it is still exact in 2D case and allows a coherent estimate of the full K-Space. For stochastic case, namely the periodic media affected by disturbances, such as the noise, non-periodic measurements and so on, it requires to be explored further using mathematical statistics optimization methods and the experimental cases is required to implement to validate the effectiveness of INCOME further.

## References


[1] Hussein M I, Leamy M J, Ruzzene M. "Dynamics of Phononic Materials and Structures: Historical Origins, Recent Progress, and Future Outlook" J. Applied Mechanics Reviews, 66(4):040802, 2014, doi: 10.1115/1.4026911.

[2] Droz C , Zhou C , Ichchou M, J.-P. Lainé "A hybrid wave-mode formulation for the vibro-acoustic analysis of 2D periodic structures" J. Journal of Sound & Vibration, vol. 363, pp. 285-302, February 2016, Available from: https://doi.org/10.1016/j.jsv.2015.11.003.

[3] R.F. Boukadia, E. Deckers, C. Claeys, M. Ichchou, W. Desmet. "A wave-based optimization framework for 1D and 2D periodic structures" J. Mechanical systems and signal processing, vol.139 pp: 106603.1-106603.23. May. 2020, doi: 10.1016/j.ymssp.2019.106603.

[4] Kay S M, Marple S L. "Spectrum anameadlysis—A modern perspective" J. Proceedings of the IEEE, vol. 69, pp:1380-1419, 1981, doi: 10.1109/PROC.1981.12184.

[5] Ichchou M N, Berthaut J, Collet M. "Multi-mode wave propagation in ribbed plates: Part I, wavenumber-space characteristics" J. International Journal of Solids & Structures, Vol.45, pp: 1179-1195, March 2008. Available from: https://doi.org/10.1016/j.ijsolstr.2007.09.032.

[6] Junyi L, Balint D S. "An inverse method to determine the dispersion curves of periodic structures based on wave superposition" J. Journal of Sound and Vibration, vol.350, pp: 41-72, August 2015, doi: 10.1016/j.jsv.2015.03.041.