Quasi-normal modes of resonant phononic structures

V. Laude[1], Y.F. Wang[2]

[1]*Institut FEMTO-ST, Université Bourgogne Franche-Comté, CNRS, 25030 Besançon, France*

[2]*Department of Mechanics, School of Mechanical Engineering, Tianjin University, 300350 Tianjin, China*

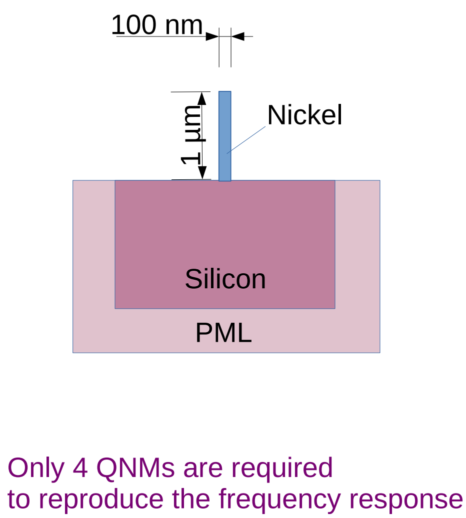
Resonant structures supporting elastic waves attached to a substrate suffer from radiation loss. As a result, instead of the normal modes of closed systems having purely real eigenfrequencies, open systems possess quasi-normal modes (QNMs) characterized by complex valued eigenfrequencies and diverging power at infinity. Because of the eigen-expansion theorem, that states that the response of a system to a given excitation is a superposition of all eigenmodes, there is strong interest in obtaining all quasi-normal modes. Quasi-normal modes have been widely discussed in photonics [1,2], but less for acoustic and elastic waves [3]. Of special interest are the determination of complex eigenvalues and eigenmodes, and the definition of an adequate modal volume and elastic equivalent of the Purcell effect [4].

Following Reference [1], we make use of the unconjugated form of the reciprocity relation for elastic waves in order to obtain a relation between the solution to the time-harmonic elastodynamic wave equation and the discrete set of quasi-normal modes. Of significance is the fact that the total energy of QNMs is unbounded, so that usual normalization relations for normal modes can not be employed. Instead, a multipole expansion with complex coefficients is naturally obtained. In passing, a complex modal volume is defined.

We then consider a practical way to obtain all quasi-normal modes of an elastic resonating structure. Our technique relies on the stochastic method to obtain all possible resonances from the response of the system to a random force [5] and extends it to the quasi-normal mode problem. The technique yields poles in the response that can be continued in the complex frequency plane. Starting from an initial guess along the real frequency axis, a fast iterative algorithm based on the power method for eigenvectors is implemented. Once the quasi-normal modes have been obtained, the response of the system to a given excitation can be computed efficiently from the expansion theorem, by forming the projection of the excitation on each quasi-normal mode as a function of frequency.

We implement an equivalent of the photonic techniques presented in [1] to phononic structures, including perfectly matched layers to transform the semi-infinite domain to a finite, but complex-valued, domain. We obtain numerically the modal volume of open elastic systems. As a test system, we consider a nanopillar on a surface and then a pair of coupled nanopillars forming a kind of tuning fork, resonating in the hundreds of MHz to a few GHz range. We obtain the quasi-normal modes in both cases and verify that the response of the system to an excitation of the nanopillars is well accounted for by the superposition of the first quasi-normal-modes. The first quasi-normal modes for the case of an isolated nano-pillar are shown in Figure 1, together with a comparison of the time-harmonic response of the system with the expansion over the first few quasi-normal modes.

As will be discussed, the same strategy can be followed to obtain quasi-normal modes for open acoustic systems, as well as for coupled acoustic and elastic systems.



QNM	Frequency (GHz)	Q	V (μm^2)
1	0.0681	3196	0.02726-2.6e-06i
2	0.4142	64	0.02594+0.002272i
3	0.6096	4	0.04812+0.008197i
4	1.1142	23	0.02706-0.001394i

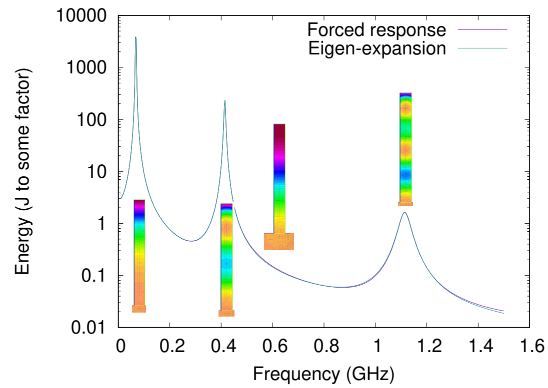


Figure 1: Two-dimensional model of a nickel pillar on a silicon surface. Quasi-normal modes are obtained including a viscoelastic loss model for both materials and taking into account radiation to the bottom of the substrate. Radiation is implemented numerically as a frequency-dependent perfectly matched layer (PML). The frequency response of the vibrating pillar can be estimated from the superposition of the first few quasi-normal modes.

References

1. C. Sauvan, J. P. Hugonin, I. S. Maksymov, and P. Lalanne, Phys. Rev. Lett. **110**, 237401 (2021), doi:10.1103/PhysRevLett.110.237401.
2. E. A. Muljarov and W. Langbein, Phys. Rev. B **94**, 235438 (2016), doi:10.1103/PhysRevB.94.235438.
3. Yan-Feng Wang, Shu-Yan Zhang, Yue-Sheng Wang, and Vincent Laude, Phys. Rev. B **102**, 144303 (2020). doi:10.1103/PhysRevB.102.144303.
4. Mikołaj K. Schmidt, L. G. Helt, Christopher G. Poulton, and M. J. Steel, Phys. Rev. Lett. **121**, 064301 (2018). doi:10.1103/PhysRevLett.121.064301.
5. Vincent Laude and Maria E. Korotyaeva, Phys. Rev. B **97**, 224110 (2018). doi:10.1103/PhysRevB.97.224110.