Glide-Reflection Symmetric Topological Phononic Crystal Waveguide

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Abstract: The topological backscattering immunity of waves guided along a domain wall is associated with symmetry protection in valley-Hall and quantum-Hall phononic crystal waveguides. This desirable property results from a topological transition at a Dirac point of the 2D crystal, leading to the opening of an initially closed band gap but to a limited available bandwidth compared to wide band gap topological trivial crystals. Introducing a glide-symmetric dislocation of a square lattice crystal of circular inclusions, we show that a pair of wide-bandwidth, single-mode, and symmetry-protected guided waves are created, without resorting to bulk boundary correspondence. A demonstration experiment is performed with acoustic waves in water, at ultrasonic frequencies. The concept further applies to different types of waves, including elastic waves in solids, but also optical and electromagnetic waves.

On the one hand, the concepts of topological phononics unleash unprecedented wave properties. Originally inspired by topological insulators [1], they have paved the way to achieve waveguides that are uni-directional and backscattering-free. Topological guided waves can propagate along a domain wall between two crystal phases [2], leading to symmetry-protected, single-mode guided waves.

On the other hand, non topological waveguides are normally based on the coupling of resonances of defects inside a phononic crystal with a complete band gap. They are generally multimodal. Furthermore they possibly present limitations such as gaps for the guided modes or very flat bands with very limited bandwidth [3].

Here, we show that the introduction of a glide-symmetric dislocation in a topologically trivial crystal causes the creation of a pair of waves guided along the domain wall. This phenomenon occurs in all Bragg band gaps and lead to symmetry protection. Using ultrasonic acoustic waves in water, a series of experiments are performed to demonstrate the operation of the waveguide.

The chosen topological trivial crystal is based in a simple square 2D lattice, as shown as a super cell in figure 1(a), that also portrays the bandstructure presenting a complete band gap. When the glide operation is applied, guided waves appear inside the band gap as shown in 1(b). These guided modes are symmetry protected against backscattering and almost close the gap. Furthermore, a degeneracy by pairs of all bands appears at the X point of the first Brillouin zone, when the glide parameter, g, is exactly half the lattice constant, a. This is a result of the combination of the space group symmetry of the waveguide and of its periodicity along the domain wall. For $g \neq a/2$, there is still a pair of guided modes but with reduced bandwidth and symmetry protection is lost, generating a mini gap for guided modes. This is a signal that for a continuous parametric change in g, we have a topological phase transition.

The 2-periodic phononic crystal and the wave guide were fabricated and tested experimentally. We retrieve the acoustic wave transmission using the ultrasonic echo technique. Different values of the glide operator were used to experimentally observe the closing of the gap at $g = a/2$ and the reopening for $g \neq a/2$, with good agreement with numerical calculations. Since the gap is fully opened for $g = 0$ and closed for $g = a/2$, and the glide parameter can be continuously tuned with periodicity a, a continously-tunable transmission filter is obtained.

We anticipate that the symmetry principles involved extend the existence of glide-reflection symmetric crystal waveguides to different types of waves.

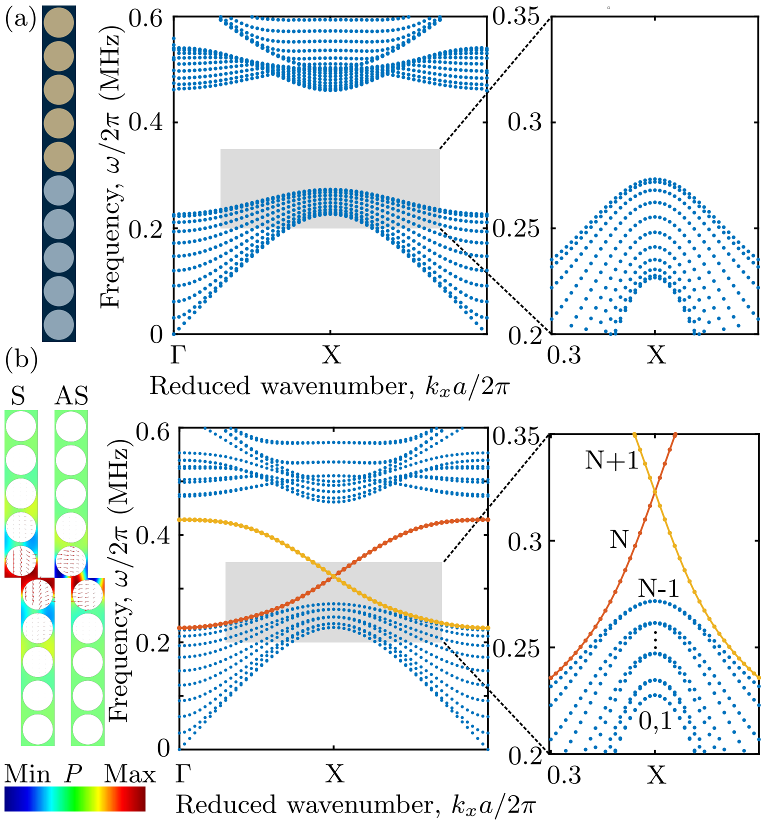

Fig. 1: Band structure topology of the glide-reflection waveguide, computed for a supercell made of $N = 10$ unit cells of the crystal. (a) For $g = 0$, the supercell simply repeats N times vertically the primitive cell of the 2D square-lattice crystal. The band structure of the waveguide is obtained from the projected band structure of the 2D crystal. (b) For $g = a/2$, the bands group by pairs of symmetric (S) / anti-symmetric (AS) Bloch waves, with respect to the glide reflection symmetry. They are degenerate at the Brillouin zone edge (the X point), causing a pair of guided waves to appear inside the complete band gap. The modal distributions of the S (red color band) and AS (yellow color band) guided waves are shown on the left for $k_x a/\pi = 0.8$.

References

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