Computer-assisted enumeration and classification of multi-qubit doilies

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Abstract

For $N \geq 2$, an N-qubit doily is a doily living in the N-qubit symplectic polar space. These doilies are related to operator-based proofs of quantum contextuality. Following and extending the strategy of [SdBHG21] that focused exclusively on three-qubit doilies, we first bring forth several formulas giving the number of both linear and quadratic doilies for any $N > 2$. Then we present an effective algorithm for the generation of all N-qubit doilies. Using this algorithm for $N = 4$ and $N = 5$, we provide a classification of N-qubit doilies in terms of types of observables they feature and number of negative lines they are endowed with.

The doily is the unique triangle-free self-dual finite geometry composed of 15 points and 15 lines, with three points on a line and, dually, three lines through a point.

This talk [MSG$^+$22a] is about doilies labeled with N-qubit observables, to be called *N-qubit doilies* or *multi-qubit doilies*, that are related to Kochen–Specker operator-based proofs of quantum contextuality. We describe an algorithm for the enumeration of all N-qubit doilies, for any $N \geq 2$, and an optimized C implementation of this algorithm. We also present several classification criteria of N-qubit doilies, that are their types of observables, their linear or quadratic nature and their configurations of negative lines. As the numbers of N-qubit doilies increase rapidly with N, the enumeration program can in practice only be executed for small numbers of qubits. We use it to classify N-qubit doilies for

$N = 4$ and $N = 5$, according to these criteria. We thus produce precise tables for the number of doilies in each category. Before this work, the classification was only known for $N = 3$ [SdBHG21]. The presented results are published in [MSG+22b].

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References

[MSG+22a] Axel Muller, Metod Saniga, Alain Giorgetti, Henri de Boutray, and Frédéric Holweck. Computer-assisted enumeration and classification of multi-qubit doilies. In *Journées Informatique Quantique 2022 - JIQ'22*, Paris, France, 2022. `https://quantcert.github.io/doilies/jiq2022handout.pdf`.

[MSG+22b] Axel Muller, Metod Saniga, Alain Giorgetti, Henri de Boutray, and Frédéric Holweck. Multi-qubit doilies: Enumeration for all ranks and classification for ranks four and five. *Journal of Computational Science*, 64:101853, 2022. `https://doi.org/10.1016/j.jocs.2022.101853`.

[SdBHG21] Metod Saniga, Henri de Boutray, Frédéric Holweck, and Alain Giorgetti. Taxonomy of Polar Subspaces of Multi-Qubit Symplectic Polar Spaces of Small Rank. *Mathematics*, 9(18):2272, 2021. `https://doi.org/10.3390/math9182272`.