Vibration propagation analysis in plates using the irrotational intensity field

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Abstract
Structural intensity is a technique that is used to indicate the magnitude and the direction of vibration energy flow within a structure. In this work, the structural intensity of flat plates subjected to harmonic excitation is conducted using the finite element method. The intensity field provides information on the energy transmission path including the position of the sources and the sinks. However, a rotational component appears that is masking the vibration energy flow and makes its understanding difficult. Thus, a methodology is proposed to filter out the rotational component of the structural intensity field. The approach is based on the decomposition of the structural intensity field into rotational, irrotational and harmonic components. Only the irrotational component is needed to describe the path of vibration energy. The proposed methodology combines a dynamic response computation and a diffusion equation solving. Its efficiency is illustrated on plates, providing a deep understanding of the energy transmission paths at medium and high frequencies.

1 Introduction

Understanding the vibration behavior of automotive structures is increasingly important to meet the increased expectations of customers in terms of comfort. Hence, it is essential to look for new methods facilitating the understanding of the propagation of vibration energy, particularly at medium and high frequencies. A specific attention is paid here on plates which are major elements of interest to structural dynamics specialists in many fields, especially in the automotive industry. It is therefore desirable for engineers to have useful methods for predicting the energy flow in plates at medium and high frequencies.

The structural intensity analysis allows to describe the amplitude and direction of the vibration energy carried by elastic waves in a structure. This type of method has been used in several numerical [1, 2, 3] and experimental [4, 5] studies to analyze dominant transmission paths in structures. However, the understanding of the structural intensity field becomes more and more difficult when frequency increases. It has been noticed that the structural intensity vector field basically consists of a rotational and an irrotational part [6]. The irrotational intensity indicates how the energy flows from the sources towards the sinks and the rotational part shows the local energy loops. A direct application has been proposed on plates [7] where the irrotational intensity field has been used to identify and locate energy sources in a plate. To this purpose, a numerical method based on the discrete Fourier transform to calculate this vector field was introduced. Alternatively, a method to calculate the irrotational part of the structural intensity to asses the vibrational sources and sinks in a plate from experimental measurements using TFS (Test Functional Series) has been proposed [8]. This technique was later compared with the method based on the Fourier transform [9].

The proposed filtering methods are difficult to implement in the case of complex structures. This paper presents a new numerical method to extract the irrotational component of the structural intensity field based on the finite element method. In this way, the method can be extended easily to complex structures. Section 2 gives a brief overview of the mathematical background of the power and the structural intensity. The power flow analysis in a plate using the structural intensity is presented in the third section to highlight this phenomenon of loops and energy circulation in the field. In section 4, the fundamental concepts rotational intensity filtering are discussed. The last part is devoted to the analysis of the power flow using the irrotational intensity field in a single plate and plates assembly.

2 Theoretical formulations

The mathematical formulations concerning the structural intensity field and the power [1, 4, 10, 11] are clearly documented in the literature. Therefore, they are just briefly reviewed here.

2.1 Power

The mean power is defined as the time-averaged product of generalized forces and related in-phase velocities. For harmonic analysis, the mean power injected into the structure by a point force can be described using complex numbers by

$$P_{inj} = \frac{1}{2}\Re(\vec{F}.\vec{v}^{*}) \tag{1}$$

where $\vec{F}$ is the excitation force and $\vec{v}^{*}$ is the conjugate of velocity at the excitation point. $\Re$ denotes the real part.

2.2 Dissipated power in a structure

Structural (hysteretic) damping model is considered in this study. The power dissipated per surface unit within a structure vibrating in harmonic regime at frequency ω is usually expressed as [12]:

$$P_{diss} = 2\eta\omega E_d \tag{2}$$

where η denotes the hysteretic damping and E_d stands for strain energy density.

2.3 Structural intensity

The instantaneous structural intensity is a time-dependent vector quantity corresponding to the variation of the energy density in an infinitesimal volume [11]. When some power is injected into a structure at one or more locations, the power travels along the structure and it is dissipated by various damping phenomenon. The instantaneous structural intensity at a point M of the structure and at a time t is mathematically defined as:

$$\vec{\Pi}(M,t) = -\bar{\bar{\sigma}}(M,t)\vec{v}(M,t) \tag{3}$$

where $\bar{\bar{\sigma}}$ and $\vec{v}$ correspond to the stress tensor and the vector velocity, respectively. The time average of the instantaneous intensity represents the net energy flow in a structure:

$$\vec{\Pi}(M) =< \vec{\Pi}(M,t) > \tag{4}$$

where $< ... >$ denotes time average.

In the case of steady-state harmonic vibrations, for a given frequency, the time average of the structural intensity over a period can be expressed as:

$$\vec{I}(M) = -\frac{1}{2}\Re(\bar{\bar{\sigma}}.\vec{V}^*). \tag{5}$$

$\bar{\bar{\sigma}}$ and $\vec{V}^*$ are respectively the stress tensor and the conjugate of velocity at point M.

2.4 Structural intensity formula of a flat thin plate

The time average of the structural intensity in the case of a flat thin plate in the (x, y) plane is expressed as [13]:

$$\begin{cases} I_x = -\frac{w}{2}\Im(N_x u^* + N_{xy} v^* + Q_x w^* + M_x \theta_y^* - M_{xy}\theta_x^*) \\ I_y = -\frac{w}{2}\Im(N_y v^* + N_{xy} u^* + Q_y w^* - M_y \theta_x^* + M_{xy}\theta_y^*) \end{cases} \tag{6}$$

I_x, I_y represent the component of the structural intensity field along x and y directions, respectively. N_x, N_y and N_{xy} are the complex membranes forces. M_x ,M_y and M_{xy} are complex bending and torsional moments. Q_x ,Q_y are the complex shear forces. The displacement components of the plate are denoted by u, v and w, and the angular displacements by θ_x and θ_y. $\Im$ stands for the imaginary part.

3 Power flow analysis in a plate using structural intensity

The objective of this part is to implement the calculation of the structural intensity in the case of a simple plate to highlight the limitation of this field at medium and high frequencies in the understanding of the vibration transfer paths. The plate is simply supported, with properties shown in the table 1.

Table 1: Plate properties

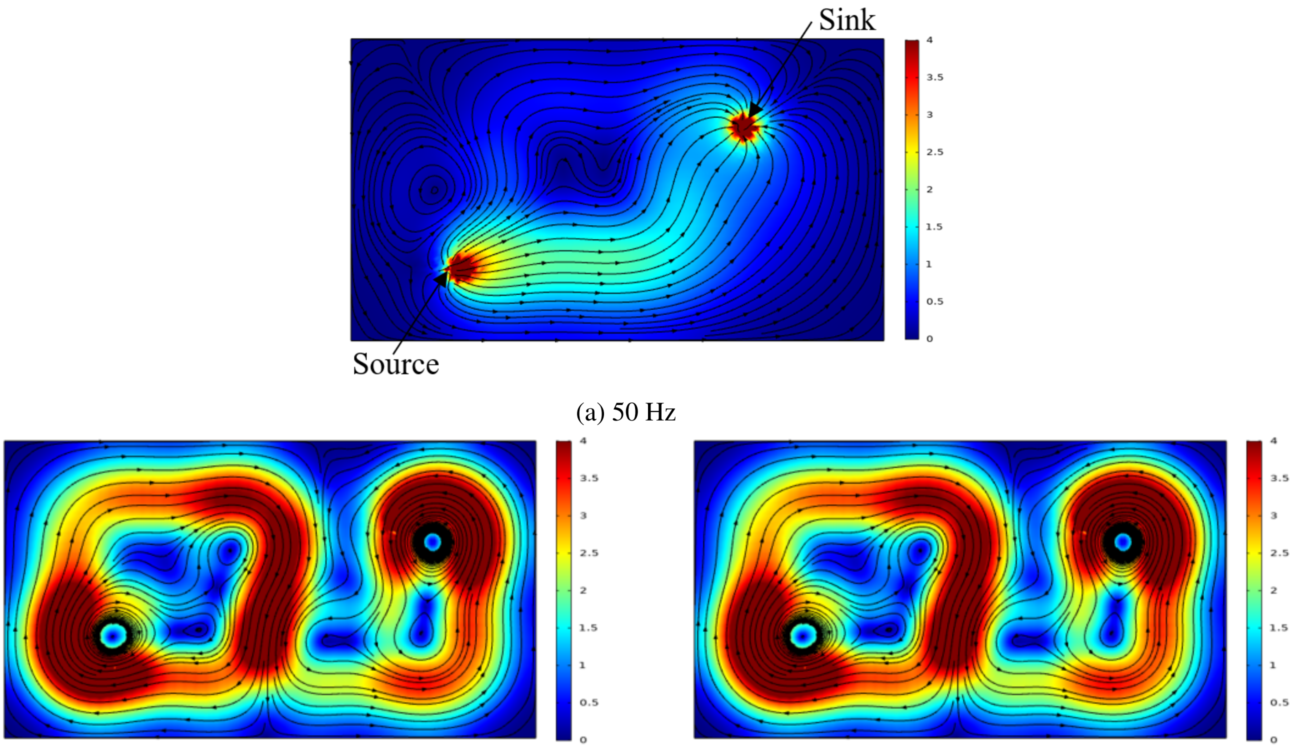
Length (mm)	3 000
Width (mm)	1 700
Thickness (mm)	10
Elastic Modulus (MPa)	210 000
Poisson ratio	0.3
Density (kg/m^3)	7 800

The plate is subjected to harmonic excitation at location (600 mm, 400 mm). A point viscous damping of 100 N.s/m is applied at location (2200 mm, 1200 mm).

The plate model consists of quadratic finite elements. The response under harmonic excitation provides generalized forces and displacements. The structural intensity is calculated by substituting these quantities in equation (6). Both quantities must be expressed at the same point. It is more convenient to interpolate the displacements at the center of the element rather than calculate the forces at the nodes in order to maintain a better accuracy in the calculations. The field is graphically presented using streamlines, as they allow a better visualization of the path than arrows.

The structural intensity field at 50 Hz as shown in figure 1a, which provides information on the path taken by the vibration energy. It clearly indicates the source of the energy and the sink, which is only the viscous damping in this case.

Figures 1b and 1c show that streamlines become complex to read at higher frequencies. It is no longer possible to understand the path taken by energy and to recognize the position of the source and the sink from the structural field. The rotational intensity is more dominant and the field tends to spin. Thus, it does not express clearly the path of the vibration energy. In the following section, a method to filter this rotational component is proposed.



(a) 50 Hz

(b) 100 Hz

(c) 150 Hz

Figure 1: Structural intensity field distribution in a single plate

4 A method for filtering the rotational component of the structural intensity field

The Hodge-Helmholtz decomposition [14] is a fundamental theorem in mathematics. It has many applications, especially in fluid mechanics to simulate flows [15]. According to this theorem, any continuous vector field can be decomposed to a sum of three components: a rotational component (divergence-free); an irrotational component (curl-free) and a harmonic component (divergence-free and curl-free).

Applying this theorem to the structural intensity vector field render possible to decompose the field $\vec{I}$ into a rotational intensity $\vec{I_r}$; an irrotational intensity $\vec{I_{ir}}$ and a harmonic part $\vec{h}$:

$$\vec{I} = \vec{I_r} + \vec{I_{ir}} + \vec{h} \tag{7}$$

$\vec{I_r} = \vec{\nabla} \wedge \vec{A}$ describes the vortices and energy circulation phenomena in the field with $\vec{A}$ the associated potential vector.

$\vec{I_{ir}} = \vec{\nabla}\psi$ describes the transport of energy from energy sources to dissipation zones. It is the component of interest. ψ is the scalar potential of active intensity.

$\vec{h}$ is the harmonic component, it is divergence-free and curl-free at the same time. It may be a constant vector field in 2D cases.

The vibration energy flow through a closed section S of a structure is

$$\oiint_S \vec{I}.\vec{n}\, ds \tag{8}$$

where $\vec{I}$ is the structural intensity and $\vec{n}$ is the the outward pointing unit vector normal to the surface element ds.

The flow of energy across any given closed surface is equivalent to the rate of change of the structural intensity inside the surface that encloses the volume (divergence theorem):

$$\oiint_S \vec{I}.\vec{n}\, ds = \iiint_V \vec{\nabla}.\vec{I}\, dv. \tag{9}$$

V is the volume bounded by the surface S.

Combining the Helmholtz decomposition (7) and the divergence theorem (9) yields

$$\begin{aligned}
\oiint_S \vec{I}.\vec{n}\, ds &= \iiint_V \vec{\nabla}.\vec{I}\, dv \\
&= \iiint_V \vec{\nabla}.(\vec{I}_r + \vec{I}_{ir} + \vec{h}) \\
&= \iiint_V \vec{\nabla}.\vec{I}_r + \iiint_V \vec{\nabla}.\vec{I}_{ir} + \iiint_V \vec{\nabla}.\vec{h} \\
&= \iiint_V \vec{\nabla}.\vec{I}_{ir}.
\end{aligned}$$

Thus, the flow of energy across any closed section can be written as

$$\oiint_S \vec{I}.\vec{n}\, ds = \oiint_S \vec{I}_{ir}.\vec{n}\, ds. \tag{10}$$

Therefore, the power flow is expressed only as a function of the irrotational intensity $\vec{I}_{ir}$. This means that the rotational intensity and the harmonic component do not contribute to the quantification of the power flow. It is reasonable then to describe the propagation of vibration energy using only the irrotational component of the structural intensity.

To extract the irrotational intensity, the divergence operator can be applied to the Helmholtz decomposition of the structural intensity:

$$\begin{aligned}
\vec{\nabla}.\vec{I} &= \vec{\nabla}.(\vec{I}_r + \vec{I}_{ir} + \vec{h}) \\
&= \vec{\nabla}.(\vec{\nabla} \wedge \vec{A}) + \vec{\nabla}.\vec{\nabla}\psi + \vec{\nabla}.\vec{h} \\
&= \Delta\psi.
\end{aligned}$$

Thus,

$$\vec{\nabla}.\vec{I} = \Delta\psi. \tag{11}$$

The local power balance in steady-state harmonic vibrations of an elastic medium is equivalent to the variation of the structural intensity field [11]:

$$\vec{\nabla}.\vec{I} = P_{inj} - P_{diss}. \tag{12}$$

By combining Eqs. (11) and (12), a new equation governing the vibration energy flow is obtained. The solving of this equation is required to get the irrotational intensity vector field:

$$\Delta\psi = P_{inj} - P_{diss}. \tag{13}$$

ψ is the potential scalar, the unknown quantity, and its gradient $\vec{\nabla}\psi$ represents the irrotational intensity field. The equation is formally identical to the equation that governs heat transfer for stationary thermal conduction problems.

To solve equation (13), the computation of a dynamic response under harmonic excitation is first required to calculate the power injected P_{inj} and the power dissipated locally in each element of the structure P_{diss}. The solving of the equation is carried out by finite element method using a thermal solver.

Solving (13) not only requires the power injected to the structure and the distribution of the locally dissipated power, but also the boundary conditions of the considered domain. The normal flow of energy is zero at the boundaries which describes the total reflection at the boundary. There is no energy transfer to the outside of the structure. Thus, the boundary conditions write

$$\vec{\nabla}\psi.\vec{n} = \frac{\partial \psi}{\partial n} = 0, \tag{14}$$

where $\vec{n}$ is the unit vector normal to the considered boundary.

5 Power flow analysis using the irrotational intensity field

5.1 Single plate

The purpose of this part is to investigate the vibration energy path in a single flat plate using the irrotational intensity. The plate has the same properties as the one presented in section 3. Three viscous damper with damping rates of 100 N.s/m are attached to the plate in arbitrarily chosen points. To take into account the dissipation due to material, a uniform structural damping of 1% is considered. The plate is simply supported along its edges and excited by a harmonic force of 1N at a specific point, as shown in Figure (2).

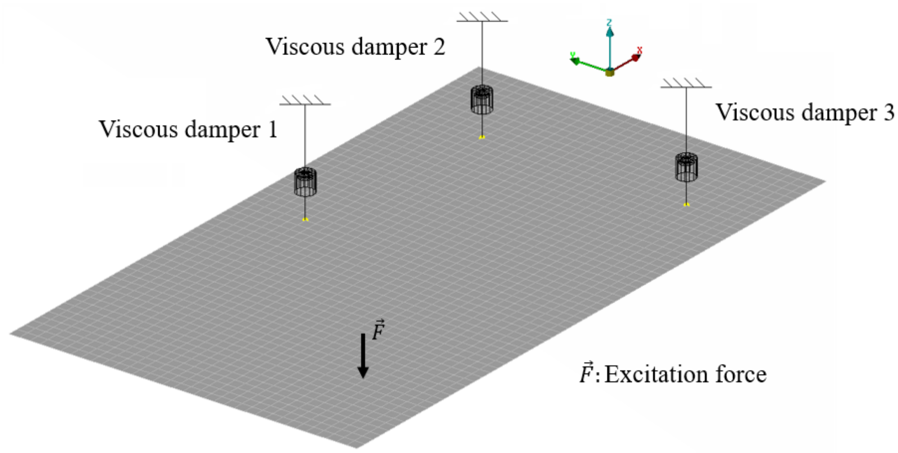
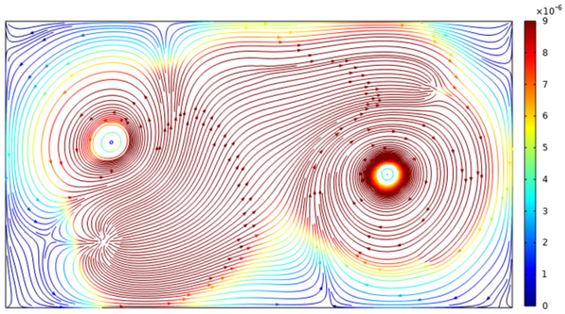
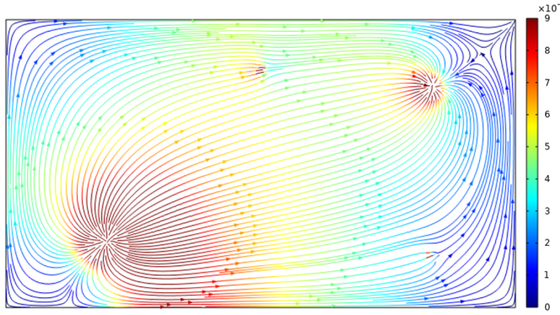


Figure 2: The finite element model of the plate

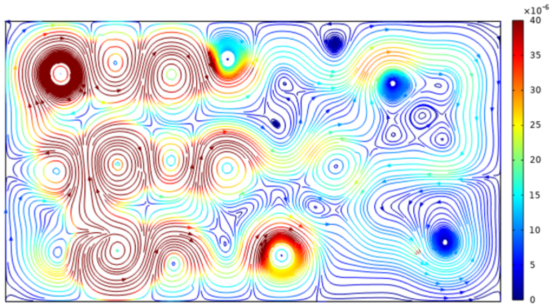
Figure 3 shows the structural intensity field and the associated irrotational intensity field at different frequencies, calculated using equation (13). The fields are drawn using streamlines and the color describes the amplitude of the fields.



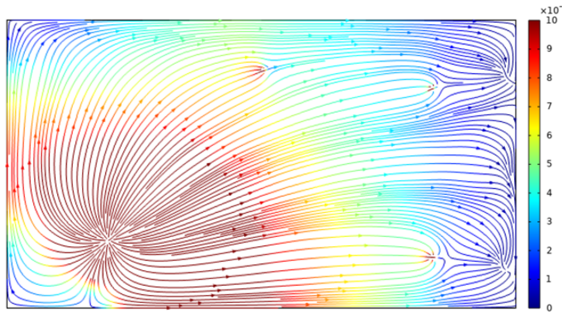
(a) Structural intensity field at 50 Hz



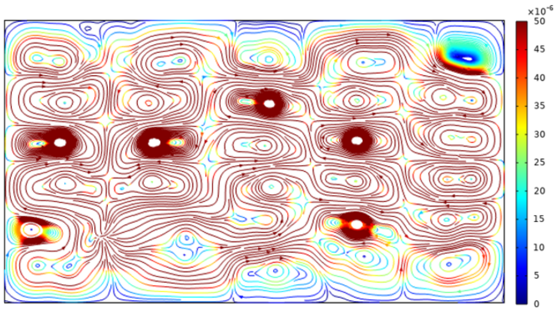
(b) Irrotational intensity field at 50 Hz



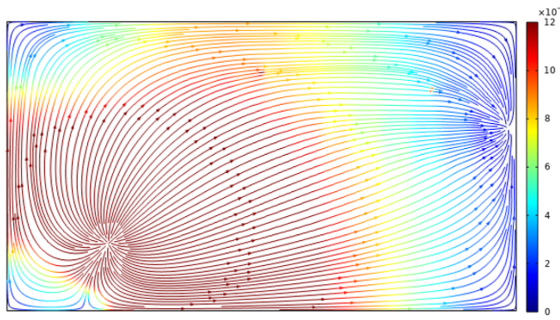
(c) Structural intensity field at 300 Hz



(d) Irrotational intensity field at 300 Hz



(e) Structural intensity field at 800 Hz



(f) Irrotational intensity field at 800 Hz

Figure 3: Structural intensity field and irrotational intensity field distributions in the plate

The number of vortices in the structural intensity field increases as the modal density is higher, making the field difficult to understand. The irrotational intensity field shows a smooth configuration and offers a better understanding of the energy propagation in the structure: the energy flows outward from the source and it is dissipated throughout the plate by the structural damping and the viscous damping. At higher frequencies, the energy is mostly dissipated by structural damping: streamlines do not converge to the positions of the viscous damping. The irrotational intensity characterizes perfectly the positions of the source and the sinks of the energy.

As expected, streamlines are parallel to the boundaries and no energy crosses the edges as the plate is simply supported and the normal flow at the edges is zero. No energy is absorbed by the supports.

Figure 4 illustrates the power dissipated by the structural and the viscous damping at different frequencies. As the frequency increases, the viscous damping dissipates less power than the structural damping, which can also be concluded directly from the irrotational intensity field.

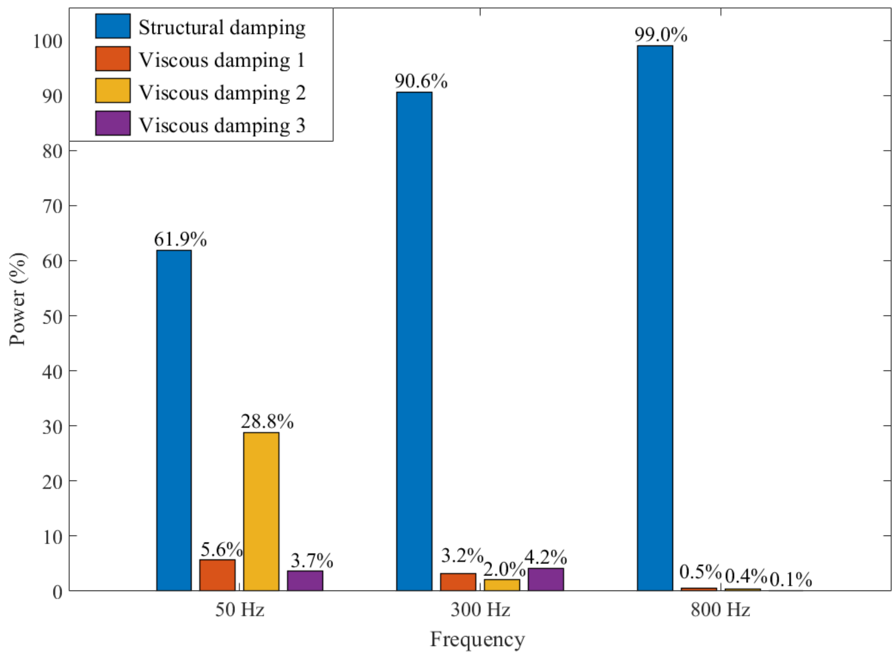


Figure 4: Power dissipated by structural and viscous damping in the plate

5.2 Plate assembly

In this section, the filtering method is applied to a structure constituted by two identical coupled plates (same properties as in section 3). The plates are coupled by three beams as shown in figure 5. The common edge is free and the other edges are simply supported. The first plate is excited by a harmonic force of 1N. For both plates a hysteretic damping of 1% is considered.

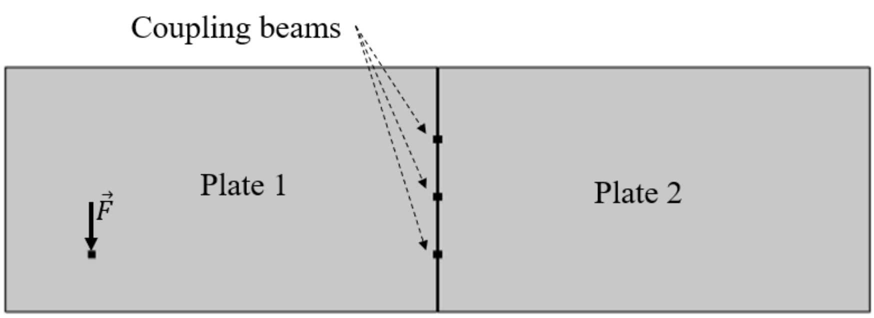
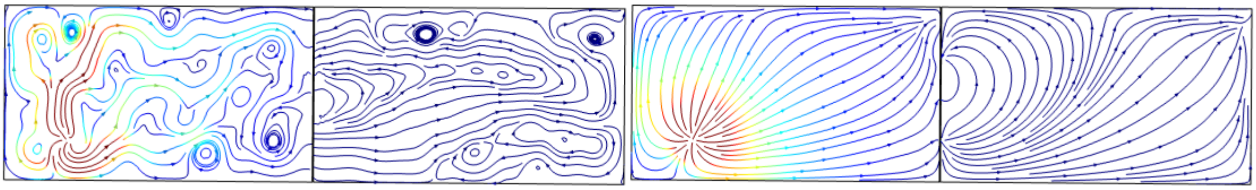


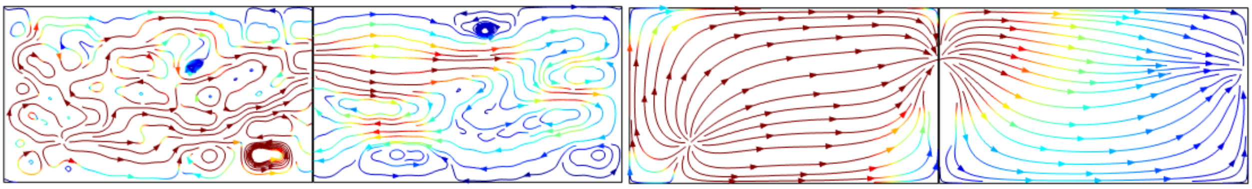
Figure 5: Two coupled plates

Figure 6 shows the structural intensity and the irrotational intensity distributions for different frequencies of force excitation. The irrotational intensity field clearly shows the path of the energy in the plates and in which beam the energy is mostly transmitted. At medium frequencies (300 Hz), one can notice that energy is exchanged between the two plates: the energy propagates in both directions from plate 1 to plate 2 and vice versa. At high frequencies, the energy is diffusive. It propagates only in one direction from plate 1 to plate 2. For the considered frequencies it can be assumed that a major part of the energy is dissipated in plate 1.



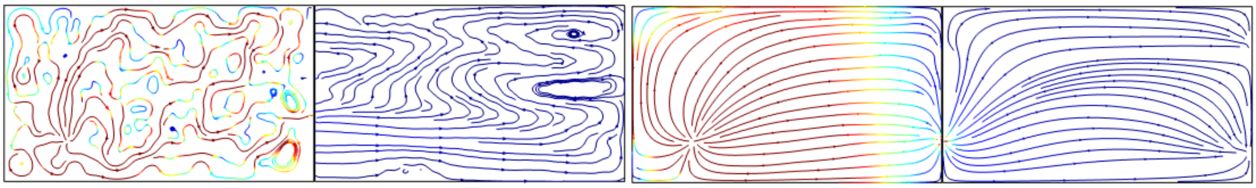
(a) Structural intensity field at 300 Hz

(b) Irrotational intensity field at 300 Hz



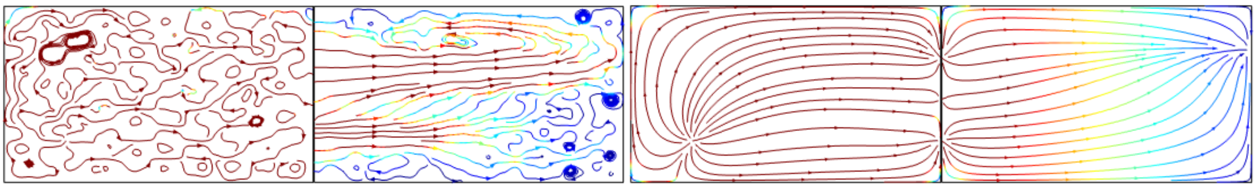
(c) Structural intensity field at 500 Hz

(d) Irrotational intensity field at 500 Hz



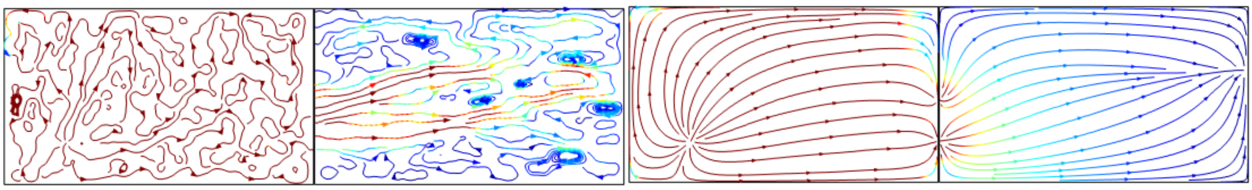
(e) Structural intensity field at 1000 Hz

(f) Irrotational intensity field at 1000 Hz



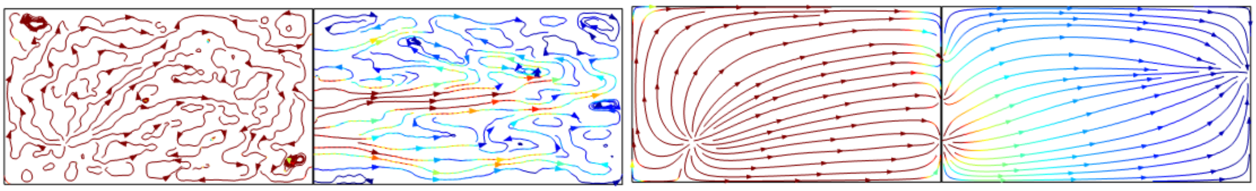
(g) Structural intensity field at 1500 Hz

(h) Irrotational intensity field at 1500 Hz



(i) Structural intensity field at 2000 Hz

(j) Irrotational intensity field at 2000 Hz



(k) Structural intensity field at 3000 Hz

(l) Irrotational intensity field at 3000 Hz

Figure 6: Structural intensity field and irrotational intensity field distributions in two coupled plates

6 Conclusion

In this paper, a method to calculate the irrotational intensity field is provided. It is based on harmonic response computation and diffusion equation solving. Extracting the irrotational component of the structural intensity field has a significant advantage. It allows the visualization of the energy flow without the masking effects of energy loops that are associated with the rotational intensity. The filtering method has been applied to a single plate and plates assembly. It provides a better understanding of vibration propagation and it is shown that it can perfectly characterize sources and sinks of energy in the plates.

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