

1 Generation of extremely high angle Bessel 2 beams

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8 **Abstract:** We present a setup to generate tightly focused Bessel beams, composed of a
9 half-ball lens coupled with a relay lens. The system is simple and compact in comparison with
10 conventional imaging of axicons based on microscope objectives. We experimentally demonstrate
11 the generation of a Bessel beam with 42 degrees cone angle at 980 nm in air with a typical beam
12 length of 500 μm and a central core radius of about 550 nm. We studied numerically the effects
13 of misalignment of the different optical elements and the range of tilt and shift that are acceptable
14 to obtain a regular Bessel beam.

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16 1. Introduction

17 The Bessel beam finds application in numerous fields thanks to its peculiar properties. It was first
18 recognized as a propagation-invariant solution to the Helmholtz equation in 1987 by Durnin *et*
19 *al.* [1]. The beam is generated by the interference of plane waves in a conical geometry, with an
20 angle of interference called *cone angle* [2]. Its transverse profile is invariant during propagation
21 and is featured by an intense central core surrounded by weaker concentric rings. The central
22 core of the beam can reach a very high aspect ratio, since the radius of the core and its length
23 depend on two independent parameters, in contrast with the Gaussian beam where the waist and
24 Rayleigh range cannot be independently controlled. The large depth of field of the Bessel beam
25 and the high contrast between the central core and the rings fit well the needs of the processing of
26 transparent material, like drilling [3–5] and cutting [6]. Moreover, high-aspect ratio central core
27 allows a long working range that is useful when the position focus-sample is critical, as in the
28 process of curved surfaces [7] and in the welding of different materials [8]. Another advantage of
29 the Bessel beam is the self-reconstruction property that allows the regeneration of the beam after
30 an obstacle. This last feature is particularly beneficial for microscopy techniques, one example
31 is the light sheet microscopy [9, 10]. It can be used also to study non-linear processes [11], in
32 optical trapping [12], in the generation of warm dense matter over a long distance [13], or in
33 creating long plasma tracks to guide discharges [14].

34 Some applications, such as material processing and particle acceleration, require a very high
35 intensity in the central core. This can be reached at constant input pulse energy by increasing
36 the focusing angle of the Bessel beam. It has been shown theoretically and experimentally that
37 a higher angle Bessel beam allows obtaining channels in transparent materials thanks to the
38 reduction of the screening effect of the plasma [15]. In fact, to cut or drill transparent materials,
39 the Bessel beams used have angles in the range between 17° and 26° in air, that lead due to
40 refraction to angles between 10° and 16° in a medium with refractive index ~ 1.5 [3, 16]. A
41 higher angle Bessel beam leads to a smaller central core radius as the FWHM of the central peak
42 follows: $d_{\text{FWHM}} = 0.36 \frac{\lambda}{\sin \theta}$.

43 The theoretical studies on the generation of large angle Bessel beam provided interesting
44 setup configurations. For example, the use of a 90° apex-angle concave conical mirror with a
45 radially polarized beam allows to generate a really high aspect-ratio Bessel beam with a length of

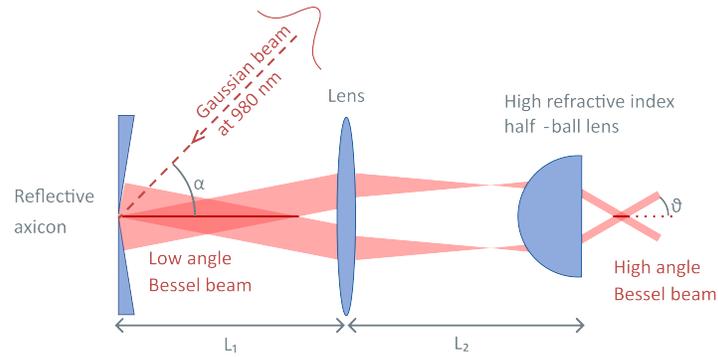


Fig. 1. Schematic of the setup for the generation of high angle Bessel beam. The Gaussian beam at 980 nm generates a low angle Bessel beam thanks to a reflective axicon of 2 degrees, the Bessel beam is then demagnified with a telescopic system composed by a lens of 35 mm focal length and a half-ball lens. The half-ball lens has a diameter of 8 mm and it is made of S-LAH79 (refractive index: 1.97 at 980 nm).

46 50000λ and a central core dimension of 0.36λ [17]. However, the use of a reflective axicon with
 47 very high angle is incompatible with the insertion of a sample. Another theoretical setup, that
 48 overcomes this problem, uses a combination of a binary phase plate and a doublet including a
 49 high NA 0.95 lens [18]. The computed Bessel beam has a central core size (FWHM) of 0.39λ
 50 (cone angle of 67.6°).

51 The standard technique to generate a high-angle Bessel beam uses a telescopic system: a
 52 primary low angle Bessel beam, created using an axicon, a spatial light modulator or a fluidic
 53 axicon [19], is imaged using a high magnification $2f$ - $2f$ telescope. To reach high magnifications,
 54 the second lens is a microscope objective [20]. However, not only the overall setup is very long
 55 because of the length of the telescope, but also the intensity distribution impinging on the second
 56 optical element, the microscope objective, is in the shape of a thin ring. The high intensities used
 57 for laser materials processing can easily lead to damage of a costly optical element. Another
 58 technique, that takes advantage of the combination of three axicons, allows to reach energies
 59 up to the Joule level, but it is limited to a cone angle of 26 degrees (FWHM 0.82λ) by the
 60 manufacturability of high base angle axicons [16, 21].

61 Here, we propose a compact experimental setup that overcomes the limitations of the previous
 62 techniques using a half-ball lens. The half-ball lens helps to generate very high angle Bessel
 63 beams, up to 42° , thanks to the high refractive index and the high curvature. Simultaneously,
 64 the absence of glue in the half-ball lens leads to a higher damage threshold when compared to
 65 a conventional microscope objective. Since strong focusing is conventionally associated with
 66 delicate alignment, we analyze the different positioning constraints on the system.

67 2. Experimental setup

68 We generate the high angle Bessel beam with the setup shown in Fig. 1, using as a laser source
 69 a laser diode at 980 nm. The Gaussian beam has a waist of ≈ 3 mm (diameter at $1/e^2$) and
 70 impinges on a reflective axicon of 2 degrees base angle (Canunda axicon AX-2-25-S) at an
 71 incidence angle of $\alpha = 30$ degrees. This allows generating a primary Bessel beam with a cone
 72 angle of 4 degrees. Then, the beam cone angle is increased using a telescopic system composed
 73 of a lens (focal length 35 mm) and a 8 mm diameter half-ball lens. The material of the half-ball
 74 lens is S-LAH79 which is a high refractive index dielectric, with a value of 1.97 at the used
 75 wavelength. The shaped beam illuminates only a restricted area of the half-ball lens, shaped as a
 76 thin ring of thickness $200 \mu\text{m}$. This is in agreement with the commonly accepted rule of thumb

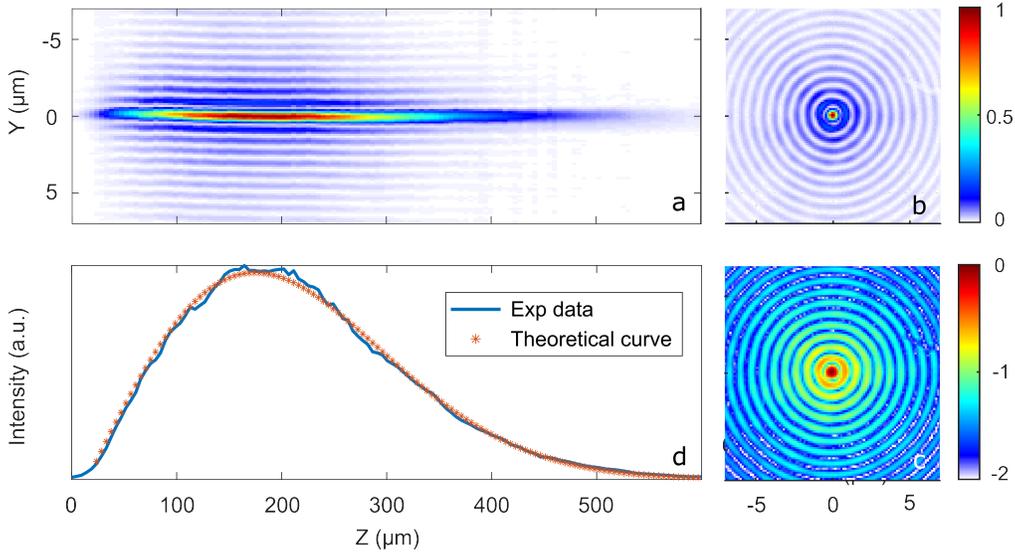


Fig. 2. Intensity distribution of the Bessel beam, with wavelength 980 nm, featured by a central core of 523 nm (FWHM), corresponding to an angle of 43 degrees, and a length of about 500 μm (FWHM 250 μm). (a) Section of the scan; (b) Cross-cut in linear scale at $z = 160 \mu\text{m}$; (c) Cross-cut in logarithmic scale $z = 160 \mu\text{m}$. (d) Experimental and theoretical data of the evolution of the central core intensity;

77 of using lenses with beam diameters of only 1/10 of the focal length to avoid aberrations. Here,
 78 the limit is therefore $\sim 500 \mu\text{m}$. For this reason, the high curvature does not introduce measurable
 79 spherical aberrations and all optical rays are identically refracted.

80 The ideal distance between the axicon and the relay lens (L_1) is 90 mm and the one between
 81 the relay lens and the half-ball lens (L_2) is 30 mm. We will see in the last section that these
 82 distances are relatively flexible to produce a Bessel beam.

83 We image the beam using an imaging system composed of a microscope and relay lens that
 84 leads to a magnification of 110. The full imaging system is placed on a motorized translation
 85 stage, which allows us to scan the beam along the propagation direction.

86 3. Experimental results

87 We show our experimental results in Fig. 2. In Fig. 2(a), we plot the longitudinal cross-section of
 88 the intensity distribution as a function of propagation distance, where we see the parallelism of
 89 the lobes and that the Bessel beam length is of about 500 μm equal to 510λ (FWHM 250 μm or
 90 255λ). Fig. 2(b) shows the transverse cross-section at a distance corresponding to the peak of
 91 the longitudinal distribution ($z = 160 \mu\text{m}$). The same distribution is shown with a logarithmic
 92 scale in Fig. 2(c), which features the high degree of symmetry and quality of the Bessel beam
 93 produced. The dimension of the central core changes slightly along the beam, which means that
 94 the angle of the Bessel beam is not perfectly constant. In our case, the first part of the beam is
 95 featured by a central core of 523 nm (FWHM), *i.e.*, 0.53λ , which corresponds to an extremely
 96 high angle of $\theta = 42$ degrees (measured at $z = 90 \mu\text{m}$), and the end of the beam has a central
 97 core of 554 nm (FWHM) *i.e.*, 0.56λ , which corresponds to an angle of $\theta = 40$ degrees (measured
 98 at $z = 325 \mu\text{m}$). We obtained the dimension of the central core by fitting its intensity with a
 99 zeroth-order Bessel function with an error of 1 degree on the angle estimations.

100 In Fig. 2(d), we compare the evolution of the on-axis intensity distribution to the one of an
 101 ideal Bessel-Gauss beam, which follows [3, 22]:

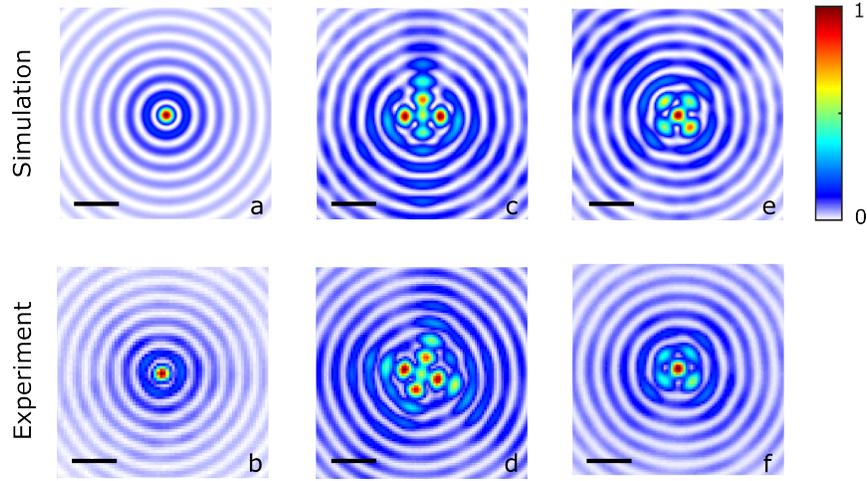


Fig. 3. Comparison between the simulations obtained with Zemax OpticStudio™ and the experimental results. We compared the undistorted Bessel beam (Fig. a simulated and Fig. b experimental), an axicon tilt of 0.14° (Fig. c simulated and Fig. d experimental) and the effect of $50\ \mu\text{m}$ shift of the half-ball lens (Fig. e simulated and Fig. f experimental). Scale bars are $2\ \mu\text{m}$.

$$I(z) = \frac{8\pi P_0 z \sin^2 \theta}{\lambda w^2} \exp \left[-2 \left(\frac{z \sin \theta}{w} \right)^2 \right]$$

102 where P_0 and w are respectively peak power and the waist of the input Gaussian beam, θ is the
 103 cone angle of the Bessel beam. We can see that the experimental data is in excellent agreement
 104 with the theoretical one.

105 4. Analysis of the setup

106 We have simulated our setup from the reflective axicon to the high angle Bessel beam using
 107 the software Zemax OpticStudio™. This analysis aimed to investigate how critical is the
 108 positioning of each element and the effects on the final beam shape. For the simulations, we
 109 have modelled the axicon using an ideal off-axis reflective axicon with a phase profile given
 110 by $\Phi(x, y) = (2\pi/\lambda) \tan \beta \sqrt{\cos^2(\alpha) x^2 + y^2}$ [20]. We used the optical design of the relay lens
 111 provided by the supplier and we modeled the half-ball lens with the high refractive index 1.97.
 112 The system was optimized to produce the desired cone angle and the parallelism of the rays of
 113 the Bessel beam, which means a constant angle for the entire beam. We used the Zemax Point
 114 Spread Function, based on Huygens wavelet principle, to compute the final intensity distribution
 115 in the Bessel zone. We provide below the maximum deviations in tilt or shift from the ideal
 116 position before which the high angle Bessel beam starts to lose its characteristic cylindrical
 117 symmetry, particularly in the first lobe (see Fig. 3). We compare in Fig. 3, the simulated and the
 118 experimental results. For comparison, the undistorted beam is shown in (a) for the simulation
 119 part and in (b) for the experimental one.

120 The axicon has been manufactured to be operated under an incidence angle of $\alpha = 30^\circ$. Our
 121 simulations show that, to maintain an acceptable symmetry of the intensity distribution, the
 122 maximum deviation from the ideal angle is about 0.08° : the increase in the deviation leads to a
 123 change in the shape of the first ring into four lobes as we can see in Fig. 3 (c) and (d) where the
 124 tilt applied to the axicon is 0.14° for the simulated and the experimental results, respectively.

125 This distortion progressively grows from the end of the beam, as can be expected from an axicon
126 illuminated under oblique incidence [23].

127 The distance between the axicon and the relay lens (L_1) is not critical. It can be varied over
128 a range of about 40 mm around the ideal position of 90 mm ($\pm 22\%$), while the only impact on
129 the Bessel beam is to slightly change its cone angle by 1° to 2° . In this case, the cone angle is
130 still homogeneous along the beam. The distance between the relay lens and the half-ball lens
131 (L_2) is also not critical within a smaller range of 1 mm to 2 mm, but a deviation from the ideal
132 position of 1 mm induces a variation of the angle between the beginning and the end of the
133 Bessel beam of 1° to 2° over the $200\ \mu\text{m}$ length of the Bessel beam. The small error in the
134 longitudinal positioning of the two lenses introduces a quadratic term. The consequence is that
135 the wavevectors undergo slightly different refraction and are not perfectly parallels. The result
136 is a varying angle along the propagation direction. This case corresponds to the "accelerating"
137 or "decelerating" Bessel beams first reported in Ref. [24]. The half-ball lens allows having a
138 deviation from the ideal position of 1° for the tilt. In contrast, the centering of the half-ball lens
139 is relatively critical as it requires a precision of typically $30\ \mu\text{m}$. In Fig. 3 (e), we show the result
140 of the beam for a de-centering of $50\ \mu\text{m}$ and the corresponding experimental result (f). In this
141 case, the first side lobe gets split into four parts. We finally note that our simulation results are in
142 good agreement with our experimental observations.

143 In addition to the positioning constraints, we studied the flexibility of the system regarding
144 different wavelengths. We could obtain a high cone angle Bessel beam for a range of wavelength
145 that varies from $0.5\ \mu\text{m}$ to $2\ \mu\text{m}$. Due to the chromatic dispersion, the positioning of the different
146 lenses had to be adjusted for each case to obtain a regular Bessel beam.

147 5. Conclusion

148 We have experimentally demonstrated a novel design for Bessel beam generation with a high
149 focusing angle using a simple optical setup. We were able to experimentally reach an extremely
150 high angle of 42° (equivalent numerical aperture of NA 0.67). This was made possible by the
151 use a half-ball lens with a high refractive index. This configuration brings different advantages:
152 the setup is at least 5 times more compact in comparison with a standard demagnification system
153 and more cost-effective. Moreover, the usable wavelength range varies from the VIS to the near
154 IR. Importantly, the damage threshold of the half-ball lens is much higher compared to the one of
155 a microscope objective, particularly because of the absence of glue.

156 In addition, we have analyzed, using numerical simulations the constraints on the positioning
157 of the optical elements. Our analysis reveals that only two parameters are relatively critical: the
158 incidence angle on the reflective axicon should be accurate within 0.08° and the centering of the
159 half-ball within $30\ \mu\text{m}$. Finally, our approach is also compatible with spatial filtering [25], as
160 well as polarization [18] or transversal shaping techniques [26, 27]. We believe that this simple
161 and compact setup can provide new perspectives for the study of laser-matter interaction with
162 Bessel beams in unconventional configurations.

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170 **Data Availability Statement.** Data underlying the results presented in this paper are not publicly available
171 at this time but may be obtained from the authors upon reasonable request.

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