

Topology optimization of piezoelectric structures : micro-actuators and energy harvesters

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Abstract— This paper reviews the primary research efforts conducted within the AS2M department of the FEMTO-ST institute, focusing on topology optimization of piezoelectric structures. The paper primarily highlights the principle and the possibilities offered by topology optimization with a specific emphasis on the SIMP approach (Solid Isotropic Material with Penalization). Then, the design processes of piezoelectric micro-actuators and energy harvesters are described. The optimized piezoelectric structures are presented and the improvements over classical designs are assessed. Finally, the paper discusses the feasibility and the potential of multi-material topology optimization.

Index Terms— Piezoelectric micro-actuator, piezoelectric energy harvester, topology optimization

I. INTRODUCTION

The interest of miniaturized systems is considerable and well established [1]. Based on smart materials like piezoelectric materials, they can change their inherent properties in response to external stimuli in a controllable manner. Taking this advantage, they are widely used in several applications such as: biomedical, optics, fluidics, car industry, energy harvesting, electronics, etc. However, due to their size and density of integration, their design remains challenging because it requires taking into account the coupling between the structure and its mechanisms through a global design strategy. This requirement is induced by smart materials that play a significant role in the technological design of these systems. To address this challenge, various design methodologies have been proposed such as optimal arrangement of actuators/sensors [2]–[4], interval method [5], [6] or blocks method [7], [8]. Nevertheless, most of these methods lack generalization since they act only on the geometric parameters of the structure. This limits in advance the shape of the active mechanisms (actuation and measurement) and consequently that of the resulting structure.

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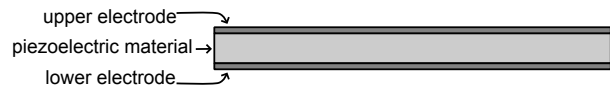


Fig. 1. Piezoelectric material sandwiched between two electrodes.

In this regard, topology optimization [9], and particularly the SIMP (Solid Isotropic Material with Penalization) method seems to be a promising solution. Unlike classical optimization methods, it gives rise to efficient structures in response to requirement specifications. Its principle is mainly based on optimal material distribution within a specified design domain. Presented initially by Sigmund et al. [9]–[11], this powerful method is suitable for the design of passive structures. Since becoming a conceptual design tool, it has been particularly applied to design smart structures based on piezoelectric materials [12]. However, it remains challenging to handle due to the non-intuitive and non-unified integration of piezoelectric materials.

To tackle this limitation, the AS2M department has been actively working since 2018 to enhance the SIMP method by extending it to include piezoelectric materials. The objective is to provide a straightforward strategy for integrating the physics of the piezoelectric materials within the SIMP method. This gave rise to several challenges related to: smart materials modeling, finite-elements formulation, computational and numerical implementation. All these challenges have been or are being investigated at AS2M/FEMTO-ST institute.

This paper provides a comprehensive overview of the research that has been conducted at the AS2M department/FEMTO-ST institute, the works that are currently underway, and the potential directions for future advancements. Two main developments based on piezoelectric materials will be presented and discussed. The first concerns the design of piezoelectric actuators, while the second concerns energy harvesters. The rest of the paper will discuss the current works and the prospects with a specific focus on multi-material topology optimization.

II. TOPOLOGY OPTIMIZATION

A. SIMP approach

SIMP is a mathematical design methodology aiming to find an optimal layout within a limited design domain [9]. Based on material distribution, the method allows minimizing or maximizing an objective function while subjected to one or several constraints. Its key principle consists of introducing a density penalization law. The

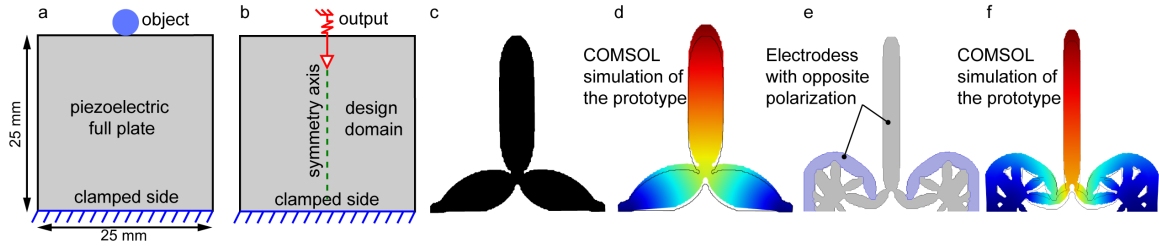


Fig. 2. Topology optimization of a piezoelectric micro-actuators. a) Problem definition, b) Problem formulation, c) Optimized layout without polarity, d) Simulated layout without polarity, e) Optimized layout with polarity, f) Simulated layout with polarity.

method is largely integrated into several design softwares such as COMSOL, ALTAIR Inspire, Ansys Discovery, SOIIDWORKS, etc. As a global and systematic approach, it is largely used in the engineering and design of passive mechanical structures because it offers several advantages such as weight reduction while enhancing performance and efficiency.

The method has also been applied for the topological design of active structures in particular piezoelectric structures [12]. However, the existing methodology lacks some mathematical development regarding the optimization of the polarity in addition to the topology. These mathematical limitations include the explicit formulation of the sensitivity analysis. Moreover, the realization of the optimized topologies of the piezoelectric structures received a very little attention in the literature. We addressed these limitations by (i) developing analytical and theoretical aspects of topology optimization of piezoelectric structures, (ii) developing algorithms and computer codes and (iii) fabricating and investigating experimentally the obtained structures. The common underlying factors in these developments were piezoelectric material modeling and numerical implementation.

B. Piezoelectric modeling

Our primary investigations focused on planar piezoelectric structures. Thus, the starting design domain consists of a piezoelectric layer sandwiched between two electrodes as illustrated in Fig. 1. Its modeling involves several simplifying assumptions [13], [14]. They particularly enable us to derive a 2D model from the IEEE 3D model [15] of piezoelectric material. To discretize the design domain and obtain the finite element modeling, the four-node rectangular element is employed as shown in Fig. 4-(a). With the developed finite element model, it is possible to formulate the optimization problem.

III. PIEZOELECTRIC MICRO-ACTUATORS

The use of piezoelectric materials to actuate microbotics systems is of particular interest. As a smart material, they have several advantages such as: high displacement resolution, large output force, high dynamics response and significant scaling-down possibilities [16]. However, due to their crystalline arrangement, they provide a low relative deformation (0.1% of actuator's size) that limits their stroke [17]. To overcome this limitation, we employed topology optimization framework [14] to optimize both the topology and the polarity of the actuator. This simultaneous

optimization allows combining material expansion and compression in order to increase the stroke of the actuator without using any passive amplification mechanism. This enables the miniaturization of the optimal design. Two 1D actuators were designed starting from a full domain considered as a basic reference piezoelectric actuator. The first design considered only the optimization of topology while the second one took into account the optimization of the topology and polarization profile simultaneously. This section recaps the problem formulation, the optimization and the main results of this study. To find out more theoretical details, readers can refer to [13], [14].

A. Problem formulation

Figures 2-(a,b) illustrates the definition and the mechanical formulation of 1D piezoelectric actuator. The bottom side of the domain is clamped while the middle point of the top side is considered as the actuator output. In addition, the actuator-object interaction is modeled as a spring that modulates the actuator displacement: a lower stiffness value results in a higher displacement and vice versa. Using this configuration, two optimized designs are obtained where the difference lies in whether or not the electrodes are optimized. In both cases, the volume fraction is set to 0.3, meaning that only 30% of the initial domain is used for the optimized designs.

B. Algorithm, optimization and simulation

Following the modeling and formulation of the problem, an optimization algorithm was developed and implemented under MATLAB [13]. The application of this algorithm leads to the designs depicted in Figs. 2-(c,e). Layout (c) comprises a uniform electrode while layout (e) comprises two different electrodes with opposite polarities. The second design comprises two regions with inverse polarities. When one region retracts the other extends resulting in a considerable improvement of output displacement. This analysis is confirmed by FEA simulations illustrated in Figs. 2-(d,f) where the obtained results show that the displacement of the design with optimized polarity is almost twice the displacement of the design with uniform polarity. More comparison results between the full actuator plate (reference actuator) and the optimized designs are reported in Table I.

C. Fabrication and experimental validation

Starting from a piezoelectric plate, the three prototypes shown in Fig. 3 were fabricated. The fabrication process started by cutting the designs from piezoelectric plates

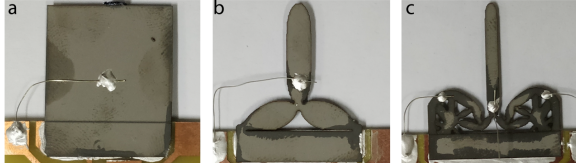


Fig. 3. Fabricated prototypes, a) Full plate (reference actuator), b) Prototype without polarity optimization, c) Prototype with polarity optimization.

TABLE I
SUMMARY OF SIMULATION AND EXPERIMENTAL RESULTS [14]

	Simulation (Input voltage = 5V)		
	Full plate	Opt without pol	Opt with pol
Displacement (nm/V)	57	81	161
Displacement gain w.r.t.f.p	-	1.42	2.82
Blocking force (N)	2.56	0.21	0.18
Blocking force gain w.r.t.f.p	-	0.08	0.07
Energy density (J/m^3)	4.55	1.81	3.10
Energy density gain w.r.t.f.p	-	0.39	0.68
	Experiment (Input voltage = 5V)		
	Full plate	Opt without pol	Opt with pol
Displacement (nm/V)	62	86	174
Displacement gain w.r.t.f.p	-	1.38	2.8

* w.r.t.f.p : with respect to full plate

(commercial piezoelectric material PSI-5H4E from Piezo Systems Inc) using a laser machine (Siro Lasertec GmbH, Pforzheim, Germany). Then, the wires are glued to the electrodes of the PZT plates. Moreover, to follow the polarization profile, the top electrode is divided into two sections to avoid charge cancellation. An experimental bench was set and a series of measurements were performed under a maximum excitation voltage of 5V which respects the linear assumption of the piezoelectric model. The resulting average displacements are reported in Table I. As expected, there is a satisfying agreement between the experimental and the simulation results. In addition, the superiority of the optimized designs versus the full piezoelectric plate in terms of stroke is observed.

D. Discussion

The developed algorithm reduces drastically the material amount while enhancing the actuator energy density and stroke. Indeed, only 30% of the material was optimally distributed in order to provide a displacement greater than the displacement of an actuator with a uniform polarization. Although the actuator output force decreased, the optimization led to a compact and economical design. This is particularly interesting in the context of miniaturization since the non-occupied space can be utilized to implement additional functionalities such as sensors or electronic circuits.

IV. PIEZOELECTRIC ENERGY HARVESTERS

In parallel to actuation, piezoelectric materials are widely used in energy harvesting applications. Converting vibration to electrical energy, these devices, i.e. Piezoelectric Energy Harvesters (PEHs) offer a potential alternative to batteries in low-power-wireless devices such as wireless sensors [18], small-scale robots [19], etc. Thanks to the direct effect of piezoelectricity, they can convert mechanical to electrical energies with a simple mechanism. This simplicity makes the piezoelectric energy harvester

more efficient than their rivals like electromagnetic and triboelectric at small scales. At AS2M department, we mainly worked on the optimization of the mechanical structures of PEHs.

Mostly known and still used configuration for the vibrational PEH is the cantilever configuration with tip attachment due to its largely produced strains and feasibility of fabrication. Considering this configuration as the first approach to increase the efficiency of the cantilever PEH, we proposed to have in-span attachments in addition to tip attachment in order to harvest the energy from higher modes and resonance frequencies [22]. Based on an analytical approach to find the output voltage, we proposed a neural network-based genetic algorithm (GA) approach to optimize the placement and geometry of the in-span attachments. However, the major problem with cantilever configuration is that it is one degree of freedom configuration, which can absorb the energy from one direction of excitation. This will restrict the possible applications of the cantilever PEHs, where the excitation can come from different directions. There are some designs for multi-directional PEHs in the literature [23], [24]. However, the miniaturization of these mechanism-based designs is challenging. To tackle this problem, we employed SIMP topology optimization to obtain new and previously unknown configurations for the PEH.

A. single-layer piezoelectric energy harvester

1) *Modeling & problem formulation:* Utilizing the piezoelectric constitutive equations, first, a 2D finite element model of a single piezoelectric plate sandwiched between two electrodes (Fig. 1) is developed. The goal is to design a two degrees of freedom piezoelectric plate energy harvester that can harvest the energy from external in-plane harmonic force coming from different directions. In this regard, the configuration of load and boundary conditions in Fig. 4-(a) is proposed. The most challenging problem in this case is the charge cancellation due to a combination of tension and compression in different parts of the plate. Therefore, an optimization problem is formulated to find the best possible layout and polarization profile of the piezoelectric plate to maximize the electrical output and overcome the problem of charge cancellation. The volume fraction (optimized design volume/full plate volume) is decreased to decrease the stiffness of the piezoelectric plate against in-plane forces.

2) *Algorithm & optimization:* After the definition of the optimization problem, we used the gradient-based optimizers like optimality criteria (OC) and Method Moving Asymptotes (MMA) [25]. To implement the gradient-based optimizer, the sensitivity of the objective function with respect to the optimization variables i.e. density and polarization profile is calculated analytically with the help of the adjoint method. After performing the sensitivity analysis, we developed the optimization algorithm as it is mentioned in [20], [21]. Then, we developed our MATLAB code to implement the topology optimization algorithm [26].

3) *Numerical results, simulation & experiment:* In panels (b) and (c) of the same figure, the final optimized layout

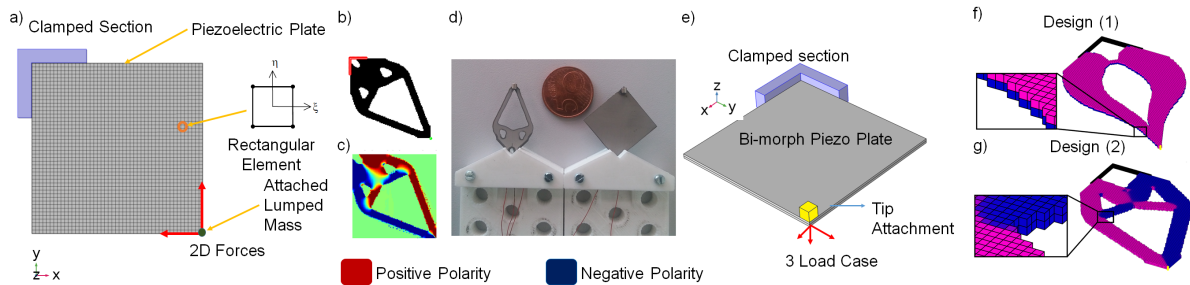


Fig. 4. Piezoelectric Energy Harvesters designed by topology optimization. a) single-layer piezo plate modeled by 2D finite element method [20]. b) Optimized topology, c) Optimized polarity, d) Fabricated prototype, e) Bi-morph piezo plate modeled by 3D finite element method [21], f) Optimized topology without polarization optimization, g) Optimized topology with polarization optimization

and polarization profile for PZT plate under excitation of two harmonic forces in two directions can be seen [20]. In panel (c), the red color and blue color represent positive and negative polarization in the z direction.

To analyze the performance of the optimized design, COMSOL multiphysics is used to compare the performance of the optimized design with the full plate. The simulation results proved the superiority of the optimized designs over the classical full plate while having less amount of material [20]. On the other hand, the amount of produced voltage and electrical power is not the same for every direction of the force. This is due to the fact that the stiffness of the plate in different directions is not the same. For the sake of brevity, we do not present the simulation results here. Interested readers are referred to the published paper [20].

The fabrication process is similar to what has been explained for the piezoelectric actuators. The difference here is that magnets are attached at the tip of the beam to generate vibrations force when excited by an electromagnet as it is shown in figure 4-(d). The magnets are attached in two different directions so they can excite the designs in two different directions.

Experimental results demonstrated that for an excitation frequency equal to 20 Hz, the voltage and power of the optimized design are 8.75 and 7.54 times higher than the full plate. These improvements are due to the fact that the optimized design is having better strain distribution and more importantly, it has separated electrodes that avoid charge cancellation.

B. Bi-morph piezoelectric energy harvester

In the next phase of our research, a bi-morph piezoelectric plate instead of the single-layer piezoelectric plate is considered as a design domain to consider out-of-plane forces and deformations [21].

1) *Modeling & Problem formulation:* Similar objective and constraints from single-layer PEH are considered in the optimization problem of the multi-directional Bi-morph PEH i.e. reduction of weight while maximizing the efficiency of the harvested energy from excitation coming from different directions. In the case of bi-morph PEH, the configuration of the boundary condition remains the same while a 3-load case is applied at the tip of the structure (Fig. 4-(e)). The bi-morph plate consists of 3 electrodes on the top, middle and bottom surfaces of the plate. The finite

element modeling of the system is done by discretizing the design domain with a finite number of 3D hexahedron elements.

2) *Algorithm & optimization:* The sensitivity analysis and optimization algorithm for 3D and 2D finite element modeling is formulated similarly. However, the implementation MATLAB code changes considerably to include the third dimension and application of electrical boundary conditions regarding the existence of several electrodes.

3) *Numerical results, simulation & experiment:* The results of the optimization for two cases are shown in Fig. 4-(f,g) [21]. The optimized design (1) is the result of optimization without optimizing the polarity and design (2) is the result of optimization with optimizing polarity. In design (1), in the case of planar forces, there will be charge cancellation due to compression and tension in different parts of the layer. To remedy, in design (2), the polarity is optimized as well. For the realization of this polarization profile, the top and bottom electrodes are divided into two sections to simulate the polarization profile. As such, the design has 2 electrodes on top, 2 electrodes on bottom and one electrode in the middle.

To assess experimentally the performance of the optimized designs, their electrical to mechanical efficiency is compared with a classical full plate. The experimental investigation demonstrated that the optimized design with optimized polarity can have up to 2 times better voltage output than the piezoelectric full plate while having less amount of mass [21].

C. Frequency tuning & optimization of mass

The best efficiency of a vibrational PEH can be obtained when it is excited at its resonance frequency. Frequency matching is therefore very crucial for every PEH since only 2% deviation of resonance frequency from excitation frequency will drop the electrical output power by 50%. Moreover, the available excitation frequency in real applications is generally between 10 to 30 Hz, which is below the normal resonance frequency of the PEHs. The classical and conventional method to match the resonance frequency with the low excitation frequency is to attach a lumped mass at the tip of the cantilever PEH [30].

In our recently published work [29], we combined topology optimization and frequency tuning technique to raise further the efficiency of PEH. The idea consists to define a constraint on the fundamental frequency of

TABLE II

SUMMARY OF PUBLICATIONS REGARDING TOPOLOGY OPTIMIZATION OF PIEZOELECTRIC STRUCTURES IN AS2M DEPARTMENT

Year	Publication	Structure	Approach	Contribution
2017	[2]	Uni-morph PEH	Parametric gradient-based optimization	Explicite cost function to find optimal thickness
2018	[27]	Amplification mechanism	SIMP approach	Increasing the stroke of stack piezo actuator
2020	[20]	single-layer PEH	SIMP approach	Optimization of polarization and topology
2020	[21]	Bi-morph PEH	SIMP approach	Multidirectional PEH/avoiding charge cancelation
2020	[26]	single-layer piezo	SIMP approach	First MATLAB code published for TOM of piezo
2020	[14]	single-layer piezo pusher	SIMP approach	Increasing stroke by optimizing the polarization
2020	[22]	cantilever PEH	Neural network & genetic algorithm	In-span attachment mass
2022	[28]	single-layer piezo pusher	SIMP approach	Considering voltage uncertainty
2023	[29]	Bi-morph PEH	SIMP approach	Tuning resonance frequency/mass optimization

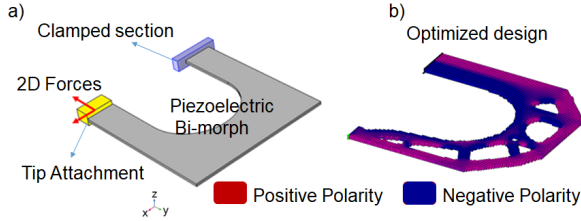


Fig. 5. a) New configuration for frequency tuned piezoelectric energy harvester. b) Topology optimized design [29].

PEH. To tackle the challenges of eigenfrequency tuning within the topology optimization approach, we defined the attachment's mass as a new optimization variable in addition to the density and polarity. Aiming for low weight piezoelectric energy harvester, a new configuration is proposed (Fig. 5-(a)) to minimize the fundamental resonance frequency and the mass of the attachment simultaneously. The obtained result (Fig. 5-(b)) in MATLAB and COMSOL Multiphysics demonstrated that the algorithm successfully restricted the fundamental frequency close to the desired one while respecting the mass and volume constraints of the vPEH.

Simulation results prove the superiority of the optimized design in Fig. 5-(b) in comparison with the previously optimized design of Fig. 4-(g) while having less amount of attachment mass. This is an interesting achievement that we restricted the first resonance frequency while at the same time having a lower amount of weight. On the other hand, the stress analysis reveals a higher amount of stress in the newly proposed configuration (Fig. 5-(a)) in comparison with the previous configuration of the PEH (Fig. 5-(g)).

V. TOWARD MULTI-MATERIAL TOPOLOGY OPTIMIZATION

In pursuit of advancing the application of topology optimization to piezoelectric structures, AS2M department embarked on a new venture. Building upon the proven success of topology optimization using single material, this new trajectory seeks to simultaneously distribute active (piezoelectric) material and passive material.

The utilization of multi-material topology optimization presents an avenue for the full exploitation of the inherent advantages of using different materials to enhance structural performance. The multi-material approach leads to an increase in the degrees of freedom in force, displacement and energy transduction particularly in the context of

piezoelectric materials [31]. It is therefore desirable and intriguing to consider a design with multiple materials especially for multi-physics and multi-functional structures.

To realize this, a formulation for the material distribution scheme has to be developed based on the power law. This formulation will enable a simultaneous creation of three distinct phases within the optimized structures: active regions, passive regions and void regions. In the work by Sigmund [32], the concept of multi-material topology optimization for the distribution of electro-thermo-mechanical materials in micro-electro-mechanical systems design is discussed. The paper revolutionized the field by proposing various schemes for optimizing material distribution. The first scheme presented distributed only two materials without introducing a void. The second scheme introduces two materials and also a void.

The second scheme formulation by [32] has gained significant utilization in the field of topology optimization problems that involve two different passive materials [33], [34]. This scheme can be used as a starting point to develop a multi-material topology optimization that includes active and passive materials. As demonstrated in [35] simultaneous distribution of piezoelectric and passive materials was achieved. In this case, the piezoelectric actuators can be placed strategically within the structure making it possible for a response in a controlled manner to external excitations and mitigate undesired structure response. The approach thus enables the structure to efficiently react to external forces while reducing reliance on passive materials.

With the growing feasibility and enhanced design flexibility provided by multi-material topology optimization, the highlighted approaches can be extended to the design of multi-material piezoelectric actuators. This extension involves several steps, including the selection of a proper material interpolation and penalization scheme, multiphysics modeling and formulation of the sensitivity analysis. By following these steps, designers can optimize the distribution of materials in piezoelectric actuators to achieve the desired performance characteristics and functional integration. In addition to the design considerations for two-dimensional multi-material structures, three-dimensional multi-material structures can be pursued based on the discussed approaches. Furthermore, non-linear characteristics of the piezoelectric material can be accounted for to accurately represent their behavior in designs leading to development of advanced multi-material piezoelectric structures.

VI. CONCLUSION

This paper reviewed and discussed the approaches developed at AS2M/FEMTO-ST institute for the topological design of piezoelectric structures. The summary of the publications and the introduced contribution is reported in Table II. We demonstrated that topology optimization methodology can be employed as a design tool to obtain miniaturized piezoelectric structures with enhanced performances.

Extending the SIMP to piezoelectric material paves the way for promising perspectives. The first perspective would concern multi-material topology optimization including active and passive material. The other perspectives would concern multi-degrees of freedom structures and large deformations.

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