Port-Hamiltonian Control Design for an IPMC Actuated Highly Flexible Endoscope

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***Abstract*— This paper deals with modelling and control of an endoscope actuated by Ionic Polymer Metal Composites (IPMC) patches. The endoscope is modelled by a nonlinear partial differential equation (PDE) capable to describe large deformations. The dynamics of the flexible structure and of the IPMC patches are in port-Hamiltonian form, with the actuators interconnected to the mechanical device in power-conserving way. Thus, the complete model is a port-Hamiltonian system in which a PDE with fixed boundary conditions is coupled with a set of ordinary differential equations. The control inputs are the voltages applied to the patches, and the feedback law is designed within the Interconnection and Damping Assignment Passivity-based Control (IDA-PBC) framework. Asymptotic stability of the closed-loop system is proved, and the effectiveness of the design procedure is illustrated by a numerical example.**

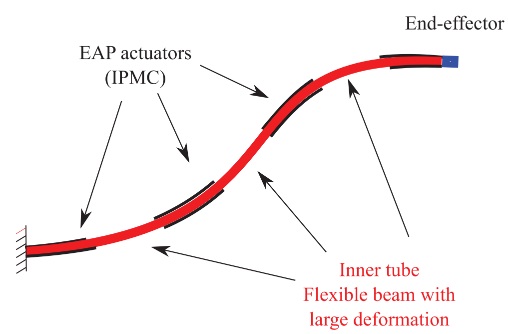

Fig. 1. IPMC actuated flexible beam with large deformation.

I. Introduction

In clinical practice, the utilisation of endoscopes for minimally invasive surgery is becoming increasingly prevalent to reduce patient discomfort. Thanks to the recent technology advance, in [1] a micro-endoscope for endonasal skull base surgery which employs Ionic Polymer Metal Composites (IPMC) actuators for endoscope bending is proposed. The most important advantage of the IPMC actuator is that it exhibits a large bending with a very low applied voltage [2]. Nonetheless, modelling and control design pose significant challenges due to the intricate nature of the material and to the behaviour of the flexible structure.

In this paper, the modelling and control problem of the IPMC actuated endoscope subject to large deformations is considered. To deal with multi-physical phenomena and the coupling between the IPMC patches and the endoscope, the study is carried out within the port-Hamiltonian framework, [3]–[5]. Key benefits of such a framework is that it allows for a systematic and modular approach not only for model development, but also for control design driven by a clear physical interpretation based on energy, see for example the energy shaping and control by interconnection and damping assignment (IDA-PBC) paradigms, [6]–[8].

The port-Hamiltonian approach for the modeling of the IPMC actuator has been investigated in [9]. In [10], instead, a lumped parameter model for an IPMC actuated flexible structure has been proposed, and the IDA-PBC paradigm has been employed to control the position of the flexible structure. A linear distributed parameter model of the same system has been proposed in [11]. However, the model can only describe the flexible beam with small deformations. The main contributions of this paper are as follows: we propose a nonlinear distributed parameter model for a flexible endoscope under the port-Hamiltonian framework. Our model takes into account the large deformations of the flexible endoscope. Moreover, we model the IPMC patches using a simple lumped parameter port-Hamiltonian system, which is then interconnected to the nonlinear flexible beam model in a power-preserving manner along the spatial domain. Following that, we extend the IDA-PBC control synthesis methodology to this port-Hamiltonian system, whose dynamics is described by a nonlinear partial differential equation (PDE) describing the flexible structure and coupled with a certain number of linear lumped-parameter sub-systems (i.e., described by ordinary differential equations – ODEs), each associated to an IPMC patch.

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II. Port-Hamiltonian model

The model of the IPMC actuated endoscope is developed within the port-Hamiltonian framework, [3], [4]. The flexible structure is described by a nonlinear, distributed-parameter port-Hamiltonian system with one-dimensional domain [5], while each IPMC patch as a lumped-parameter port-Hamiltonian system. Each patch is interconnected in power-conserving way to the mechanical structure along the spatial domain of the flexible link (see Fig. 1), and the control input is the collection of the applied voltages to all the IPMC actuators. From a mathematical point of view, the final model is formulated as a set of coupled PDEs and ODEs.

A. Nonlinear model of a flexible link

The PDE model of the flexible beam has been developed to be simple enough for control design, but at the same time to be able to describe large deformations in the 3D space, [12]. The idea is rather simple. Let $z \in [0, \ell]$ be the spatial coordinate along the unstressed configuration. Thus, the position and orientation of each cross-section with respect to the inertial frame $\mathbb{E}_0$ is denoted by $h_b^0(z) \in SE(3)$, where the subscript b refers to the body frame $\mathbb{E}_b$ moving with the cross-section. The motion of the cross-section is described by a twist $T \in se(3)$. Such a motion is caused by the elastic forces due to difference in "velocity" between infinitesimally closed cross-sections. The deformation of the structure is described by a generalised displacement $q \in se(3)$ and generates a "force", more precisely a wrench $W \in se^*(3)$. Thus, the difference between the two wrenches applied to the cross-section is responsible for its motion, described by a generalised momentum $p \in se^*(3)$. These considerations lead to the following equations, see [12] for more details:

$$\begin{cases} \dfrac{\partial q}{\partial t} = \dfrac{\partial T}{\partial z} + \mathrm{ad}_{q+n} T \\ \dfrac{\partial p}{\partial t} = \dfrac{\partial W}{\partial z} - \mathrm{ad}^*_{q+n} W + p \wedge T \end{cases}. \tag{1}$$

In (1), all the quantities are in the body frame $\mathbb{E}_b$. Besides, ad_v denotes the adjoint representation of the Lie algebra $se(3) \ni v$, while ad^*_v is its dual, [13], the term $p \wedge T = \mathrm{ad}^*_T p$ takes into account the fact that the dynamics is written in the body frame, and $n \in se(3)$ is a "twist" that describes how the unstressed configuration evolves in the z coordinate. The system is equipped by two boundary ports, i.e. $(T(\ell), W(\ell))$ and $(T(0), W(0))$, while T and W are related to p and q via the constitutive equations, see (4).

The model used for control design is (1), but re-written in coordinates and with the addition of a *port* along the domain to which the IMPC actuators are attached. For simplicity, the focus is on the *planar motion* only. Thus, given $v \in se(2)$, with some abuse in notation we write $v = (v_x, v_y, v_\theta) \in \mathbb{R}^3$, so for the adjoint representation of the algebra we have

$$\mathrm{ad}_v \Leftrightarrow \begin{pmatrix} 0 & -v_\theta & v_y \\ v_\theta & 0 & -v_x \\ 0 & 0 & 0 \end{pmatrix} \in \mathbb{R}^{3\times 3} \tag{2}$$

and

$$v\wedge \Leftrightarrow \begin{pmatrix} 0 & 0 & v_y \\ 0 & 0 & -v_x \\ -v_y & v_x & 0 \end{pmatrix} \in \mathbb{R}^{3\times 3}. \tag{3}$$

To get the dynamical equations it is necessary to specify the constitutive relations. Let m and J be the mass and moment of inertia of the cross section for unitary length, and let $M = \mathrm{diag}(m, m, J)$ be the inertia matrix. Besides, $C = C^{\mathrm{T}} > 0$ is the stiffness matrix. Then, the constitutive equations are

$$W(t,z) = Cq(t,z) \qquad T(t,z) = M^{-1} p(t,z). \tag{4}$$

The quantities q and p are usually referred as energy variables, while W and T are the co-energy variables.

If $x = (q,p) \in \mathbb{R}^6$, we extend (1) with the addition of the distributed inputs $u_d \in \mathbb{R}^N$, thus we get the nonlinear PDE

$$\begin{aligned} \frac{\partial x}{\partial t}(t,z) = P_1 \frac{\partial}{\partial z}\big(\mathcal{L}x(t,z)\big) \\ + P_0(x(t,z))\big(\mathcal{L}x(t,z)\big) + G_d(z) u_d(t) \end{aligned} \tag{5}$$

in which

$$P_1 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad P_0(x) = \begin{pmatrix} 0 & A(q) \\ -A^{\mathrm{T}}(q) & P^\wedge(p) \end{pmatrix}$$
$$G_d(z) = \begin{pmatrix} 0 \\ \bar{G}_d(z) \end{pmatrix} \qquad \mathcal{L} = \begin{pmatrix} C & 0 \\ 0 & M^{-1} \end{pmatrix}.$$

and $\bar{G}_d \in \mathbb{R}^{3\times N}$. The PDE (5) extends the class of linear, port-Hamiltonian boundary control systems introduced in [14] to the nonlinear case. With some abuse in notation, $A(q)$ and $P^\wedge(p)$ are the expressions of ad_{q+n} and $p\wedge$ reported in (2) and (3), respectively. We have assumed that the unstressed configuration is a straight line along the x direction of the body frame, and so $n = (1, 0, 0)$. Besides, the IPMC patches apply a bending torque to the mechanical structure, thus u_d collects the applied torques, and $\bar{G}_d^{\mathrm{T}}(z) = (0, 0, G_\theta^{\mathrm{T}}(z))$, with $G_\theta \in \mathbb{R}^{1\times N}$. Each (integrable) function $G_{\theta i}$ describes how the torque u_{di} generated by the i-th IPMC actuator acts on the flexible beam.

The Hamiltonian function of (5) is $H(x) = \frac{1}{2}\int_0^\ell x^{\mathrm{T}} \mathcal{L} x \, \mathrm{d}z$, and its variation along the system's trajectories is

$$\begin{aligned} \dot{H}(x(t,\cdot)) = W^{\mathrm{T}}(t,\ell)T(t,\ell) - W^{\mathrm{T}}(t,0)T(t,0) \\ + y_d^{\mathrm{T}}(t) u_d(t) \end{aligned}$$

where $y_d(t) = \frac{1}{J}\int_0^\ell G_\theta^{\mathrm{T}}(z) p_\theta(t,z)\,\mathrm{d}z$ is the power-conjugated output. The quantity $y_d^{\mathrm{T}} u_d$ gives the power flow through the spatial domain, while the other two terms specify the power flow through the boundary ports in 0 and ℓ. In the remaining part of the paper, we assume that the compliance matrix C is diagonal, i.e.:

$$C = \begin{pmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & C_\theta \end{pmatrix}, \text{ with } C_x, C_y, C_\theta > 0,$$

and that the boundary conditions are

$$\begin{aligned} T_x(t,0) &= b_x W_x(t,0) \qquad & T_\theta(t,0) &= 0 \\ T_y(t,0) &= b_y W_y(t,0) \qquad & W(t,\ell) &= 0 \end{aligned} \tag{6}$$

with b_x and b_y positive. So, the beam is (approximately) in the free-clamped configuration.

B. Model of the IPMC actuator

From a physical point of view, the bending of the IPMC due to an applied voltage is caused by the cation flux and polar solvents in the polymer membrane diffusion between the electrodes. Motivated by the requirement that the model of such a phenomenon is employed for control design, rather than the multi-scale description proposed in [15], the simpler formulation proposed in [10] is adopted. If the mechanical dynamics is assumed to be part of the flexible structure, only the electrical one is of importance and is equivalent to a RLC circuit. For the i-th IPMC patch, we write

$$\begin{cases} \begin{pmatrix} \dot{\varphi}_i \\ \dot{Q}_i \end{pmatrix} = \begin{pmatrix} -r_i & -1 \\ 1 & -g_i \end{pmatrix} \begin{pmatrix} \frac{\partial H_{ai}}{\partial \varphi_i}(\varphi_i, Q_i) \\ \frac{\partial H_{ai}}{\partial Q_i}(\varphi_i, Q_i) \end{pmatrix} + \begin{pmatrix} u_i \\ u_{ai} \end{pmatrix} \\ \begin{pmatrix} y_i \\ y_{ai} \end{pmatrix} = \begin{pmatrix} \frac{\partial H_{ai}}{\partial \varphi_i}(\varphi_i, Q_i) \\ \frac{\partial H_{ai}}{\partial Q_i}(\varphi_i, Q_i) \end{pmatrix} \end{cases} \tag{7}$$

where

$$H_{ai}(\varphi_i, Q_i) = \frac{1}{2}\frac{\varphi_i^2}{L_i} + \frac{1}{2}\frac{Q_i^2}{C_i} \tag{8}$$

is the energy (Hamiltonian). In (7), φ_i is the magnetic flux, Q_i the charge in the capacitor, r_i and g_i^{-1} the resistances, u_i the applied voltage to the actuator, y_i the current in the inductance, and y_{ai} the voltage across the capacitor. The bending torque is proportional to y_{ai}, with a constant coefficient k_i. So, if $u_a = (u_{a1}, \ldots, u_{aN})$ and $y_a = (y_{a1}, \ldots, y_{aN})$, the power-conserving interconnection between the flexible structure and the N IPMC actuators is given by

$$\begin{pmatrix} u_d(t) \\ u_a(t) \end{pmatrix} = \begin{pmatrix} 0 & K \\ -K & 0 \end{pmatrix} \begin{pmatrix} y_d(t) \\ y_a(t) \end{pmatrix}, \tag{9}$$

where $K = \operatorname{diag}(k_1, \ldots, k_N)$. Note that u_{ai} is a current caused by the structure deformation. The overall dynamics is in port-Hamiltonian form, in which a PDE is coupled with N ODEs. If $Q = (Q_1, \ldots, Q_N)$ and $\varphi = (\varphi_1, \ldots, \varphi_N)$, then $x_e = (x(z), Q, \varphi)$ denotes the state variable. The Hamiltonian function is $H_e(x_e) = H(x) + \sum_{i=1}^N H_{ai}(\varphi_i, Q_i)$, the input $u = (u_1, \ldots, u_N)$ groups the voltages applied to the IPMC actuator, and $y = (y_1, \ldots, y_N)$ is the output collecting the currents in the patches.

III. Equilibrium configurations

The working principle of the IPMC actuated flexible beam is simple. The bending is modified by the voltage applied to each patch, and the controller is designed to make the desired configuration asymptotically stable. Thus, a preliminary step consists in determining the achievable equilibria. Such equilibria are now denoted by $\cdot^\star$. From (5), it is immediate to see that $p_\theta^\star(z)$ is an equilibrium configuration for the beam if $\frac{\partial p_\theta^\star}{\partial z}(z) = 0$, i.e. $p_\theta^\star(z)$ is constant for all $z \in [0, \ell]$. Besides, for all patches, we get from (7) that

$$\begin{aligned} 0 &= -r_i \frac{\varphi_i^\star}{L_i} - \frac{Q_i^\star}{C_i} + u_i^\star \\ 0 &= \frac{\varphi_i^\star}{L_i} - g_i \frac{Q_i^\star}{C_i} - k_i \frac{p_\theta^\star}{J} \int_0^\ell G_{\theta i}(\sigma)\,\mathrm{d}\sigma \end{aligned} \tag{10}$$

Thus, if

$$\frac{\varphi_i^\star}{L_i} = g_i \frac{Q_i^\star}{C_i} \tag{11}$$

then from (10) we get

$$u_i^\star = (1 + r_i g_i)\frac{Q_i^\star}{C_i} \qquad p_\theta^\star = 0, \tag{12}$$

where $u_i^\star$ is the control action to be provided in steady-state, with $i = 1, \ldots, N$. For the PDE model (5), at the equilibrium we have that $\frac{\partial p_x}{\partial t}(t,z) = \frac{\partial p_y}{\partial t}(t,z) = 0$, and so

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{pmatrix} C_x q_x^\star(z) \\ C_y q_y^\star(z) \end{pmatrix} + \begin{pmatrix} 0 & q_\theta^\star(z) \\ -q_\theta^\star(z) & 0 \end{pmatrix} \begin{pmatrix} C_x q_x^\star(z) \\ C_y q_y^\star(z) \end{pmatrix} = 0.$$

that has to hold for some $q_\theta^\star(z)$. However, because of the free boundary condition in $z = \ell$ reported in (6), we have that $C_x q_x^\star(\ell) = C_y q_y^\star(\ell) = 0$, and so $q_x^\star(z) = q_y^\star(z) = 0$ for all $z \in [0, \ell]$. This result, together with $\frac{\partial q_x}{\partial t}(t,z) = \frac{\partial q_y}{\partial t}(t,z) = 0$ at the equilibrium, implies that

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{pmatrix} p_x^\star(z) \\ p_y^\star(z) \end{pmatrix} + \begin{pmatrix} 0 & q_\theta^\star(z) \\ -q_\theta^\star(z) & 0 \end{pmatrix} \begin{pmatrix} p_x^\star(z) \\ p_y^\star(z) \end{pmatrix} = 0$$

with $p_x^\star(0) = m b_x C_x q_x^\star(0) = 0$ and $p_y^\star(0) = m b_y C_y q_y^\star(0) = 0$ because of the boundary conditions in $z = 0$ stated in (6). As before, we get that $p_x^\star(z) = p_y^\star(z) = 0$ for all $z \in [0, \ell]$. The last equilibrium coordinate results from the requirement $\frac{\partial p_\theta^\star}{\partial t}(t,z) = 0$. We get $C_\theta \frac{\mathrm{d}q_\theta^\star}{\mathrm{d}z}(z) + \sum_{i=1}^N k_i \frac{Q_i^\star}{C_i} G_{\theta i}(z) = 0$, in which $q_\theta^\star(\ell) = 0$ because of the last condition in (6). If $\mathcal{G}_{\theta i}(z) = \int_z^\ell G_{\theta i}(\sigma)\,\mathrm{d}\sigma$, $i = 1, \ldots, N$, it is easy to see that

$$q_\theta^\star(z) = \frac{1}{C_\theta} \sum_{i=1}^N \frac{k_i Q_i^\star}{C_i} \mathcal{G}_{\theta i}(z). \tag{13}$$

So, the "desired" equilibrium is $x_e^\star = (x^\star(z), Q^\star, \varphi^\star)$, in which $x^\star(z) = \left(0, 0, \frac{1}{C_\theta}\sum_{i=1}^N \frac{k_i Q_i^\star}{C_i}\mathcal{G}_{\theta i}(z), 0, 0, 0\right)$, while $Q^\star$ and $\varphi^\star$ are defined in (11) and (12).

IV. IDA-PBC control design

The IDA-PBC design paradigm aims at obtaining a state-feedback control action able to map the open-loop dynamics into a target one characterised by a desired energy function and "structural properties," such as the internal dissipation. More details in [7], [8] or, for a problem similar to the one discussed here, in [10]. The results of this paper are an extension of the lumped-parameter case tackled there. The closed-loop Hamiltonian function is selected with a global minimum at the desired equilibrium configuration, in this case $x_e^\star$. This specifies the steady-state in closed-loop. The transient is tuned by changing the internal dissipation and the equivalent stiffness of the system which determine how fast the energy decreases towards its minimum, i.e. to $x_e^\star$. A possible choice for the desired Hamiltonian is

$$\begin{aligned} H_e'(x_e) &= \frac{1}{2}\sum_{i=1}^N \left(\frac{\tilde{\varphi}_i^2}{L_i'} + \frac{\tilde{Q}_i^2}{C_i'} + \Gamma_i \tilde{\varphi}_i \tilde{Q}_i\right) \\ &\quad + \sum_{i=1}^N \left[\int_0^\ell (\alpha_i \tilde{Q}_i + \beta_i \tilde{\varphi}_i)\tilde{q}_\theta\,\mathrm{d}z + \frac{1}{2}\left(\int_0^\ell \gamma_i \tilde{q}_\theta\,\mathrm{d}z\right)^2\right] \\ &\quad + \frac{1}{2}\int_0^\ell \left[p^{\mathrm{T}} M^{-1} p + C_x q_x^2 + C_y q_y^2 + C_\theta \tilde{q}_\theta^2\right]\mathrm{d}z, \end{aligned} \tag{14}$$

where $\tilde{\varphi}_i = \varphi_i - \varphi_i^\star$, $\tilde{Q}_i = Q_i - Q_i^\star$, and $\tilde{q}_\theta = q_\theta - q_\theta^\star$, while the C_i', L_i' and Γ_i are parameters, and the α_i, β_i and γ_i functions to be determined later. Note that $H_e'(x_e^\star) = 0$.

As far as the target dynamics is concerned, with an eye on (7) and (9) for each IPMC patch we have that

$$\begin{cases} \dot{\varphi}_i = -r_i' \dfrac{\partial H_e'}{\partial \varphi_i} - \dfrac{\partial H_e'}{\partial Q_i} \\ \dot{Q}_i = \dfrac{\partial H_e'}{\partial \varphi_i} - g_i \dfrac{\partial H_e'}{\partial Q_i} - k_i \displaystyle\int_0^\ell G_{\theta i} \frac{\delta H_e'}{\delta p_\theta}(x_e)\,\mathrm{d}z, \end{cases} \tag{15}$$

where $r_i' > 0$ is a "desired" resistance. The function $\frac{\delta H_e'}{\delta p_\theta}$ is the variational derivative of H_e' with respect to p_θ, see [5] for more details related to the port-Hamiltonian description of infinite-dimensional systems, or [16] for an application in control design. If the first relations in (7) and (15) are compared, the expression of the stabilising law is deduced:

$$u_i(x_e) = r_i \frac{\varphi_i}{L_i} + \frac{Q_i}{C_i} - r_i' \frac{\partial H_e'}{\partial \varphi_i}(x_e) - \frac{\partial H_e'}{\partial Q_i}(x_e). \tag{16}$$

As far as the flexible structure is concerned, because of the choice (14), only the last equation is changed into

$$\begin{aligned}\frac{\partial p_\theta}{\partial t} &= \frac{\partial}{\partial z}\frac{\delta H_e'}{\delta q_\theta}(x_e) - C_x q_x q_y + C_y (q_x + 1) q_y \\ &\quad + \sum_{i=1}^{N} k_i \int_0^\ell G_{\theta i} \frac{\delta H_e'}{\delta Q_i}(x_e)\,\mathrm{d}z\end{aligned} \tag{17}$$

since H_e' differs from H_e in the φ_i, Q_i and q_θ coordinates only, while the boundary conditions (6) remain the same, except for the last one that turns into $\frac{\delta H_e'}{\delta q_\theta}(x(\ell), Q, \varphi) = 0$.

From the second relations in (7) and (15), since we have that $\frac{\delta H_e'}{\delta p_\theta}(x_e) = J^{-1} p_\theta$, after few passages we obtain that

$$\begin{aligned}0 &= \left(\tilde{L}_i^{-1} - g_i \Gamma_i\right)\varphi_i - \left(g_i \tilde{C}_i^{-1} - \Gamma_i\right) Q_i \\ &\quad - \left(L_i'^{-1} - g_i\Gamma_i\right)\varphi_i^\star + \left(g_i C_i'^{-1} - \Gamma_i\right) Q_i^\star \\ &\quad + \int_0^\ell \left(\beta_i - g_i \alpha_i\right)\left(q_\theta - q_\theta^\star\right)\mathrm{d}z,\end{aligned}$$

where $\tilde{L}_i^{-1} = L_i'^{-1} - L_i^{-1}$ and $\tilde{C}_i^{-1} = C_i'^{-1} - C_i^{-1}$. Then,

$$\begin{aligned}\Gamma_i &= g_i \tilde{C}_i^{-1} & \beta_i(z) &= g_i \alpha_i(z) \\ \tilde{L}_i^{-1} &= g_i \Gamma_i = g_i^2 \tilde{C}_i^{-1}\end{aligned} \tag{18}$$

with $i = 1, \ldots, N$, since $L_i'^{-1} - g_i \Gamma_i = L_i^{-1}$ and $g_i C_i'^{-1} - \Gamma_i = g_i C_i^{-1}$, and because of (11).

In a simliar way, from the last equations in (5) and (17), after few passages we get that

$$\begin{aligned}0 = \sum_{i=1}^{N} \Bigg[& \left(k_i \tilde{C}_i^{-1} G_{\theta i} + \frac{\mathrm{d}\alpha_i}{\mathrm{d}z}\right)(Q_i + g_i \varphi_i) \\ & - g_i \left(k_i \tilde{C}_i^{-1} G_{\theta i} + \frac{\mathrm{d}\alpha_i}{\mathrm{d}z}\right)\varphi_i^\star \\ & - \left(k_i C_i'^{-1} G_{\theta i} + \frac{\mathrm{d}\alpha_i}{\mathrm{d}z}\right) Q_i^\star \\ & + k_i G_{\theta i} \int_0^\ell \alpha_i \tilde{q}_\theta\,\mathrm{d}z + \frac{\mathrm{d}\gamma_i}{\mathrm{d}z}\int_0^\ell \gamma_i \tilde{q}_\theta\,\mathrm{d}z \Bigg] - C_\theta \frac{\mathrm{d}q_\theta}{\mathrm{d}z}\end{aligned}$$

which implies that $\frac{\mathrm{d}\alpha_i}{\mathrm{d}z}(z) = -k_i \tilde{C}_i^{-1} G_{\theta i}(z)$, $i = 1, \ldots, N$. To satisfy the matching condition, because $q_\theta^\star$ is defined as in (13), an effective choice is $\alpha_i(z) = k_i \tilde{C}_i^{-1} \mathcal{G}_{\theta i}(z)$ and $\gamma_i(z) = k_i \sqrt{\tilde{C}_i^{-1}}\mathcal{G}_{\theta i}(z)$. The result is that all the elements that appear in the desired Hamiltonian function (14) have been determined. Similarly to the lumped-parameter case treated in [10], such parameters are strongly coupled and the degrees of freedom are the constants C_i'. Using a similar approach as in [10], it is possible to check that they can be selected sufficiently small so that H_e' is lower bounded and $x_e^\star$ is a global minimum.

Before concluding this part, it is interesting to analyse the structure of the feedback law with an eye on the physical parameters of the plant. In a real-world scenario, see Table I, the admittance g_i is almost 0, thus (16) simplifies into

$$\begin{aligned}u_i(x_e) &= -\frac{r_i'}{L_i}(\varphi_i - \varphi_i^\star) - \frac{1}{C_i'}(Q_i - Q_i^\star) \\ &\quad + k_i \tilde{C}_i^{-1}\int_0^\ell \mathcal{G}_{\theta i}(q_\theta - q_\theta^\star)\,\mathrm{d}z + \frac{r_i}{L_i}\varphi_i + \frac{Q_i}{C_i} \\ &= -(r_i' - r_i)\frac{\varphi_i}{L_i} + \left(\frac{1}{C_i'} + \frac{g_i r_i}{C_i}\right) Q_i^\star \\ &\quad - \frac{1}{\tilde{C}_i}\left[Q_i - k_i \int_0^\ell \mathcal{G}_{\theta i}(q_\theta - q_\theta^\star)\,\mathrm{d}z\right]\end{aligned} \tag{19}$$

since $L_i' \simeq L_i$ if $g_i \simeq 0$, see (18). For the same reason, also $g_i r_i C_i^{-1} \simeq 0$. Now, from (7), we get that

$$I_i(t) = \dot{Q}_i(t) + \frac{k_i}{J}\int_0^\ell G_{\theta i}(z) p_\theta(t,z)\,\mathrm{d}z \tag{20}$$

being $I_i = L_i^{-1}\varphi_i$ the measured current flowing in the i-th patch, and where we have assumed $g_i \simeq 0$. If we take the time derivative of the last term in (19), from (5) we obtain

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\int_0^\ell \mathcal{G}_{\theta i}(z) q_\theta(t,z)\,\mathrm{d}z &= \frac{1}{J}\int_0^\ell \mathcal{G}_{\theta i}(z)\frac{\partial p_\theta}{\partial z}(t,z)\,\mathrm{d}z \\ &= \frac{1}{J}\left[\mathcal{G}_{\theta i}(\ell) p_\theta(t,\ell) - \mathcal{G}_{\theta i}(0) p_\theta(t,0)\right] \\ &\quad - \frac{1}{J}\int_0^\ell \frac{\mathrm{d}\mathcal{G}_{\theta i}}{\mathrm{d}z}(z) p_\theta(t,z)\,\mathrm{d}z \\ &= \frac{1}{J}\int_0^\ell G_{\theta i}(z) p_\theta(t,z)\,\mathrm{d}z.\end{aligned} \tag{21}$$

In (21), we have used the fact that $\mathcal{G}_{\theta i}(\ell) = 0$ by construction, $p_\theta(t,0) = 0$ because of the clamped boundary condition in $z = 0$ specified in (6), and $G_{\theta i}(z) = -\frac{\mathrm{d}\mathcal{G}_{\theta i}}{\mathrm{d}z}(z)$. As a consequence, from (20), we can rewrite (19) as

$$u_i(I(t)) = -(r_i' - r_i) I(t) - \frac{1}{\tilde{C}_i}\int_0^t I(\tau)\,\mathrm{d}\tau + \frac{Q_i^\star}{C_i'} \tag{22}$$

so the state-feedback control action obtained thanks to the IDA-PBC design is approximated by a output-feedback law that does not require to measure the bending of the beam. Note that (22) has a PI-like structure. This result extends [10, Proposition 4] to the distributed-parameter scenario.

V. Stability analysis

The stability analysis relies on two assumptions. The first one is that both the open- and closed-loop systems are well-posed. This means that trajectories exist, thus it makes sense to evaluate e.g. how the energy function changes as long as the system evolves. As a matter of fact, it seems reasonable to extend the machinery in [17, Appendix B] to the class of boundary control systems (5). Note that for the flexible beam model discussed here, it has been shown in [18] that it can be equivalently described as a quasi-linear hyperbolic system. The second assumption is about the pre-compactness of the closed-loop trajectories. This is a fundamental requirement to apply the LaSalle's invariance principle in the distributed parameter scenario, [19, Theorem 3.64].

With few passages, and because of the control law (16) in closed-loop we still have a port-Hamiltonian system, for the target energy-function (14), we obtain that

$$\begin{aligned}\dot{H}_e'(x_e) = &- b_x W_x^2(0) - \sum_{i=1}^{N} r_i' \left(\frac{\partial H_e'}{\partial \varphi_i}(x_e)\right)^2 \\ &- b_y W_y^2(0) - \sum_{i=1}^{N} g_i \left(\frac{\partial H_e'}{\partial Q_i}(x_e)\right)^2 \leq 0.\end{aligned} \tag{23}$$

Besides, for construction, H_e' is a Lyapunov function. From (23), we deduce that a steady state is reached and, because H_e' is radially unbounded, that the trajectories of the closed-loop system are bounded, e.g. in the L^2 norm for the part of the state modelled via the nonlinear PDE (5).

As far as the steady-state is concerned, from the first relation in (15), we have that $\dot{\varphi}_i = 0$, i.e. a constant value for φ_i is reached for each patch. Besides, since

$$\begin{aligned}\frac{\partial H_e'}{\partial \varphi_i}(x_e) &= \frac{\tilde{\varphi}_i}{L_i'} + g_i \frac{\tilde{Q}_i}{\tilde{C}_i} + \frac{g_i k_i}{\tilde{C}_i} \int_0^\ell \mathcal{G}_{\theta i} \tilde{q}_\theta \, \mathrm{d}z = 0 \\ \frac{\partial H_e'}{\partial Q_i}(x_e) &= \frac{\tilde{Q}_i}{C_i'} + + g_i \frac{\tilde{\varphi}_i}{\tilde{C}_i} + \frac{k_i}{\tilde{C}_i} \int_0^\ell \mathcal{G}_{\theta i} \tilde{q}_\theta \, \mathrm{d}z = 0\end{aligned} \tag{24}$$

we see that Q_i is also constant, and so are the bending torque applied by each pach to the mechanical structure and the integral term in (24). Besides, from (15), we get that

$$\int_0^\ell G_{\theta i} p_\theta \, \mathrm{d}z = \text{const.}, \text{ with } i = 1, \ldots, N. \tag{25}$$

To study the steady-state of the beam, inspired by [20], a multiplier approach based on the auxiliary functional $W(x) = \int_0^\ell \eta \, (q_x p_x + q_y p_y) \, \mathrm{d}z$ is considered, where $\eta \in C^1(0, \ell; \mathbb{R})$ is a function defined later. It is easy to verify that, for $\varepsilon > 0$ and sufficiently small, $H_e' + \varepsilon W$ is a Lyapunov function for the closed-loop system, and that

$$\begin{aligned}\dot{W}(x) = &\frac{1}{2} \left[\eta \left(\frac{p_x^2 + p_y^2}{m} + C_x q_x^2 + C_y q_y^2 \right) \right]_{z=0}^{z=\ell} \\ &+ \int_0^\ell \left[\eta \Omega(x) - \frac{\mathrm{d}\eta}{\mathrm{d}z} \left\| (q_x, q_y, p_x, p_y) \right\|^2 \right] \mathrm{d}z\end{aligned}$$

with $\Omega(x) = C_y Q_\theta q_x q_y + m^{-1} q_\theta q_y p_x - J^{-1} p_\theta p_y$. If we select η so that $\eta(\ell) = 0$ and $\frac{\mathrm{d}\eta}{\mathrm{d}z}(z)$ is sufficiently large, since the trajectories of the closed-loop system are bounded, we get that there exist $\kappa_0, \kappa_\eta > 0$ so that

$$\begin{aligned}\dot{W}(x) \leq &\kappa_0 \eta(0) \left\| (q_x, q_y, p_x, p_y)(0) \right\|^2 \\ &- k_\eta \int_0^\ell \left\| (q_x, q_y, p_x, p_y) \right\|^2 \, \mathrm{d}z.\end{aligned} \tag{26}$$

A possible choice is $\eta(z) = \mathrm{e}^{\bar{\eta}(z-\ell)} - 1$, with $\bar{\eta} > 0$. Now, if we consider the Lyapunov function $H_e' + \varepsilon W$, for ε sufficiently small the (possibly) positive contribution in (26) is dominated by the boundary dissipation due to the boundary conditions (6). In fact, with an eye on (23), we have that

$$\begin{aligned}b_x W_x^2(0) + b_y W_y^2(0) = &\tfrac{1}{2} \left[b_x W_x^2(0) + b_y W_y^2(0) \right] \\ &+ \tfrac{1}{2} \left[b_x^{-1} T_x^2(0) + b_y^{-1} T_y^2(0) \right],\end{aligned}$$

thus it is sufficient that ε is selected so that $2\kappa_0 \eta(0) \, \varepsilon \leq \min(b_x C_x^2, b_y C_y^2, b_x^{-1} m^2, b_y^{-1} m^2)$. So, in steady state, q_x, q_y, p_x and p_y go to 0 for all $z \in [0, \ell]$.

Finally, the focus is on the deflection dynamics. From the LaSalle's invariance principle [19, Theorem 3.64], the trajectories converge to the largest invariant set contained in $\dot{H}_e'(x_e) = 0$. From (5) and the previous considerations, we obtain that in steady state

$$\begin{cases} \dfrac{\partial q_\theta}{\partial t}(t, z) = \dfrac{1}{J} \dfrac{\partial p_\theta}{\partial z}(t, z) \\ \dfrac{\partial p_\theta}{\partial t}(t, z) = \dfrac{\partial}{\partial z} \dfrac{\delta H_e'}{\delta q_\theta}(q_\theta(z), p_\theta(z)) + \bar{\tau} \end{cases}, \tag{27}$$

with the boundary conditions $p_\theta(0) = 0$ and $\frac{\delta H_e'}{\delta q_\theta}(\ell) = 0$, and where $\bar{\tau}$ is the constant bending torque applied by the patches on the mechanical structure. With few passages, it turns out that (27) is a wave equation with constant boundary conditions. As a matter of fact, the only invariant solution compatible with the property (25) of the steady-state is that $p_\theta(z)$ is constant for all $z \in [0, \ell]$, i.e. it is equal to zero because of the clamped configuration in $z = 0$. Thus, from the second relation in (27), we can check that $q_\theta = q_\theta^\star$, and so from (24) we finally get that $\varphi_i = \varphi_i^\star$ and $Q_i = Q_i^\star$, with $i = 1, \ldots, N$. The conclusion is that the unique invariant solution contained in $\dot{H}_e'(x_e) = 0$ is the equilibrium $x_e^\star$, thus proving that the feedback law (16) makes such a configuration asymptotically stable.

TABLE I
PARAMETERS OF THE IPMC ACTUATED ENDOSCOPE, [10].

parameter	value	parameter	value
ℓ	0.16 m	m	$4.5 \cdot 10^{-5}$ kg·m^{-1}
J	$0.19 \cdot 10^{-5}$ kg·m	C_x	10 N·m^{-2}
C_y	0.80 N·m^{-2}	C_θ	0.26 Nm·m^{-1}·rad^{-1}
C_i	$5.8 \cdot 10^{-2}$ F	L_i	0.01 H
r_i	29.75 Ω	g_i	$1.4 \cdot 10^{-3}$ Ω^{-1}
k_i	0.98 Nm·V^{-1}		

VI. NUMERICAL SIMULATIONS

In this section, the shape control for an IPMC actuated flexible beam with the proposed control methodology is considered. The numerical values of the parameters for the simulation are listed in Table I. The IPMC patches are assumed to be equal. The aim is to use the IPMC actuators to drive the flexible beam to the desired shape. The flexible beam has $N = 4$ IPMC actuators attached to it, located at the clamped side, 1/4 and 3/4 of the length away from the clamped side, and at the tip end side. The length of each actuator is 2 cm. So, the $G_{\theta i}(z)$ functions appearing in (5) that describe the way in which the actuators bends the mechanical structure have been selected to be equal to 1 in a

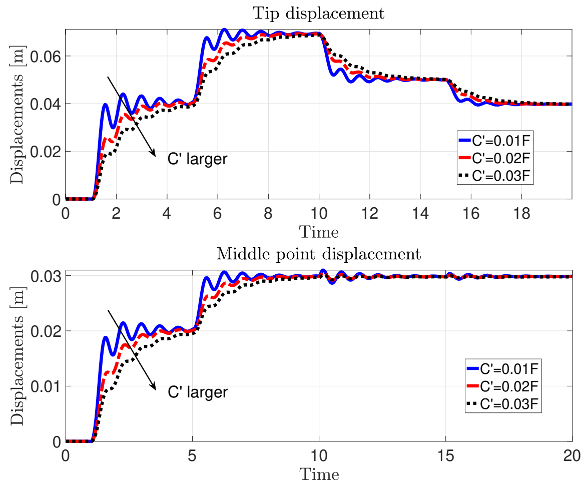

Fig. 2. Beam tip and middle point displacements under actuation with $r'_i = 10\Omega$ and different C'_i.

subset of the domain $[0, \ell]$ of length 2 cm and centered in the application point, and 0 elsewhere. The desired displacement of the end tip point is 4 cm, and for the middle point is 3 cm. To achieve the desired configuration, we activate the four IPMC actuators one after another, starting from the clamped side. The first actuator is activated at 2 s, the second one at 5 s, the third one at 10 s, and the last one at 15 s.

In Fig. 2, we show the beam tip (upper figure) and the middle point (lower figure) displacements. As far as the controller parameters are concerned, we have fixed $r'_i = 10\Omega$ fixed, while different values for C'_i have been tested. Thus, the IPMC actuators share the same control law. The blue solid line shows the displacements when $C'_i = 0.01\,\mathrm{F}$. When such a parameter becomes larger, the response turns slower (red line $C'_i = 0.02\,\mathrm{F}$, and black dashed line $C'_i = 0.03\,\mathrm{F}$).

VII. Conclusions and Future Activities

In this paper, a model and a control law for a deformable endoscope actuated by a set of IPMC patches by using the port-Hamiltonian framework have been presented. The flexible structure is modelled by a nonlinear PDE with fixed boundary conditions that is able to describe large deformations in space. The IPMC patches are modelled by a lumped-parameter port-Hamiltonian system, and are interconnected to the spatial domain of the PDE model. The resulting system is still port-Hamiltonian, with the dynamics characterised by coupled PDEs and ODEs. The control design is based on the IDA-PBC paradigm, and exploits the electro-mechanical coupling to get the closed-loop Hamiltonian function.

There are several points that require further investigation. Among them, a more realistic model of the IPMC patch could be employed. Such a model should take into account the polymer gel diffusion, thus making the control design even more challenging. From an experimental point of view, the validity of the proposed modelling and control design framework has to be validated. For this reason, we are realising a laboratory setup.

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