Physics-Informed Neural Networks for Inverse Problems in Structural Dynamics

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ABSTRACT

This study introduces an innovative approach that employs Physics-Informed Neural Networks (PINNs) to address inverse problems in structural analysis. Specifically, we apply this technique to the 4-th order PDE of Euler-Bernoulli formulation to approximate beam displacement and identify structural parameters, including damping and elastic modulus. Our methodology incorporates partial differential equations (PDEs) into the neural network's loss function during training, ensuring it adheres to physics-based constraints. This approach simplifies complex structural analysis, even when specific boundary conditions are unavailable. Importantly, our model reliably captures structural behavior without resorting to synthetic noise in data. This study represents a pioneering effort in utilizing PINNs for inverse problems in structural analysis, offering potential inspiration for other fields. The reliable characterization of damping, a typically challenging task, underscores the versatility of methodology. The strategy was initially assessed through numerical simulations utilizing data from a finite element solver and subsequently applied to experimental datasets. The presented methodology successfully identifies structural parameters using experimental data and validates its accuracy against reference data. This work opens new possibilities in engineering problem-solving, positioning Physics-Informed Neural Networks as valuable tools in addressing practical challenges in structural analysis.

Keywords: PINN, Machine learning, Identification

1. INTRODUCTION

The application of PINNs in the mechanics community has expanded recently. Among others, Roy and Guha[1] introduced a PINN framework for solving plasticity problems under plane-strain assumption. The physics were introduced into the neural network training process using a loss function that includes constitutive relations and boundary conditions. As'ad and Farhat[2] put forward a computational strategy for training a mechanics-informed neural network, with the objective of modeling nonlinear viscoelastic problems. Rezaei et al.[3] compared the potential of PINNs as solvers in solid mechanics with the FE method. An interesting approach was proposed by Lai et al.,[4] who used neural ordinary differential equations. These works represent a bird's eye view of the most recent studies focusing on the application of PINNs to solve forward problems in mechanical systems — where inputs, parameters, and physical models are known, and the objective is to employ PINN algorithms to determine the corresponding outputs. Here, PINNs are used for solving inverse problems in mechanical systems.

Haghighat et al.[5] explored the application of PINNs for modeling nonlinear elastoplasticity and simultaneously conducting parameter identification. Yucesan et al.[6] introduced a PINN framework to refine a torsional vibration damper model based on experimental data. In contrast to the works addressed so far that explored PDEs, they employed the neural network within a direct graph representation to update the stiffness and damping coefficients. Guo and Fang[7] proposed a parameter estimation framework incorporating training data derived from a validated finite element model, where modal parameters served as physical constraints for the PINNs.

This paper presents a physics-informed approach for inverse problems, to effectively identify beam-like structures using a PINN framework. At the moment, there are few studies on this topic in the literature. Kapoor et al.[8] applied PINNs to solve PDEs of the Euler-Bernoulli and Timoshenko models. The formulation adopted

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by the authors involves expressing the PDEs of the beam models as time-dependent, thereby significantly augmenting the volume of required training data as compared to the work proposed here. Meanwhile, Wu et al.[9] employed PINNs to address inverse problems involving linear elastic and hyperelastic materials in beam-like structures, examining the influence of hard and soft constraints in the solution accuracy. None of the studies within the beam-type structures context validated the methodologies with experimental data, nor did they consider the presence of damping in their models. Additionally, in the context of system identification, this 4-th order derivative appearing in the Euler-Bernoulli equation is known to be responsible for the high sensitivity of the procedure to noise.[10,11]

In this work, we present a PINN framework to identify the transverse displacement and complex Young modulus of a beam structure through a loss function that incorporates the PDE of the Euler-Bernoulli model and the observed data from the system under interest. Our main contribution lies in using the PINN framework to identify the complex Young's modulus, allowing the determination of system damping.

This paper is organized as follows: Section 2 introduces the PINN framework, Section 3 presents the Euler-Bernoulli differential equation to deal with, Section 4 is dedicated to the validation of the method on a numerical test-case, Section 5 shows the results of an experimental case study, and Section 6 provides final remarks.

2. ON THE PHYSICS INFORMED NEURAL NETWORKS

In PINN, the standard approach is to consider a feed-forward neural network parameterized by $\boldsymbol{\theta}$ taking as input a d-dimensional feature vector $\boldsymbol{x}_n$ from a training dataset $\mathcal{X} = \{\boldsymbol{x}_n, y_n\}_{n=1}^{N}$ where y is the expected (or observed) output. Finding $\boldsymbol{\theta}$ can be posed as a problem of minimizing an objective (or loss) function that measures the discrepancy between the neural network's output $\hat{y}$ and the target y, such as the mean squared error (MSE):

$$\mathcal{R}_d(\hat{y}(\cdot;\boldsymbol{\theta})) = \frac{1}{N}\sum_{n=1}^{N} \left|\hat{y}(\boldsymbol{x}_n;\boldsymbol{\theta}) - y(\boldsymbol{x}_n)\right|^2. \tag{1}$$

In a physics-informed context, the loss function also incorporates residual terms associated with the system under analysis. Consider the following general PDE of a mechanical system in steady state regime of motion:

$$\mathcal{D}^{\alpha}(y(\boldsymbol{x});\lambda) = f(\boldsymbol{x}), \quad \boldsymbol{x} \in \boldsymbol{\Omega}, \tag{2}$$

with $\mathcal{D}(\bullet)$ the differential operator of order α, λ the model parameters involved in the PDE, and $f(\boldsymbol{x})$ the external excitation.

From the perspective of forward analysis, a loss function related to the PDE residual, commonly known as the physics-informed loss is typically defined as

$$\mathcal{R}_f(\hat{y}(\cdot;\boldsymbol{\theta})) = \frac{1}{M}\sum_{m=1}^{M} |\mathcal{D}^{\alpha}(\hat{y}(\boldsymbol{x}_m;\boldsymbol{\theta});\lambda) - f(\boldsymbol{x}_m)|^2, \tag{3}$$

where M corresponds to the number of the discretization points linked to the PDE residual.

From a perspective of inverse problems with the PINN approach, the objective is to identify the solution $y(x)$ and the model parameters λ that minimize the loss function. Therefore, an additional neural network is proposed to approximate the parameters of interest $\lambda(\boldsymbol{x}) \approx \hat{\lambda}(\boldsymbol{x};\boldsymbol{\xi})$, where $\boldsymbol{\xi}$ represents the parameters of this second neural network. Hence, the physics-informed loss function used in this work is:

$$\mathcal{R}_f(\hat{y}(\cdot;\boldsymbol{\theta}), \hat{\lambda}(\cdot;\boldsymbol{\xi})) = \frac{1}{M}\sum_{m=1}^{M} |\mathcal{D}^{\alpha}(\hat{y}(\boldsymbol{x}_m;\boldsymbol{\theta});\hat{\lambda}(\boldsymbol{x}_m;\boldsymbol{\xi})) - f(\boldsymbol{x}_m)|^2. \tag{4}$$

3. THE EULER-BERNOULLI MODEL

Consider a cantilever beam. The equation of motion of the Euler-Bernoulli model considering a harmonic motion at an excitation frequency ω is:[12]

$$\frac{\partial^2}{\partial x^2}\left(E(x)I\frac{\partial^2 w(x,\omega)}{\partial x^2}\right) - \rho S \omega^2 w(x,\omega) = \hat{f}(x,\omega), \tag{5}$$

where $w(x)$ is the transversal deflection of the beam; I and S are the flexural inertia and the area of the cross section $h \times b$ of the beam, respectively; E and ρ are the material elastic modulus and density, respectively; $\hat{f}(x)$ is the loading.

Assuming that the Young's modulus is non-uniform and differentiable along with the length x of the beam, and the material is linear viscoelastic with loss factor $\eta = \dfrac{E_i}{E_r}$, equation (5) can be developed (subscripts i and r refer to the imaginary and real parts of the quantities of interest, respectively).

In a scenario aiming to determine both the deflection of the beam and a non-constant Young's modulus, the problem-solving framework includes a total of four independent neural networks, i.e. $\hat{w}_r \approx w_r$, $\hat{w}_i \approx w_i$, $\hat{E}_r \approx E_r$ and $\hat{E}_i \approx E_i$ with parameters $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\xi}_1$ and $\boldsymbol{\xi}_2$. Therefore, the equation can be decomposed into two cases that form the physics-informed loss function, one considering only the real terms $\mathcal{R}_f^{(\text{real})}$ and the other involving the imaginary terms $\mathcal{R}_f^{(\text{imag})}$. The residual terms related to data observations can also be defined according to their real and imaginary components:

$$\mathcal{R}_d(\hat{w}_r(\cdot,\boldsymbol{\theta}_1), \hat{w}_i(\cdot;\boldsymbol{\theta}_2)) = \frac{1}{N_x}\sum_{n=1}^{N_x}|\hat{w}_{r,n}(\boldsymbol{\theta}_1) - w_{r,n}|^2 + \frac{1}{N_x}\sum_{n=1}^{N_x}|\hat{w}_{i,n}(\boldsymbol{\theta}_2) - w_{i,n}|^2, \tag{6}$$

where $\hat{w}_{\cdot,n}$ and $w_{\cdot,n}$ are the displacements evaluated at $\boldsymbol{x}_n$, N_x is the number of displacements measured spatially along the length of the beam. The loss function is finally given by:

$$\mathcal{L} = \gamma\left(\mathcal{R}_f^{(\text{real})} + \mathcal{R}_f^{(\text{imag})}\right) + \beta\mathcal{R}_d, \tag{7}$$

where (γ, β) are constants serving as weights for the terms in the loss function.

4. NUMERICAL APPLICATION

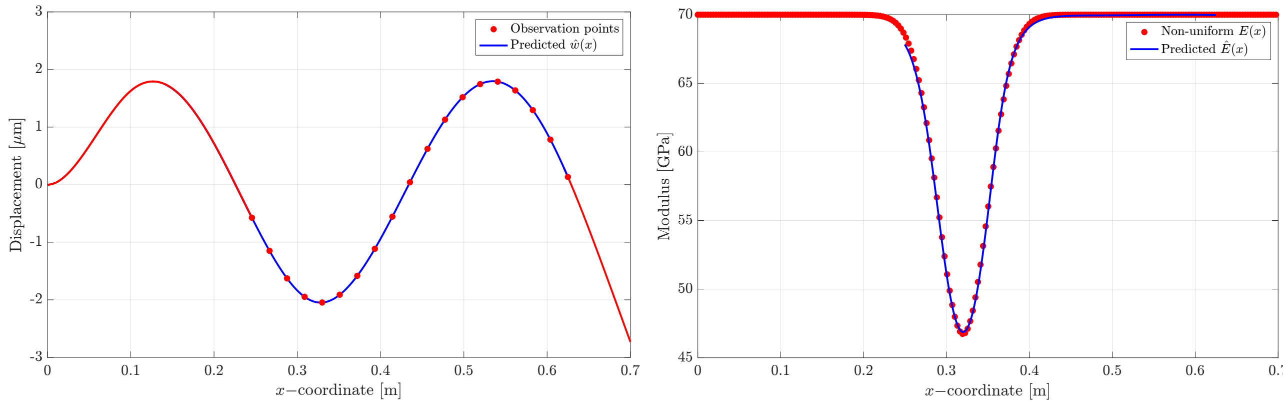
The aim of this section is to present a numerical application of a non-uniform Young's modulus identification using the PINN model. A finite element model of a clamped-free aluminum beam was constructed to generate data. A harmonic point force excites the structure. A non-uniform distribution of the elastic modulus is defined by:

$$E(x) = \left(1 - \frac{c}{u\sqrt{2\pi}} \cdot \exp\left(-\frac{1}{2}\left(\frac{x-m}{u}\right)^2\right)\right) \times E, \tag{8}$$

where the parameters c, u and m were set to 0.025, 0.03 and 0.32, respectively. For this application, the nominal E was set to 70 GPa.

Two distinct neural networks are proposed: one dedicated to capturing the displacement and the other to modeling the elastic modulus. Both networks have a similar architecture, comprising 9 hidden layers, each with 10 neurons using the hyperbolic tangent function. The Xavier Glorot method[13] initializes the neural networks, and the optimization process employs the Adam optimizer. To construct the PDE, 1024 collocation points are uniformly generated along the beam's area of interest, whereas 19 observation points are used for the data-driven loss. The chosen weights for the loss are $\gamma = 1$ and $\beta = 10^{-2}$.

In Figure 1, (a) is the predicted displacement and (b) the predicted non-uniform Young's modulus. It is noteworthy that both outputs of the physics-informed neural networks, $\hat{w}(x)$ and $\hat{E}(x)$, demonstrated precision in predicting the numerical experiment, with a normalized mean square error (NMSE) of 0.03% for the displacement and 0.29% for the non-uniform elastic modulus.



(a) Displacement, with NMSE=0.03 % (b) Non-uniform Young's Modulus, with NMSE = 0.29 %

Figure 1: Application of the PINN strategy on beam with nonuniform elastic modulus

5. EXPERIMENTAL CASE STUDY

To demonstrate the applicability of the methodology proposed in this work to solve inverse problems in beam-like structures, an experimental setup is used. The beam is made of steel, with dimensions of 1000 x 20 x 2 [mm], and experimentally determined density of 7870 kg/m^{-3}. Vibration tests were conducted under clamped-free boundary conditions, using a shaker for excitation and vibrometry measurements. A calibration procedure based on the the measured free-free modes of the beam, provided a reference Young's modulus of 205 GPa.

Two neural networks were used, one to capture the real component of the displacement and the other for the imaginary component in order to identify the elastic modulus and the loss factor. Both neural networks have the same architecture, consisting of 10 hidden layers with 9 neurons per layer. The parameters E_r and E_i, related to the elastic and loss moduli of the beam, respectively, were considered as constants and optimized together with the weights and biases of the neural networks.

Figure 2 presents the predictions of $\hat{w}_r(x)$ and $\hat{w}_i(x)$ made by the neural networks in direct comparison with the experimental data. Over the test data, the highest NMSE found was 1.29% on the imaginary component of the displacement. The evolution of $\hat{E}_r$ and $\hat{\eta}$ over the epochs converges toward values of 206 GPa for the Young's modulus (against 205 GPa identified previously) and 0.48% damping. Note that both parameters were initialized far from the values obtained at the end of training, corresponding to an initialization of 110 GPa for E_r and 1% for damping. Note also that the structural damping that may be identified from modal analysis of the clamped-free beam would be the result of a combination between the losses occurring in the clamp and the material losses in the beam. The local character of the PINN approach provides only the material-related dissipation, i.e. the loss factor of the material. A modal analysis of the free-free beam provided a modal damping ratio of the closest eigenfrequency (at 589 Hz) equal to $\zeta = 2.4044 \times 10^{-3}$ corresponding to a loss factor $\eta = 2\zeta = 4.8088 \times 10^{-3}$, which is very close to the value identified with the PINN.

6. CONCLUDING REMARKS

The framework presented in this paper provides a new way to identify structural parameters for beam-like structures. Its versatility allows the identification of non-homogeneous parameters, including damping properties thanks to the management of real and imaginary parts of frequency-based experimental data. The results obtained must now be completed with even more complex cases to illustrate its efficiency, such as multilayers composite structures (beams, plates), frequency-dependent properties of materials embedded in structures, highly damped materials.

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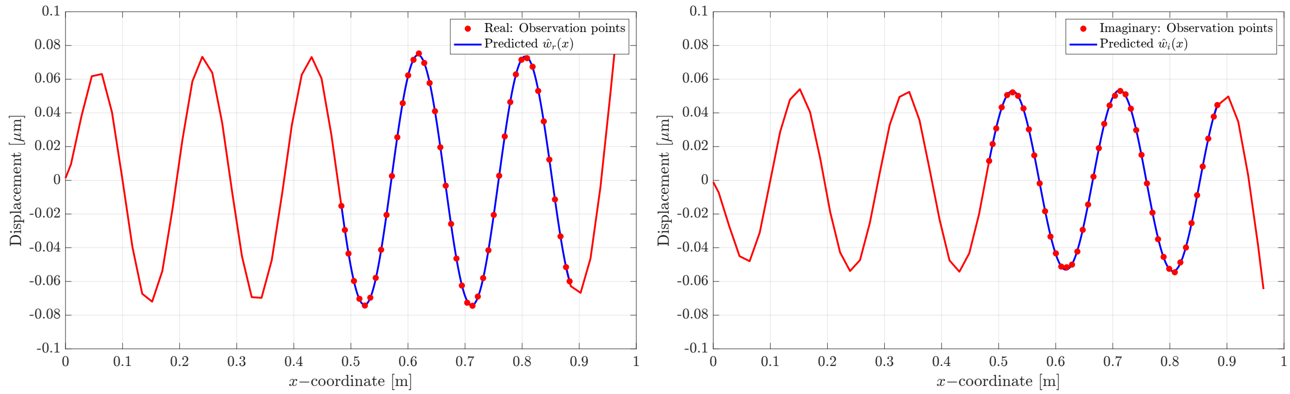

(a) Normalized mean square error: 0.69 %

(b) Normalized mean square error: 1.29 %

Figure 2: Real and imaginary parts of displacement measured (red lines), used for the training of the PINN (red dots), and predicted by the PINN (blue lines)

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