Numerical simulation of Stirling cycle heat pumps by simple adiabatic method

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Abstract - The current study adapted a numerical model called the simple model, which considers regenerator imperfection losses and heat conduction losses applied to the geometry of the FEMTO-60 engine. The simulation result shows the pressure drop in the regenerator, cooler, and heater at different crank angles and a performance comparison of the simple model with the ideal adiabatic and Schmidt analysis of the Stirling heat pump. The simulation results using Nitrogen as working fluid at a pressure of 2.0 MPa indicate a COP of 3.7, 6.2, and 16.9 for simple, ideal adiabatic, and Schmidt analyses, respectively.

Nomenclature

P pressure, Pa
M mass, kg
W work, J
Q heat, J
V volume, m^3
T temperature, K
Greek symbols
α phase advance angle, degree
β calculated angle, degree
Index and exponent
c compression
e expansion
k cooler
h heater
r regenerator
b overlap
in *input*
ch compression/heater
hr heater/regenerator
rk regenerator/cooler
ke cooler/expansion
cle clearance
mean mean
sw swept
cle clearance
wh heater wall
wk cooler wall
hp heat pump
cc cubic centimeter

1. Introduction

Currently, the energy consumption of the building has significantly risen, and the cause of greenhouse gas emissions is due to the power source of fuel for heating and cooling applications. The residential buildings contribute 25.4% of total energy use and 20% of greenhouse gas emissions in the European Union [1, 2]. Moreover, most of the building's air conditioning systems have used vapor compression heat pumps. Even though the vapor compression type of heat pump is an existing and efficient technology for heating and cooling application it has drawn back due to the working fluid. Therefore, the Stirling cycle heat pump is an alternative heating and cooling application because of its natural working fluid and in certain situations, these devices have the potential to replace the vapor compression cycle due to its theoretical maximum efficiency [3].

A Stirling heat pump operates on the Stirling cycle, which consists of four thermodynamic processes: isothermal compression, constant volume heat rejection, isothermal expansion, and constant volume heat addition. A reliable numerical model shall be developed to estimate the power input, coefficient of performance, and other characteristics of a Stirling cycle heat pump and give valuable information for future research.

The numerical modeling of Stirling cycle devices starts from the simplest Schmidt analysis up to fourth-order computational fluid dynamics analysis techniques. Schmidt developed the first analytical numerical model for the Stirling cycle device to determine pressure distribution and forecast performance by assuming isothermal compression and expansion space [4]. Martin proposed the modified Schmidt model to show the effect of pressure drop and heat loss in performance prediction [5]. Toda et al. [6] developed a modified Schmidt model for different driving mechanisms and applied his model to gamma type Stirling engine.

Finkelstein developed the first adiabatic model; it was the most significant theoretical model in the century and he introduced the concept of conditional temperature [4]. Urieli and Berchowitz developed the ideal adiabatic and simple adiabatic model and a computer code was developed for the performance prediction of Stirling engines. Generally, researchers develop different numerical modeling techniques to predict the performance of Stirling cycle devices. Therefore, the main objective of this research paper is to compare the Schmidt, ideal adiabatic and simple adiabatic model typically for the FEMTO-60 engine geometry configuration and give valuable information for future studies.

2. Schmidt analysis

Schmidt numerical model is the simplest closed form analysis of Stirling cycle devices based on the isothermal compression and expansion space. The following assumptions are considered for the analysis of this model:

- The expansion space and the cooler are at isothermal process,
- The compression space and the heater are at isothermal process,
- The potential and kinetic energy of the gas is negligible,
- There is a steady state process and the properties of the working gas are constant,
- The regenerator is perfect and there is no pressure drop in the heat exchangers,
- The temperature variation in the regenerator is linear,
- Heat is transferred to the working fluid only in the hot and cold heat exchanger,
- Heat transfer to the environment is negligible,
- The volume variation in the compression and expansion space is sinusoidal.

2.1. Development of theoretical model equations

Gustave Schmidt developed a set of equations for a special case of sinusoidal volume variation of working space for the alpha type of Stirling cycle devices. The heat pump is configured as five components namely compression space, heater, regenerator, cooler, and expansion spaces respectively as Urieli and Berchowitz have done for the adiabatic model [4], see Fig 1. The power piston and the displacement piston of a Stirling engine of the Beta type are enclosed in the same cylinder. An effective working space volume is produced when the strokes of both pistons overlap and this volume is called overlap volume.

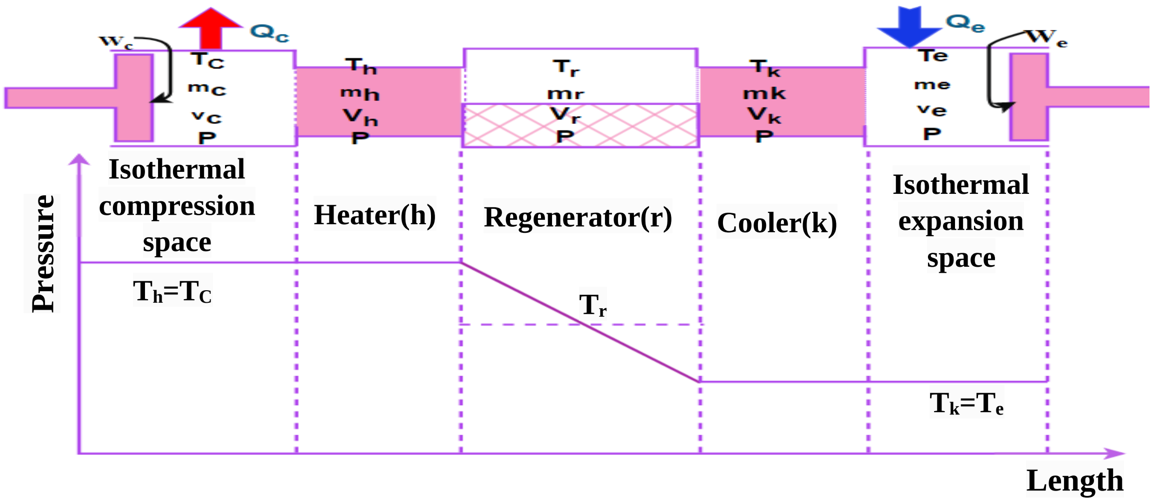

Figure 1: Isothermal temperature distribution

Parameter	Equation
Pressure and Mass	$P_{\max}=\frac{mR}{s(1-b)}, P_{\min}=\frac{mR}{s(1+b)}, P_{mean}=\frac{mR}{s(1+b^2)} \quad m=\frac{P_{mean}s\sqrt{1-b^2}}{R}$ $s=\frac{V_{swc}}{2T_h}+\frac{V_{swe}}{2T_h}+\frac{V_h}{T_h}+\frac{V_{clc}}{T_h}-\frac{V_b}{T_h}+\frac{V_r}{T_r}+\frac{V_{swe}}{2T_k}+\frac{V_{cle}}{T_k}+\frac{V_k}{T_k}$ $c=\sqrt{\left(\frac{V_{swe}-V_{swc}\cos\alpha}{2T_h}-\frac{V_{swe}}{2T_h}+\frac{V_{swe}}{2T_k}\right)^2+\left(\frac{V_{swc}\sin\alpha}{2T_h}\right)^2-\frac{V_{swe}}{T_h}\times\frac{(V_{swe}-V_{swc}\cos\alpha)}{T_k}}$ $b=\frac{c}{s}$ and $\beta=\arctan\left(\frac{\frac{V_{swe}\sin\alpha}{2T_h}}{\frac{V_{swc}\cos\alpha}{2T_h}-\frac{V_{swe}}{2T_h}+\frac{V_{swe}}{2T_k}}\right)$
Expansion volume	$V_{cle}+0.5V_{swe}(1+\cos\theta)$
Overlap volume	$\left[\frac{V_{swc}+V_{swe}}{2}\right]-\sqrt{\frac{(V_{swc})^2+(V_{swe})^2}{4}-\frac{V_{swc}V_{swe}cos\alpha}{2}}$
Compression volume	$V_{cle}+0.5V_{swc}(1+cos(\theta-\alpha))+0.5V_{swe}(1-cos\theta)-V_b$
Energy	$W_c=Q_C=\Pi P_{mean}\frac{\sqrt{1-b^2}-1}{b}\left(\mathrm{V}_{swe}(\sin\beta)-V_{swc}\sin(\beta+\alpha)\right)$ $W_e=Q_e=\Pi \mathrm{V}_{swe}P_{mean}\sin\beta\frac{\sqrt{1-b^2}-1}{b}$ $W_{in}=(W_c+W_e) \quad COP_{hp}=\frac{Q_c}{W}=\frac{T_h}{T_h-T_k}$

Table 1: *summarized equation for Schmidt Analysis [7]*

3. Adiabatic Analysis

The expansion and compression temperatures in the Schmidt numerical model were kept constant with cooler and heater, respectively. This creates a paradoxical situation in which there was no heat transfer over the cycle from either the heater or the cooler means that all the heat transfer occurs in the isothermal working spaces. Therefore, in real Stirling devices, the working spaces lead to an adiabatic process rather than the isothermal process for effective heat transfer. Finkelstein developed the first adiabatic model; it was the most significant theoretical model in the century and he introduced the concept of conditional temperature [4].

There are different classifications of adiabatic numerical modeling techniques and the ideal adiabatic and simple adiabatic analysis are the two simplest models of Stirling devices. In the ideal adiabatic model, there is no loss consideration for developing its governing equation, which is called no loss analysis. Simple adiabatic analysis is the modification of the ideal adiabatic model, which considers non-ideal heat exchangers. In addition, it also considers the work loss due to pressure drop in the heat exchangers.

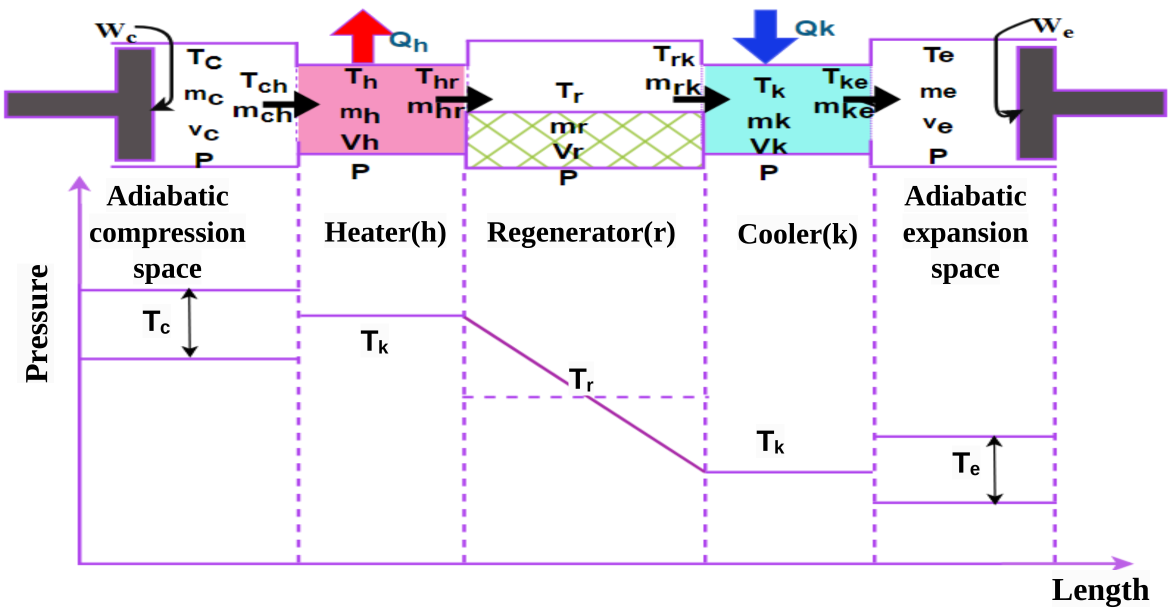

Figure 2: Adiabatic temperature distribution

The heat pump is configured by five components serially connected and the working gas in the heater/hot heat exchanger, and cooler/cold heat exchanger is kept under isothermal conditions see Fig 2. Enthalpy flows across the cells through four interfaces as a result of mass flow being compression space/heater, heater/regenerator, regenerator/cooler and cooler/expansion space.

The ideal adiabatic set of equation has 22 number of variables and 16 derivatives [7] and it is solved from $\theta = 0$ to 360°. The seven derivative equations (T_c, T_e, Q_h, Q_r, Q_k, W_c and W_e) are solved by 4th order Runge Kutta algorithm. Nine analytical variables and derivatives (W, P, Ve, V_c, m_c, m_h, m_r, m_k, and m_e) equations are solved analytically. Six conditional temperatures and mass variables (T_{ch}, T_{ke}, m_{ch}, m_{hr}, m_{rk}, and m_{ke}) take the adjacent higher value and are calculated analytically respectively.

For simple adiabatic analysis, we consider non-ideal regenerator, pressure drop in the heat exchangers and heat transfer in heater and cooler.

3.1. Non ideal regenerator: The imperfection of the regenerator is defined based on number of transfer units (NTU) and expressed as [4]:

$$\varepsilon = \frac{NTU}{NTU+1}, NTU = \frac{\vec{S}tA_{wgr}}{2A}, \vec{S}t = \frac{0.46 \times Re^{0.4}}{\Pr}, Re = \frac{|g|d}{\mu}$$

$$Q_{non-ideal,r} = q_{rloss} = \left(DQ_{rideal,\max} - DQ_{rideal,\min}\right)(1-\varepsilon)$$

Where $\varepsilon, k, h, \bar{S}t, A, Re, Pr, g, d, \mu$ are effectiveness, thermal conductivity (w/m.k), convective heat transfer coefficient (w/m^2 k), area (m^2), Stanton number, Reynolds number, Prandlt number, mass flux (kg/m^2.s), hydraulic diameter (m) and viscosity (kg/m.s) respectively

3.2. Heat transfer by conduction in the heater and cooler: The conduction loss modeled by the equation below and compute the actual heat load of the heater and cooler by adding thermal loss to the energy balance equation [8]:

$$Q_{cond} = \frac{kA\left(T_{wh} - T_{wk}\right)}{f \times L}, Q_{h,actual} = DQ_{h,ideal} + q_{rloss} + Q_{cond}, Q_{k,actual} = DQ_{k,ideal} - q_{rloss} - Q_{cond}$$

Where $DQ_{h,ideal}$ is the heat rejected by the heater in ideal adiabatic analysis and from newton, law of heating and cooling the following expression could be obtained and used to update the temperature the heater and cooler at the end of the cycle.

$$Q_{h,actual} = \frac{h_h A_h \left(T_{wh} - T_h\right)}{f}, T_h = T_{wh} - \frac{f \times Q_{h,actual}}{h_h A}\, and, Q_{k,actual} = \frac{h_k A_k \left(T_{wk} - T_k\right)}{f}, T_k = T_{wk} - \frac{f \times Q_{k,actual}}{h_k A_k}$$

The cooler and heater heat transfer coefficient could be obtained from the correlation as:

$$h_i = \frac{f_r \mu_i C_p}{2 d_i P_{ri}} = \frac{0.0791 \mu_i C_p R_{ei}^{0.75}}{2 d_i P_{ri}}, where, i = h, k, and,\ f_r = 54 + 1.43 R_e^{0.78}$$

1.3. Pressure drops in the heat exchangers: The fluid has a contact with wall the heat exchangers and this contact leads to fluid friction loss or a pressure drop loss, which affect the performance of the heat pump and modeled by [7]:

$$DW_{pdi} = \int_0^{360} \left(\Delta p_i \frac{dV}{d\theta}\right) d\theta_i, where, i = h, r, k, and, \Delta p_i = \frac{2 f_r \mu_i G_i V_i l}{m_i d_i^2}, G_i = \frac{\omega \times \left(G_{i,in} + G_{i,out}\right)}{2}$$

The actual performance of the heat pump is calculated by:

$$COP_{hp,actual} = \frac{Q_{h,acual}}{W_{actual}}, where, W_{actual} = DW_{in.} + DW_{pdi}$$

Where, $T, l, f_r, C_p, \omega, G, V$ *and* θ are temperature, length (m), friction factor, specific heat capacity at constant pressure (J/kg.k), angular frequency (rad/s), mass flow (kg/rad), volume and crank angle respectively.

4. Result and discussion

In this section, the simulation result shows the work input, energy, pressure, volume, pressure drop in the three heat exchangers and the wall and gas temperature of the working fluid at different crank angles. In addition, it shows the performance of the device for Schmidt, ideal and simple adiabatic analysis.

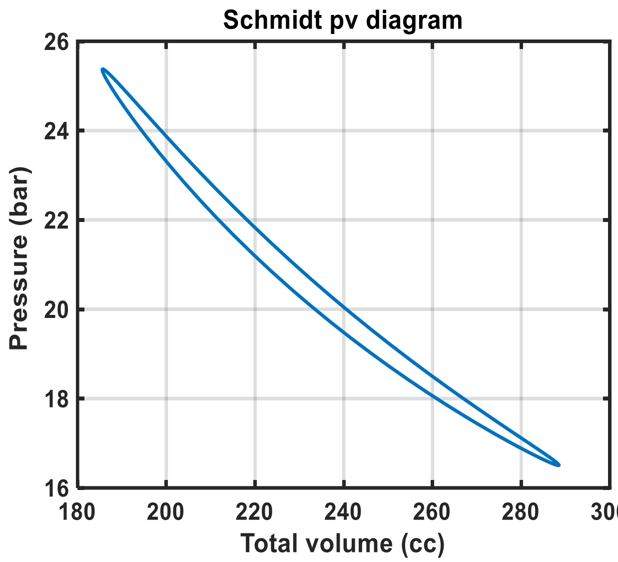

Figure 3: *Pressure vs Total volume*

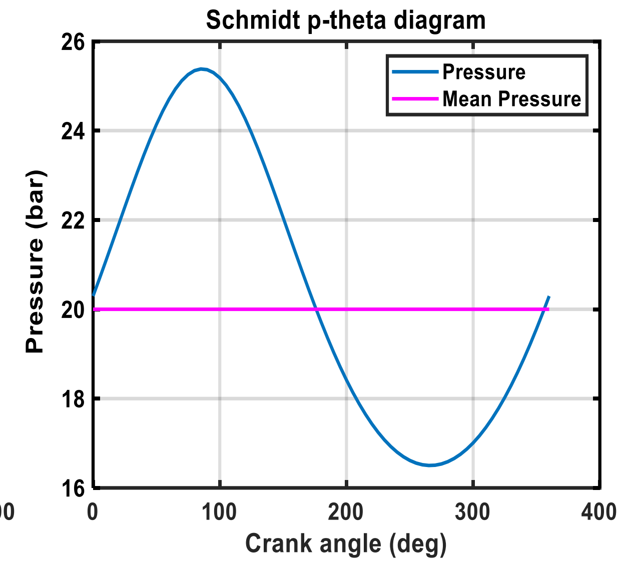

Figure 4: *Pressure vs Crank angle*

The area in figure 3 shows the work input for the machine and figure 4 shows the pressure variation in each crank angle and mean pressure of the machine. The performance of this device-based Schmidt analysis is 15.07 which similar to Carnot coefficient of performance.

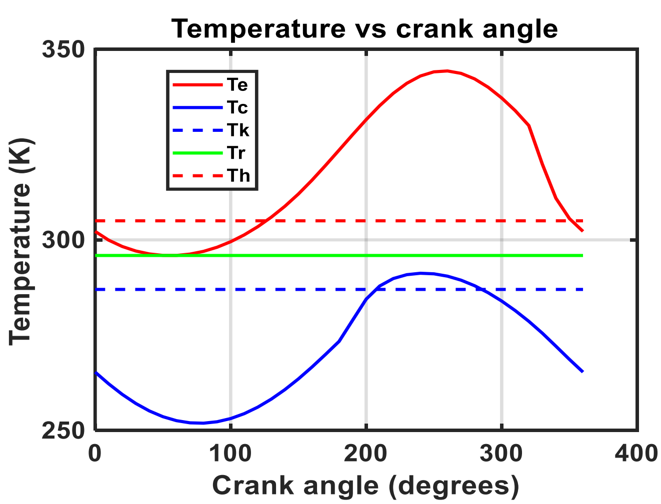

Figure 5: *Temperature vs crank angle*

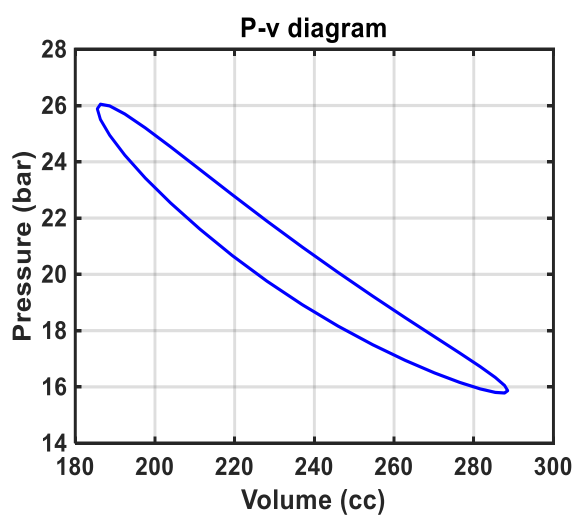

Figure 6: *Pressure vs total volume*

The temperature/crank angle diagram in Fig.5 shows a large cyclic temperature variation of the working fluid in the compression space between 302 K and 344 K and its mean values is greater than the heater temperature. Similarly, the mean gas temperature in the expansion space is less than the cooler temperature. The reduction of the performance of the device from Schmidt analysis of 16.9 to 6.2 is significant variation due to its closed form solution similar to Carnot coefficient of performance for special case of sinusoidal volume variation and no loss analysis. However, for adiabatic and simple adiabatic analysis are not a closed form of equation and the set of equation is solved by numerical methods. The accumulated energy of the three heat exchangers and work is shown in Fig.7 for ideal adiabatic analysis.

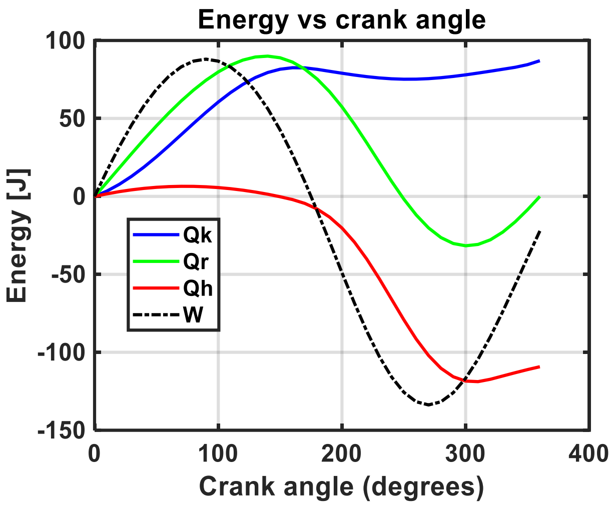

Figure 7: *Energy vs crank angle*

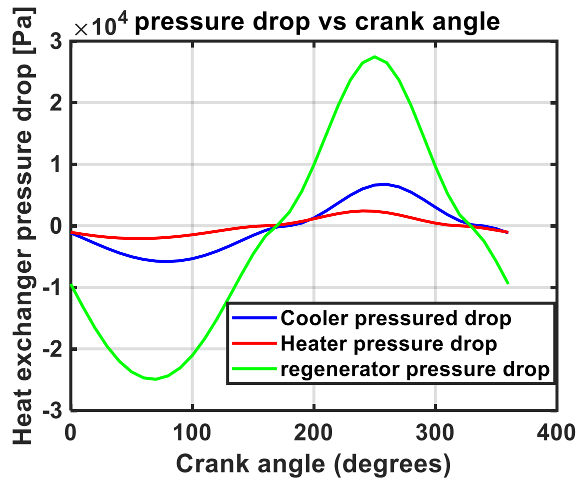

Figure 8: *Pressure drop vs crank angle*

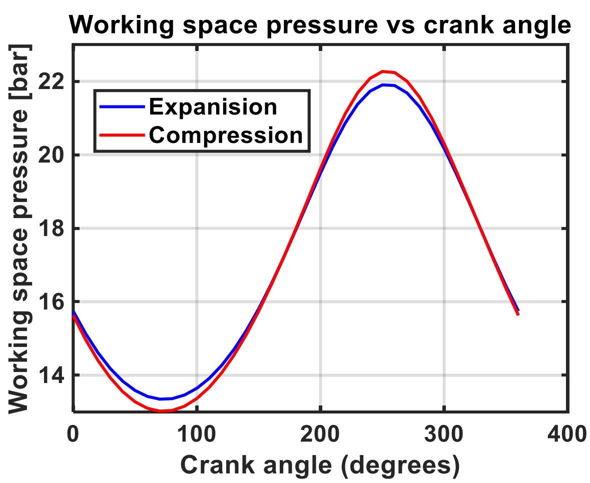

Figure 9: *Pressure vs crank angle*

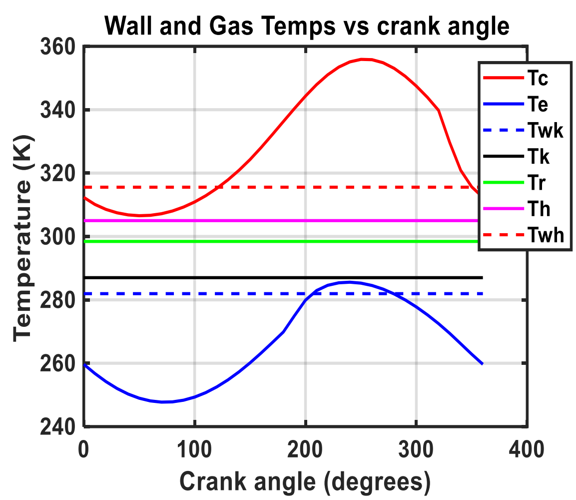

Figure 10: *Wall and gas temperature vs crank angle*

The simulation results are presented from Fig 8-10 and show the variation of various parameters with crank angle for simple adiabatic analysis. The heat exchanger pressure drop/crank angle diagram in Fig.8 shows variation of the pressure in heat exchangers and there is maximum pressure drop in the regenerator. Fig 10 shows the wall and gas temperature and the wall temperature of the heater is less than the heater temperature. The wall temperature of cooler is greater than the cooler temperature. The reduction of the performance of the device from ideal adiabatic analysis of 6.2 to 3.7 is due to its pressure drops and conduction heat transfer from the wall of the heat exchanger.

5. Conclusion

In this paper, the numerical analysis of FEMTO-60 machine based on the Schmidt, ideal adiabatic and simple adiabatic model is simulated. By applying the geometry of this machine for the given governing equations of each analysis to determine the performance and various properties of the working fluid. Schmidt analysis is a closed form and simplest analysis method as compared to adiabatic model but its simulation result is far from simple adiabatic analysis. The simple adiabatic model is the best as compared to the Schmidt and ideal adiabatic model due to consideration of pressure drop and conduction heat transfer loss in the basic equations. The performance of this machine is simulated for each model at a frequency of 7.3 Hz with nitrogen working fluid. Even though the simple adiabatic model gives better result as compared to Schmidt and ideal adiabatic model, the new polytropic model will be developed in future for betted prediction of the performance of Stirling heat pump.

6. References

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