INTERNATIONAL JOURNAL OF TOPOLOGY

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A sketch of topology

Welcome to the new open access journal: : International Journal of Topology (IJP) published by MDPI.

Mathematics underpins much of the nowadays science and technology. The most ancient disciplines are arithmetic, algebra, geometry and analysis. Topology, the investigation of properties conserved under continuous deformations is more recent. It was established on solid grounds by Henri Poincaré in his 1895 ground-breaking paper 'Analysis Situs' where he introduced the new concepts of homotopy, fundamental group, and homology, now part of algebraic topology.

Initially, algebraic topology deals with the properties of spaces that are preserved under continuous deformations, such as stretching, crumpling, and bending, but not tearing or gluing. Algebraic geometry deals with the solutions to polynomial equations, with the corresponding algebraic varieties and cohomology.

Thurston's approach to 3-manifolds emphasized the interplay between geometry and topology, advocating for a deeper understanding of how geometric structures arise from purely topological considerations. Thurston's central contribution is the geometrization conjecture (GC), formulated in the late 1970s and early 1980s, asserts that every compact 3-dimensional manifold can be decomposed into geometric pieces called 'geometries' in a canonical way. The proof of GC is due to Grogori Perelman, which also led to the proof of Poincaré conjecture as a special case.

Analysis is present in differential topology, which studies the properties of differentiable manifolds and differentiable maps between them. Key contributors after Poincaré include John Milnor, who introduced the concept of exotic spheres and Morse theory, Stephen Smale for the h-cobordism theorem and his work on the topology of higher-dimensional manifolds, René Thom for catastrophe theory, Michael Freedman and Simon Donaldson for their work about 4-dimensional topology and exotic $\mathbb{R}^4$.

At first glance, arithmetic and topology may seem like distinct areas of mathematics with little overlap. But there are deep analogies between the study of knots in 3-dimensional space and the arithmetic of prime numbers. For example, knots can be considered analogous to primes, and certain topological invariants correspond to arithmetic invariants. More advanced concepts in this direction include moduli spaces of algebraic curves and étale cohomology.

Note that while classical topology focuses on properties invariant under continuous deformations, geometric topology, piecewise-linear topology and differential topology may be extended to the study of tearing and gluing, allowing for a deeper and more versatile understanding of the structure of spaces. Cobordism, Kirby calculus and Lefschetz fibrations are part of this extended framework.

Category theory provides a powerful and unifying language for topology, enabling the abstraction and generalization of topological concepts. By treating topological spaces and continuous maps as objects and morphisms in a category, category theory facilitates the study of topological properties, constructions, and transformations in a highly structured and systematic way.

Three more items are worthwhile to mention:

Fuzzy topology is a branch of topology that extends the classical concepts of topology to fuzzy sets. Fuzzy sets, introduced by Lotfi A. Zadeh in 1965, allow for degrees of membership rather than the crisp, binary membership of classical sets. Fuzzy topology uses these ideas to study the structure and properties of topological spaces where the open sets can have varying degrees of membership.

Topology and dynamics are intertwined through various connections. Denis Sullivan received the Abel prize in 2022 for his contributions in this direction. In particular, Sullivan's contributions to topological field theory have connected algebraic topology with quantum field theory and string theory, highlighting deep connections between geometric structures and physical theories. The monodromy data of linear differential equations associated with Painlevé equations, discovered by Paul Painlevé and his colleagues in the early 20th century, can be described using character varieties. These varieties, which parameterize representations of the fundamental group of a punctured sphere into a Lie group, are deeply tied to the topology of the underlying punctured surface.

Topological quantum computing (TQC) is a theoretical approach to quantum computing that uses the principles of topological quantum field theory to encode and manipulate quantum information. Unlike traditional quantum computing architectures that rely on quantum bits as the fundamental units of information, TQC seeks to harness the topological properties of certain quantum states of matter to store and process information in a robust and error-resistant manner.

Applications of topology

For the newly launched journal IJP, we encourage interdisciplinary contributions related to topology. This includes the fields of mathematical analysis in a wide sense, physics such as quantum field theory, cosmology and condensed matter physics, data analysis and machine learning, robotics and sensor networks, computer science and network analysis, biology such as neuroscience and genomics, economics and social sciences, engineering and control theory.

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Short biography of the author

Michel Planat was a researcher at the National Center of Scientific Research in France from 1982 to 2018. From 1980 to 2001, he conducted research about nonlinear waves in piezoelectric crystals and 1/f noise in quartz resonators. He established links between $1/f$ noise and number theory an conducted research about Riemann hypothesis. He also discovered Ramanujan sums signal processing. From 2002 to 2018, he focused on quantum information theory, working on mutually unbiased bases, quantum entanglement and contextuality, and quantum computing, using mathematical tools such as finite geometries, number theory, 'dessins d'enfants' and free group theory. Since 2019, he has been publishing collaborative papers on topological quantum computing as well as genomics and transcriptomics. The main tool is the representation theory (with character varieties) of infinite groups associated with relevant DNA/RNA sequences.

Conflicts of Interest

The authors declare no conflict of interest.

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