Delay dynamics as a virtual network: Theory with Chimera states, applications with Reservoir Computing

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Abstract *Chimera states and Reservoir Computing (initially named Echo State Network and Liquid State Machine) are two concepts which independently appeared in the nonlinear dynamics and neural network computing communities respectively, surprisingly at the same time in 2002. They are both concerned by nonlinear dynamics and networks of coupled oscillators, however operating in an autonomous way for the first, generating unexpected spontaneous complex patterns, whereas the second is used in a non-autonomous way for processing an input-information according to brain-inspired principles. We will report how those two concepts met through nonlinear delay dynamics and how they cross-fertilized each other.*

Nonlinear delay dynamics form a particular class of infinite dimensional phase space dynamical systems, in the sense they do not involve partial differential equations coupling the state vector both in space and time coordinates, but they are purely temporal dynamics (and the state vector can even be scalar). Their equation of motion comprises a coupling of the state vector with itself at significantly different times with respect to the intrinsic response times (inverse of the rates of change). One of the simplest forms of a delay differential equation can be written as follows:

$$\tau.\,[\mathrm{d}X/\mathrm{d}t](t) = \mathrm{F}\,[X(t), X(t\text{-}\tau_D)],$$

where $X(t)$ is the state vector, F[.] being a nonlinear function, τ a characteristic response time for the dynamics, and τ_D a time delay (much greater then τ). Without any delay term dependence, the phase space has the same dimension than the one of the initial conditions, i.e. $X(t_0)$ the state vector at the initial time. The presence of a delay term expands the initial conditions to a functional, instead of a vector, i.e. it corresponds to all the vector state coordinates continuously evaluated over a time interval corresponding to the delay ($X(\theta) \mid \theta \in [t_0, t_0\text{-}\ \tau_D]$), thus forcing the phase space to become infinite. Despite of being purely temporal dynamical system, delay dynamics are sometimes compared or illustrated as spatio-temporal dynamics [1], due to their infinite phase space feature, and also with the objective to use space-time representation for the study of their complex motion.

Beyond those mathematical features of delay dynamics, it is also important to point out the interesting physics capability to design robust and well controlled experiments which are ruled by delay differential equations. Optics [2] and Optoelectronics [3] have provided a plethora of experiments investigating the rich dynamical properties of nonlinear delay systems. Such setups, beyond their theoretical interest, were also intensively used for well identified applications, such as physical layer encryption using chaotic light for optical telecommunications [4], high spectral purity microwave for radar sources [5], ultrafast random number generation [6], and also information processing according to brain-inspired concepts like Reservoir Computing [7]. In this last example, the conceptual idea was to make use of the high dimensional phase space of delay systems, to process a temporal input stream of information, according to the neuromorphic processing concept of Reservoir Computing. Reservoir Computing is a recurrent neural network approach which main characteristic is to perform training of a readout layer only, the input layer and the internal connectivity of the recurrent network being left to a fixed setting (usually random). This approach not only simplifies the training procedure, it also speeds it up, without any significant loss of performances as tested n many benchmark examples in the literature. Beyond this procedure simplification, another attractive advantage is the possibility to rather easily apply the RC concept to real physical networks, avoiding the energy inefficiency and the speed limitation of software solutions which typically need to heavily simulate the recurrent network architectures. Photonic RC are thus expected to provide energy efficient and ultra-fast neuromorphic hardware, as direct recurrent neural network realization, thus bringing a technological solution to new physical AI processors. Doing this with delay dynamical systems, one claims neural networks are virtually emulated in delay dynamics, as illustrated by the time division multiplexing used to distributed the input information onto temporal positions distributed along a time delay interval. The virtual emulation of a spatial network of neurons, was however essentially a conceptual interpretation, which was not clearly demonstrated theoretically.

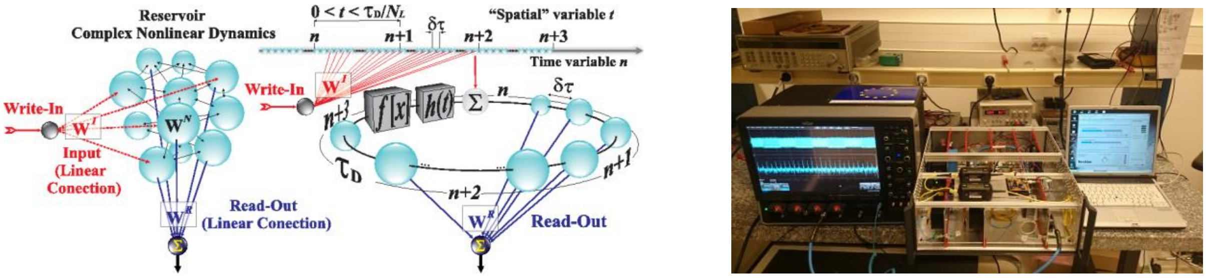

Figure 1: Delay dynamics used as a RC photonic processor (Left: Temporal processing; Right: Setup).

Delay dynamics RC was connected to Chimera states during the NOLTA conference organized in Palma de Mallorca, Spain, in 2012, where both topics had contributions. Yuri Maistrenko reported a talk about Chimera states which have just been demonstrated experimentally in spatio-temporal dynamical systems [8,9], and a very intriguing discussion started with him about the possibility of observing Chimera states in delay dynamics, since Chimera states involve network of coupled oscillators, and RC claims to emulate a network of coupled neurons… An intensive collaboration started at FEMTO-ST, which led one year later to the first discovery of Chimera states in delay dynamics [10]. Beyond the experimental and numerical demonstration, the results also allowed to obtained a mathematical approximation of the coupled oscillator network emulated in delay system, in a very close mathematical form compared to the original Kuramoto model which was used to discover Chimera states in 2002. One important result was to identify the impulse response of the bandwidth limiting filter in delay dynamics, as the coupling function between the virtual neurons distributed along the delay line.

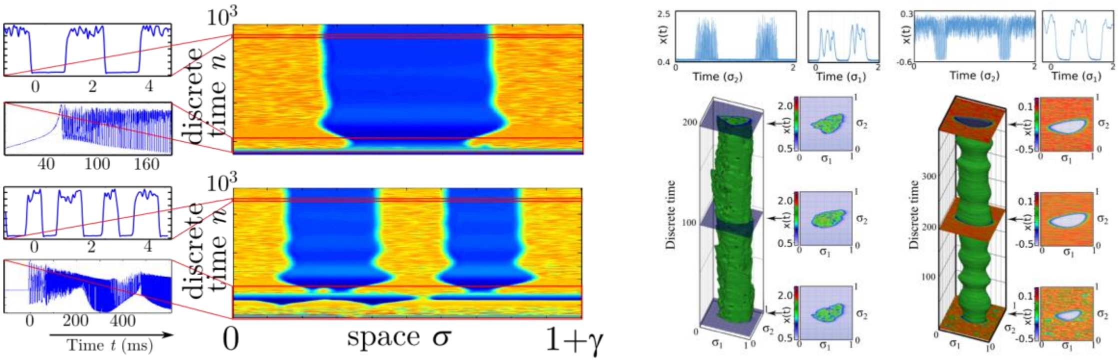

Figure 2: Chimeras states in 2D and 3D, shown in their virtual space-time representation.

The author acknowledges the financial support of the EU PHOCUS project (Grant No. 240763), the BiPhoProc project (ANR-14-OHRI-0002-02), the EIPHI Graduate School (ANR-17-EURE-0002) and the Bourgogne-Franche-Comté Region.

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