

# Uncertainty estimation in optics and optoelectronic systems

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## ABSTRACT

In this work, we present the general method to estimate the measurement uncertainty related to opto-electronic and photonic systems. It consists in a modern method, which follows the recommendations of the Guide to the Expression of Uncertainty in Measurement. After determining the uncertainty propagation equation, we review the elementary uncertainty terms separated into two distinct categories. First of all, statistical terms are studied. Then, the other elementary uncertainty terms such as those linked to the connection to international standards or manufacturer data, as well as the elementary terms linked to variations in environmental parameters such as temperature, acceleration, or hygrometry, or those linked to quantities studied such as the wavelength of lasers or that linked to the misalignment of the beam in space. We illustrate this method with two examples, an optoelectronic oscillator and that of a Brillouin Light Scattering measurement system.

**Keywords:** Optoelectronic oscillator, Brillouin Light Scattering, microwave signal, uncertainty, uncertainty analysis, error, noise.

## 1. INTRODUCTION

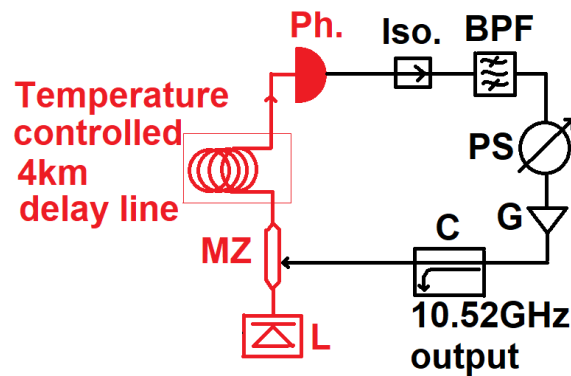
What is measurement uncertainty? It is the estimate characterizing the range of values in which the true value of a measured quantity lies. Measurement uncertainty generally includes several components. Some can be estimated based on the statistical distribution of the results of a series of measurements and can be characterized by an experimental standard deviation. The estimation of the other components can only be based on experience or other information. The uncertainty in the results of a measurement consist of several components, which may be listed as two categories according to the way in which their numerical value is estimated. It is interesting to consider how the elementary terms are grouped together for the calculation of the final uncertainty. We can see that we are dealing with two main categories of elementary uncertainty terms. The first category of terms of uncertainty is called “type A”. These terms are evaluated by statistical methods such as reproducibility, repeatability, special consideration about Fast Fourier Transform analysis, and the experimental standard deviation. The components in category A are characterized by the estimated variances. The second family of uncertainty contributions are evaluated by other means. They are called “type B”, and because various components and temperature control, experience with or general knowledge of the behaviour and properties of relevant materials and instruments, manufacturer’s specifications, data provided in calibration and other certificates (noted BR), their uncertainties assigned to reference data taken from handbooks. The components in category B should be characterized by quantities, which may be considered as approximations to the corresponding variances, the existence of which is assumed. Not everything is set in stone. The definition and interpretation of how to determine the uncertainty associated with a measurement result is still a hot topic. The two references given here provide an overview of the ongoing debate among metrologists [1,2]. Current recommendations and standards are mainly based on a modern statistical approach. The sum of variance and covariance is considered with an associated confidence level. All uncertainty components must be identified: the reference standard, the measuring means, the various components. This modern approach is based on the Guide to Expressions of Uncertainties (GUM) of the International Bureau of Weights and Measures (BIPM) [3]. Quantities are defined in standards, for example, for frequencies [4].

We are also interested in what makes optics and optoelectronics specific for uncertainty calculation. The field mainly concerns frequencies and wavelengths, powers and phase noise.

However, we are not in the same way of reasoning as for the estimation of an uncertainty linked to the determination of power, for example as for the calibration of wattmeters, or even attenuation measurements as with the calibration of an attenuator, or even the characterization of a source of radio noise. We will focus here on two examples. The first is an optoelectronic oscillator for which we must estimate the phase noise related to the delivered signal. The second is the case of estimating the frequency shift during Brillouin Light Scattering on a material for which we must characterize the speed of the propagation waves of the acoustic waves created by a power laser.

## 2. MEASUREMENT UNCERTAINTY ON THE PHASE NOISE OF AN OPTOELECTRONIC OSCILLATOR

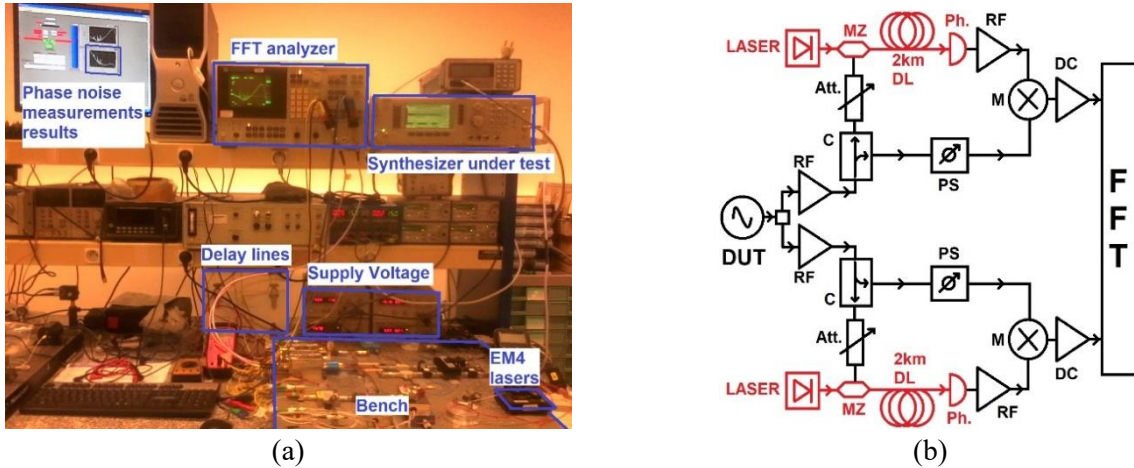
Several Optoelectronic oscillators (OEO) were designed as referenced [5-7]. A compact optical delay line OEO is designed to fit in a small volume that can be as less as 1 liter. Its source is a 1.55  $\mu\text{m}$  wavelength laser. There are also a modulator, an optical fiber consisted in a delay line, a photodetector, a 8.2 – 12.4 GHz microwave amplifier and also a driving coupler. In addition, elements of this OEO must be less sensitive to mechanical and environmental perturbations. The delivered signal is especially evaluated for its frequency stability. Measurement of phase noise is carried out using a special bench [8] and we follow a modern approach for estimating its uncertainty [9], following previous works performed on oscillators [10-13]. This section is mainly based on reference [8]. The Device Under Test (DUT) is a relatively compact OEO designed on table in the lab delivering an output signal of 5 dBm at 10.5 GHz. It is constituted a RIO laser, model ORION driven by a 125 mA signal. Modulator is with an 11 GHz bandwidth, then, a 4 km optical fiber delay line, and a DSC40S Discovery photo-detector. In the electrical part of the loop of the oscillator, there is a 54 dB gain amplifier for the microwave signal, an X-band filter, an ARRA passive phase shifter, and a buffer amplifier (AML812-1901) at the lateral arm of a microwave coupler in order to extract the output microwave signal. The OEO is represented in Figure 1. To adjust the gain of the microwave amplifier in the OEO oscillation loop, we proceed with a vector network analyser (VNA).



**Fig. 1.** OEO: Optical and electrical elements are drawn in red and black colors, respectively. L—laser; MZ—Mach Zehnder modulator; Ph—photodetector; Iso—isolator; BPF—band pass filter; PS—phase shifter; G—microwave low noise amplifier; C—coupler.

Here, we have the scheme of the OEO. It is necessary to carry out this gain adjustment by working in an open loop. It is easy to understand that the losses in the delay lines and all of the optical devices also needed to be compensated in order to ensure the continuity of the oscillation phenomenon over time [14]. We need to know the reflectance of each amplifier, as well as the noise factor, especially at the start of the amplification chain. Figure 1 describes the OEO. As mentioned previously, there are two ways to evaluate the performance of the DUT in terms of phase noise. The results are shown in Table 1 for the OEO. Figure 2 shows the setup of this measurement, explained in the caption of the Figure.

Investigating the uncertainty calculation is an old challenge of scientists working on phase noise. It firstly concerns the knowledge of the experimentally determined phase noise. It is taken close to the carrier with a negative slope of  $S'(f)$  versus the Fourier frequency noted  $f$ . Secondly, it concerns the determination of the ground noise  $f_0$  far from the carrier, mostly dependent from the power inside the loop with an approximation of  $kT/P$ , where  $k$  is the Boltzmann constant,  $T$  is the temperature, and  $P$  is the power. Sources of uncertainties were discussed [15]. We keep in mind that the frequencies of references must be as stable as possible [16-19].



**Fig. 2.** (a) Photo of the phase noise measurement bench developed at the laboratory while measuring the phase noise of a synthesizer under test. (b) Schematic view of phase noise measurement bench system using a double optical delay line. Optical elements and electrical elements are respectively drawn in red and black colors. DUT—device under test; MZ—Mach–Zehnder modulator; DL—delay line; Ph—photodetector; M—mixer; DC—DC amplifier; RF—microwave amplifier; Att.—attenuator; C—directive coupler; PS—phase shifter; FFT—fast Fourier transform analyser.

**Table 1.** Phase noise of an OEO with an output power of 5 dBm at 10.52 GHz, measured by our Instrument and by the commercial Rohde and Schwarz (R&S) bench.

Offset to the 10.52 GHz Carrier Fourier Frequency (Hz)	Measure with R&S Bench Phase Noise (dBc/Hz)	Measure with Our Bench Phase Noise (dBc/Hz)
$2 \times 10^3$	-100	-100
$4 \times 10^3$	-109	-112
$6 \times 10^3$	-115	-118
$10^4$	-119	-130
$2 \times 10^4$	-125	-140
$3 \times 10^4$	-125	-145
$4 \times 10^4$	-123	-141

Noise Floor is determined with 500 Averages with an Anritsu Synthesizer at the Input of Our Bench  $\mathcal{L}(f)$  in dBc/Hz versus Offset to the 10 GHz Carrier Fourier Frequency (Hz) decreases from -90 at 10 Hz to -170 at 10 kHz and few  $10^5$  Hz [4]. We followed a modern approach to express uncertainty in measurement as we write in the introduction. We are getting to the significant part about uncertainties. We examine each of the elementary terms as reported in [8].

Uncertainty at a  $1 \sigma$  interval of confidence is calculated as follows:

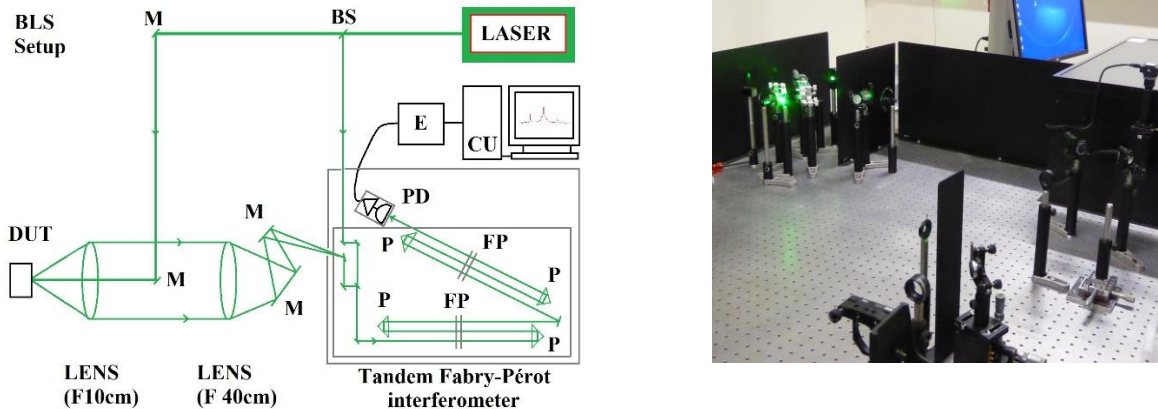
$$u_c = \sqrt{(A^2 + BR^2 + BL^2)} \quad (\text{ii})$$

According to Equation (i), it can then be considered that the whole statistical contribution is better than 0.69 dB. We deduce from (ii) that the uncertainty at 1 sigma, noted as  $u_c$ , is better than  $\sqrt{(0.692 + 0.202)}$  dB. Its leads to a global uncertainty of  $\pm 0.72$  dB at one  $\sigma$ .

We choose to keep  $U = \pm 1.44$  dB at  $2 \sigma$  for a common use of the phase noise optoelectronic instrument. This final uncertainty is defined at  $2 \sigma$ , according to the empirical rule 68.27% at  $1 \sigma$  is not enough, but 95.45% at  $2 \sigma$  is more efficient for a normal distribution in statistics.

### 3. MEASUREMENT UNCERTAINTY ON THE FREQUENCY SHIFT FOR BRILLOUIN LIGHT SCATTERING

This paper provides an experimental part and the determination of the uncertainties associated with the determination of peaks corresponding to the shift between the frequency of the signal refracted by a material and to the laser serving as an interrogation signal. The knowledge of this value, as well as the parameters of the studied materials, can also provide the value of the speed of the corresponding phononic waves, when intrinsic characteristic data of the evaluated materials are known. To lead the discussion on the uncertainty associated with the BLS, we rely on the standards of metrology. We will focus in this part on the principle of Brillouin light scattering. BLS using a 532 nm powerful Class 4 laser up to 600 mW is efficient to reveal spin wave or acoustic signals, at frequencies from few Giga Hertz to more than a hundred of Giga Hertz. Fluctuations of refractive index in a medium enables the detection and analysis of laser light scattered, thanks to BLS setup [20,21]. The general principle is to send the signal generated by the laser focusing it on the part of the sample that we want to characterize. The photons arrive in the material or in the thin layer and interact with the lattice or more generally with the material. Light helps to create phonons. These phonons propagate with speeds that may be different depending on whether the mode is transverse or longitudinal. It depends on the nature of the material, as it can be isotropic or anisotropic. The phonons in turn create light, which are shifted in frequency relatively to the wavelength of the laser. The BLS precisely consists in analysis of the refracted light emitted by a material [22]. Tandem Fabry–Perot interferometer produces peaks shifted from the frequency of the laser to characteristic frequencies depending on the material. Figure 3 gives the typical setup used for the measurement, showing the typical setup (a) and a picture of the system (b). This work is based on reference [23]. We calibrated the bench with part of the laser signal, used as the bench reference. Inside the commercial bench developed by the Swiss company "The time Stable", the light goes with six passages through two different interferometers. Each pair of mirrors is very precisely aligned during the calibration procedure.



**Fig. 3.** (a): Typical setup for BLS. BLS: Brillouin Light Scattering. DUT: device under test. M: mirror. FP: Fabry-Pérot. P: prism. PD: photodetector. E: electronics. CU: computer unit. (b): Commercial Tandem Fabry-Pérot interferometer is inside the box on the right side of this picture.

It is necessary to calibrate accurately the instrument. It is sensitive to mechanical vibrations, temperature and hygrometry. Alignment process requires an alignment of the two cavities. Each of the two cavities consists in a pair of parallel mirrors. Tandem interferometer produces two series of absorption peaks with respect to a flat noisy intensity level. We then obtain a curve providing the number of absorbed photons versus frequency.

We have measured the  $\nu_B$  for PMMA as an example of an isotropic material. The measured Brillouin frequency shift is  $\nu_B = 15.70$  GHz (longitudinal acoustic mode), with a Brillouin linewidth of 324 MHz. Estimating the uncertainty requires the knowledge of the contribution of the different fixed parameters, such as the optical index, the wavelength, the diffusion angle, the density of the material, and the longitudinal and shear modulus, but especially fluctuation of the source, mechanical stability of the setup, and environmental parameters in the room.

The Brillouin spectroscopy is a non-intrusive measurement method for bulk materials and thin films. A scanning 6-pass TFPI has been described for BLS measurement. Following the GUM, and with reference [23], we have made detailed analysis and estimation of the uncertainties in the Brillouin frequency shift measurement, which is related to the speed of propagation of phononic waves in bulk materials.

#### 4. CONCLUSION

To conclude this paper, we can see that we have estimated the uncertainty for an optoelectronic oscillator. The signal delivered at 10.5 GHz presents a relatively good performance in terms of phase noise, with a minimum of  $-145$  dBc/Hz at  $3 \times 10^4$  Hz from the carrier. The uncertainty on the phase noise of the OEO is  $\pm 1.44$  dB at  $2 \sigma$ .

On the second example, with Brillouin Light Scattering provides an expanded relative uncertainty in measured Brillouin frequency shift is estimated to be 0.26% at  $2 \sigma$ , which corresponds to an expanded uncertainty of 41 MHz for the measured frequency shift of 15.70 GHz in testing PMMA.

#### REFERENCES

- [1] Thomas von Clarmann, Steven Compennolle, Frank Hase, "Truth and uncertainty. A critical discussion of the error concept versus the uncertainty concept," Atmospheric Measurement Techniques 15, 1145-1157 (2022). <https://doi.org/10.5194/amt-15-1145-2022>
- [2] Jong Wha Lee, Euijin Hwang, Raghu N. Kacker, "True value, error, and measurement uncertainty: two views," Accreditation and Quality Assurance 27, 235-242 (2022). <https://doi.org/10.1007/s00769-022-01508-9>
- [3] GUM: Guide to the Expression of Uncertainty in Measurement, fundamental reference document, JCGM100:2008 (GUM 1995 minor corrections): <https://www.bipm.org/en/committees/jc/jcgm/publications>
- [4] "IEEE Standard Definitions of Physical Quantities for Fundamental Frequency and Time Metrology-Random Instabilities," in IEEE Std 1139-1999, pp.1-40, 21 July 1999. <http://dx.doi.org/10.1109/IEEESTD.1999.90575>
- [5] Ge Z., Hao T., Capmany J., Li W., Zhu N., Li M., "Broadband random optoelectronic oscillator," Nat. Commun. 11, 5724 (2020). <https://doi.org/10.1038/s41467-020-19596-x>
- [6] Wang Y., Li X., Zhang J., Wo J, "Spurious level and phase noise improved Fourier domain mode-locked optoelectronic oscillator based on a self-injection-locking technique," Opt. Express 29, 7535–7543 (2021).
- [7] Salzenstein P., Tavernier H., Volyanskiy K., Kim N. N. T., Larger L. and Rubiola E., "Optical mini-disk resonator integrated into a compact optoelectronic oscillator," Acta Physica Polonica A 116(4), 661-663 (2009) ). <https://doi.org/10.12693/APhysPolA.116.661>
- [8] Salzenstein P., Pavlyuchenko E., "Uncertainty Evaluation on a 10.52 GHz (5 dBm) Optoelectronic Oscillator Phase Noise Performance," Micromachines 12(5), 474 (2021). <http://dx.doi.org/10.3390/mi12050474>
- [9] Kacker R., Sommer K. D., Kessel R., "Evolution of modern approaches to express uncertainty in measurement," Metrologia 44( 6), 513–529 (2007). <http://dx.doi.org/10.1088/0026-1394/44/6/011>

- [10] Salzenstein, P., Pavlyuchenko, E., "Uncertainty calculation for phase noise optoelectronic metrology systems," PIERS 2012 Moscow, Progress in Electromagnetics Research Symposium, 1099–1102 (2012).
- [11] Salzenstein P., Cholley N., Zarubin M., Pavlyuchenko E., Hmima A., Chembo Y. K. and Larger L, "Optoelectronic phase noise system designed for microwaves photonics sources measurements in metrology application", Proc. SPIE 8071, 807111 (2011). <http://dx.doi.org/10.1117/12.886694>
- [12] Salzenstein P., Hmima A., Zarubin M., Pavlyuchenko E. and Cholley N., "Optoelectronic phase noise measurement system with wideband analysis", Proc. SPIE 8439, 84391M (2012). <http://dx.doi.org/10.1117/12.921630>
- [13] Mikitchuk K., Chizh A., Malyshev S., "Modeling and Design of Delay-Line Optoelectronic Oscillators," IEEE J. Quantum Electron. 52, 1–8 (2016).
- [14] Pavlyuchenko E., Salzenstein, P., "Application of modern method of calculating uncertainty to microwaves and opto-electronics," Laser Optics, 2014 International Conference, Saint Petersburg, Russia, June 30 2014–July 4 (2014). <http://dx.doi.org/10.1109/LO.2014.6886449>
- [15] Yanagimachi, S.; Watabe, K.; Ikegami, T.; Iida, H.; Shimada, Y. Uncertainty Evaluation of -100dBc/Hz Flat Phase Noise Standard at 10 MHz. IEEE Trans. Instrum. Meas. 2013, 62, 1545–1549, <http://dx.doi.org/10.1109/TIM.2013.2239057>
- [16] Salzenstein P., Cholley N., Kuna A., Abbé P., Lardet-Vieudrin F., Sojdr L. and Chauvin J., "Distributed amplified ultra-stable signal quartz oscillator based," Measurement 45(7), 1937–1939 (2012). <https://doi.org/10.1016/j.measurement.2012.03.035>
- [17] Sthal F., Imbaud J., Vacheret X., Salzenstein P., Cibiel G. and Galliou S., "Computation method for the short-term stability of quartz crystal resonators obtained from passive phase noise measures," IEEE Trans. on UFFC 60(7), 1530–1532 (2013). <http://dx.doi.org/10.1109/TUFFC.2013.2725>
- [18] Kuna A., Cermak J., Sojdr L., Salzenstein P., Lefebvre F., "Lowest Flicker-Frequency Floor Measured on BVA Oscillators," IEEE Trans. on UFFC 57(3), 548–551 (2010). <http://dx.doi.org/10.1109/TUFFC.2010.1446>
- [19] Salzenstein P., Kuna A., Sojdr L., Sthal F., Cholley N. and Lefebvre F., "Frequency stability measurements of ultra-stable BVA resonators and oscillators," Electronics Letters 46(10), 686–688 (2010). <http://dx.doi.org/10.1049/el.2010.0941>
- [20] Lindsay S. M., Anderson M. W., Sandercock J. R., "Construction and alignment of a high performance multipass Vernier tandem Fabry–Perot interferometer," Review of Scientific Instruments 52(10), 1478–1486 (1981). <https://doi.org/10.1063/1.1136479>
- [21] Dil J. G., van Hijningen N. C. J. A., van Dorst F., Aarts R. M., "Tandem multipass Fabry-Perot interferometer for Brillouin scattering," Applied Optics 20(8), 1374–1381 (1981). <https://doi.org/10.1364/AO.20.001374>
- [22] Hillebrands B., "Progress in multipass tandem Fabry-Perot interferometry: I. A fully automated, easy to use, self-aligning spectrometer with increased stability and flexibility," Review of Scientific Instruments 70(3), 1589–1598 (1999). <https://doi.org/10.1063/1.1149637>
- [23] Salzenstein P., Wu T. Y., "Uncertainty estimation for the Brillouin frequency shift measurement using a scanning tandem Fabry-Pérot interferometer," Micromachines 14(7), 1429 (2023). <https://doi.org/10.3390/mi14071429>