Review on the development of Stirling Heat Pump thermodynamic modeling

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Abstract. The aim of the study is to give a summary of the research that is conducted on the thermodynamic modeling methods for Stirling heat pumps.The various modeling thermodynamic techniques used for this heat pump are adapted from the Stirling engine model. The present study provides an extensive analysis of the Stirling heat pump performance using thermodynamic techniques such as isothermal, ideal adiabatic, simple adiabatic, combined adiabatic finite speed(CAFS), simple II adiabatic, polytropic with various Loss(PSVL), Polytropic finite speed thermodynamics(PFST), compressive polytropic model(CPMS) and non-ideal second order thermal model with additional loss effect. This study compared previous thermodynamic models of GPU3 Stirling engine. The review findings demonstrated that the new Non-Ideal second order model was preferable to the other thermodynamic modeling techniques due to some parameters.

Keywords: Modeling,Isothermal,Adiabatic,and Polytropic.

1 Introduction

The rapid increase in global temperature that has been observed in recent years is mostly the result of increased emissions from fossil fuels and the working fluids in the refrigeration and air-conditioning systems [15].

Heating and cooling are essential for our day-to-day activities, and most of them uses the environmentally hazardous working fluids (CFC and HCFC).Therefore, the Montreal and Kyoto Protocol were signed in 1987 and in 1997, respectively, for restricting the usage of CFC, HCFCs, Halon, PFCs, HFCs, and SF6 [9]. For that reason, it will be crucial to design a heat pump system in the future without the need of those working fluids.

Regenerative thermal machines are alternative energy conversion machines for heating and cooling at near ambient temperature. The target temperature (170-400K) and many applications between this temperature common in air-conditioning, household and commercial refrigeration, vending machines, space and water heating systems and high temperature heat pump system for industrial applications [14].

Among this regenerative thermal machines, Stirling heat pump is one alternative that can utilize existing energy more efficiently. A Stirling heat pump is operating on Stirling cycle which consists of four thermodynamic process: isothermal compression, constant volume heat rejection, isothermal expansion, and constant volume heat addition [20]. During the isothermal compression process, the refrigerant is compressed and releases heat to the high temperature region.The constant heat rejection process follows, during which refrigerant expands further, cools down, and heat is rejected to the regenerator.During the isothermal expansion process, the refrigerant in the cycle absorbs heat from the low temperature source and expands. Finally, in the constant volume heat addition process, heat is added to the refrigerant through the regenerator, and the refrigerant is heated up.

The Researchers are currently looking for alternatives to improve the performance of Stirling heat pumps and the performance prediction techniques of Stirling heat pump by thermodynamic modeling.

2 Objective of review

As compared to the Stirling engine and cryocoolers, there has been a little study done on Stirling heat pumps. Nowadays, the demand for developing thermal models for Stirling cycle devices increases in order to precisely forecast their performances. The Stirling heat pump design parameters are also being optimized in the meantime in order to enhance its performance for efficient and practical operation. Therefore, the objective of this review is to study the various thermodynamic modeling techniques of the Stirling heat pump and compare the results of each model with the experimental results.

3 Methodology

In order to perform this review, articles on the modeling and optimization of the Stirling cycle machines from peer-reviewed journals and conference proceedings were gathered and carefully examined. Zero-fourth-order analyses were used to categorize the modeling of the Stirling heat pump.

4 Thermodynamic modeling

The model reviewed was grouped based on the basic assumptions of the cyclic process.Martini [17] proposed the following thermodynamic modeling for the Stirling heat engine, and the modeling of the Stirling heat pump is adapted from this engine model.

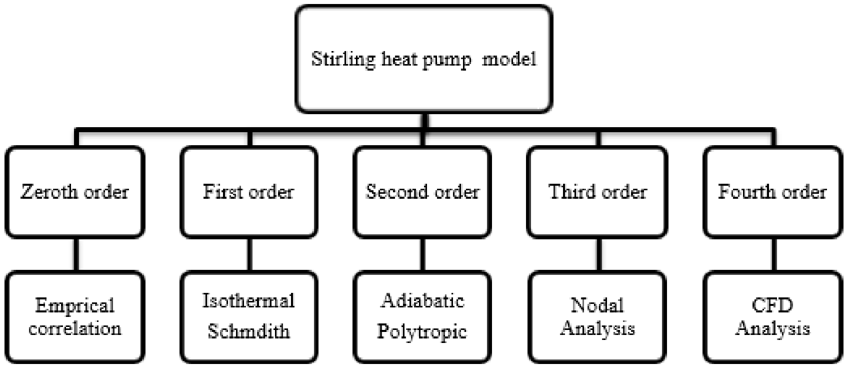

Fig. 1. Thermodynamics modeling of Stirling cycle heat pump [17]

4.1 Zero order modeling

The primary goal of this model is to create a mathematical correlation and simple computations to evaluate the power output, and performance of Stirling cycle machines. The mathematical formulation to determine the engine's power output was initially set out by William Beal:

$$W = kV_{SC}fP_{mean} \tag{1}$$

Where V_{SC} = the compression space swept volume, f =frequency and P_{mean} = the average pressure of the engine.

West [21] verified the validity of Beal's expression and suggested a modified version as:

$$W = 0.25V_{SC}fP_{mean} \tag{2}$$

This modeling approach provides a rapid overview of the performance, but cannot be utilized to develop new machines or calculate their power output and work input.

4.2 First order modeling

This model is based on isothermal assessment, and Schmidt theory was proposed by Gustave Schmidt [20].The main assumption in this study is that the gas in the compression area and heater is at a constant sink temperature, while the gas in the expansion space and cooler is at a constant source temperature. In Isothermal analysis, the following assumptions are assumed:

- Working fluid's mass is constant (there is no mass leakage),
- Applying the ideal gas equation of state,
- Considering that the machine's speed is constant, a steady state process,

– The potential and kinetic energy are neglected,
– There is no change in pressure in the system,
– The three heat exchangers (hot, cold and regenerator) are perfect.

The heat pump is configured as five component namely compression space(c), heater(h), regenerator(r), cooler(c) and expansion space(e) respectively see Fig 2.The terms w,T,V,m p represents the work, temperature of the working gas, volume of the space,mass of working gas and pressure of the working gas respectively.

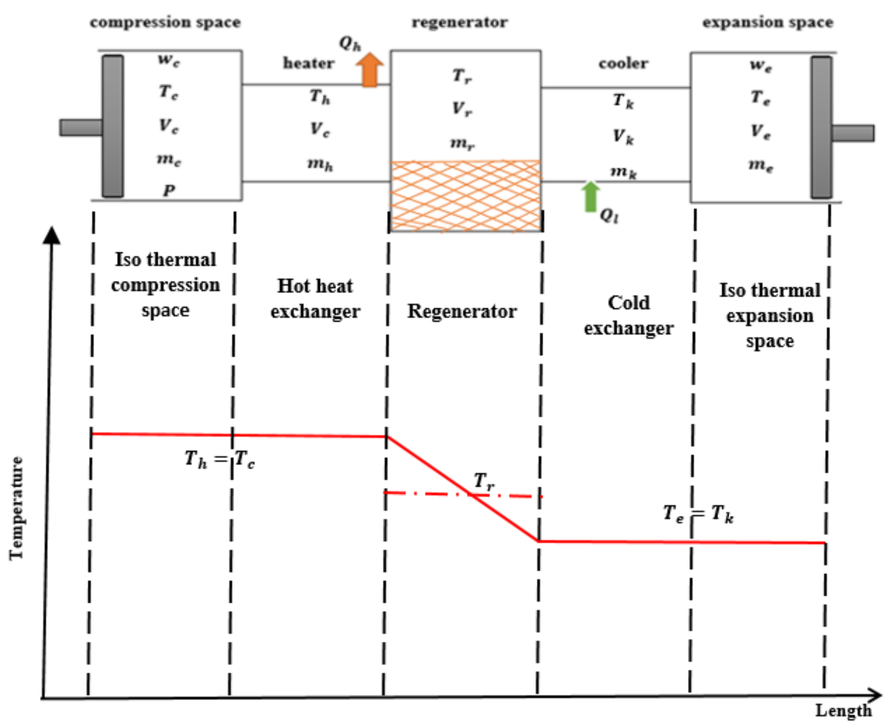

Fig. 2. Isothermal analysis of Stirling heat pump

This analysis provides a quick way to estimate the relationship between the overall size and its power output and work input, but it is not very useful as a detailed design tool for Stirling cycle machines. The calculation of power input starts with an ideal loss free analysis (no thermal and power loss) and simple correlation factor is used to find the break power input. Hence, due to the analysis was conducted under ideal conditions, the efficiency values estimated by this model varies extremely from the experimental value.

Table 1. Ideal isothermal set of equations [20].

Parameter	Equation
Pressure	$P{=}MR/[V_c/T_k+V_c/T_c+(V_r ln(T_c/T_k))/(T_c-T_k)+V_k/T_k+V_e/T_e]^-1$
Heat transfer	$Q_C\ =W_C{=}\oint PdV_C d\theta$
Heat transfer	$Q_e{=}W_e\ =\oint PdV_e d\theta$
Work input	$W_{in}{=}{-}(W_e+W_C)$
Coefficient of performance	$COP_{hp}{=}Q_C/W_{in}$

4.3 Second Order Model

This model is a modification of the first-order analysis, with the compression and expansion space being either adiabatic or polytropic rather than isothermal and including the thermal and power losses depending on the type of second order model. This model can be classified as:
4.3.1.Ideal adiabatic Analysis(model)l:An ideal adiabatic analysis is a modification of an isothermal analysis in which the compression space and expansion spaces are adiabatic rather than isothermal, there is no power or thermal loss, and also assumed a perfect heat exchangers[7].The heat pump is configured by five components serially connected and the working gas in the heater/hot heat exchanger, and cooler/cold heat exchanger is kept under isothermal conditions see Fig 4.Enthalpy flows across the cells through four interfaces as a result of mass flow being respectively compression space/heater, heater/regenerator, regenerator/cooler and cooler/expansion space.

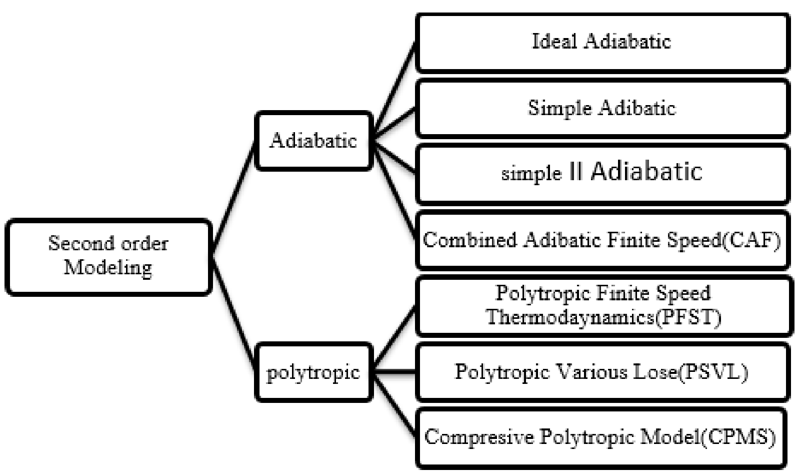

Fig. 3. Classification of second order analysis

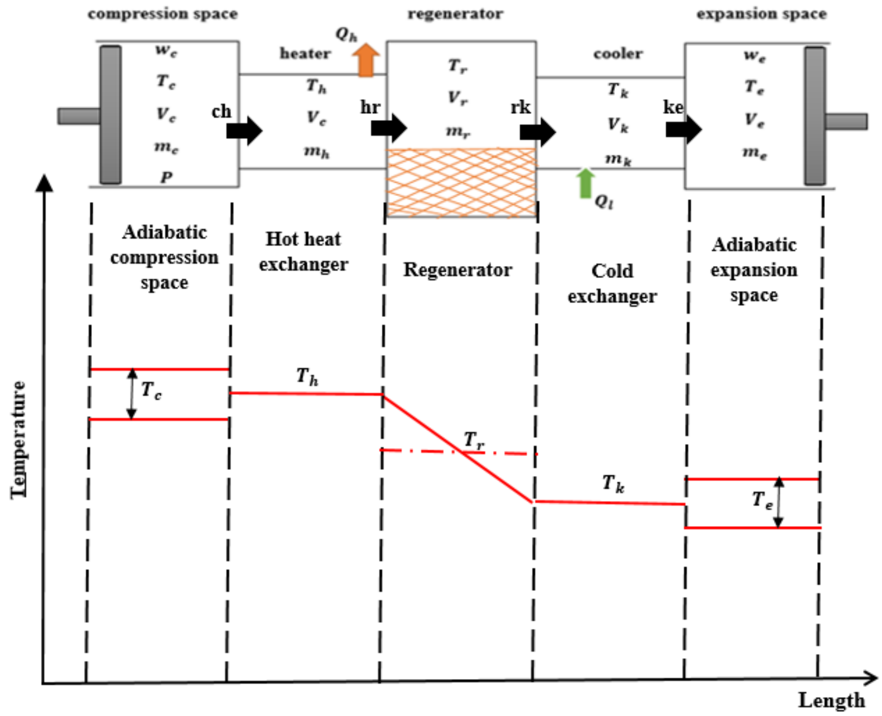

Fig. 4. Ideal adiabatic model of Stirling heat pump

The assumptions for the ideal adiabatic model analysis are:

- heater and cooler temperature is constant,
- property of the working gas is constant,
- perfect regenerator,there is no gas, leakage and pressure drop,
- the process is steady state,
- adiabatic compression and expansion space,
- linear temperature variation in the regenerator,
- The machine has constant angular velocity,
- The kinetic and potential energies of gas streams are negligible,
- Heat is transferred to the working fluid only in the hot and cold heat exchanger,
- Heat transfer to the environment is negligible.

4.3.2.Simple adiabatic model (quasi steady flow model): This model is a modification of an ideal adiabatic analysis, in which non ideal heat exchangers are applied for the heat pump analysis (see Fig 5).The heat exchanger wall temperature and gas temperature are considered to be the same in the ideal adiabatic analysis.Thus,the interface temperature of the regenerator calculated as follows:

$$T_{rh} = (3T_{r1} - T_{r2})/2 \tag{3}$$

Table 2. Ideal Adiabatic set of equations [9].

where D is the derivative factor with respect to time [20],D =dX/dT

Parameter	Equation
Pressure(P)	$P_i= MR/(V_c/T_c + V_h/T_h + V_{ir}/T_{ir} + V_k/T_k + V_e/T_e)^-1$
Pressure change(ΔP)	$dP=(-P\gamma(DV_c/T_{Ck} + DV_e/T_{he}))/(V_c/T_{ck} + (V_h/T_h + V_r/T_r + V_k/T_k) + V_e/T_{he})$
Mass(m)	$m_i=(PV_i)/RT_i$,where, i =c,h,r,k,e
Change of mass(Δm)	$Dm_i=(m_iDP)/P = (DP_i/R)(V_i/T_i)$,where, i=h,r,k
	$Dm_c=(PDV_c + (V_cDP)/\gamma)/(RT_{ch})$
	$Dm_e=(PDV_e + (V_eDP)/\gamma)/(RT_{ke})$
Flow of mass	$\dot{m}_{ch} = (dm_c)/dt$
	$\dot{m}_{ke} = (dm_e)/dt$
	$\dot{m}_{hr} = \dot{m}_{ch} - (dm_h)/dt$
	$\dot{m}_{rk} = \dot{m}_{ke} - (dm_k)/dt$
Conditional Temperature	If $\dot{m}_{ch} > 0, then T_{ch} = T_c, else, T_{ch} = T_h$
	$\dot{m}_{ke} > 0, then T_{ke} = T_k, else T_{ke} = T_e$
Temperature change(ΔT)	$dT_{ie} = T_e(dP/P) + (dV_e/V_e) - (dm_e/m_e)$
	$dT_{ic} = T_c(dP/P) + (dV_c/V_c) - (dm_c/m_c)$
Energy power	$DQ_{h,ideal} = (V_hDPC_v)/R - C_p(T_{ch}\dot{m}_{ch} - T_{hr}\dot{m}_{hr})$
	$DQ_{r,ideal} = (V_rDPC_v)/R - C_p(T_{hr}\dot{m}_{hr} - T_{rk}\dot{m}_{rk})$
	$DQ_{k,ideal} = (V_kDPC_v)/R - C_p(T_{rk}\dot{m}_{rk} - T_{ke}\dot{m}_{ke})$
	$DW_{in,ideal} = -(DW_{iC} + DW_{ie)}$
	where $DW_{iC} = PDV_C, and, DW_{ie} = PDV_e$

$$T_{rr} = (T_{r1} + T_{r2})/2 \tag{4}$$

$$T_{rk} = (3T_{r2} - T_{r1})/2 \tag{5}$$

The heat transfer to the refrigerant or working gas through forced convection and this convective heat transfer is given by the basic equation:

$$DQ_i = hA_{wg}(T_w - T) \tag{6}$$

Where h is the convective heat transfer coefficient, T_w is the wall temperature and T is the gas temperature.

Applying the equation (6) to the four heat exchanger cells, the heat transfer equation for the four heat exchanger cells as follows [20]:

$$DQ_h = h_hA_{wgh}(T_{wh} - T_h) \tag{7}$$

$$DQ_{r1} = h_{r1}A_{wgr1}(T_{wr1} - T_{r1}) \tag{8}$$

$$DQ_{r2} = h_{r2}A_{wgr2}(T_{wr2} - T_{r2}) \tag{9}$$

$$DQ_k = h_kA_{wgk}(T_{wk} - T_k) \tag{10}$$

The hot and cold heat exchanger wall temperature keept isothermally with the sink and source temperatures, respectively. The regenerator matrix temperature is given by [20]:

$$DT_{wr1} = DQ_{r1}/C_{mr} \tag{11}$$

$$DT_{wr2} = DQ_{r2}/C_{mr} \tag{12}$$

Where C_{mr} is heat capacity of each cell.

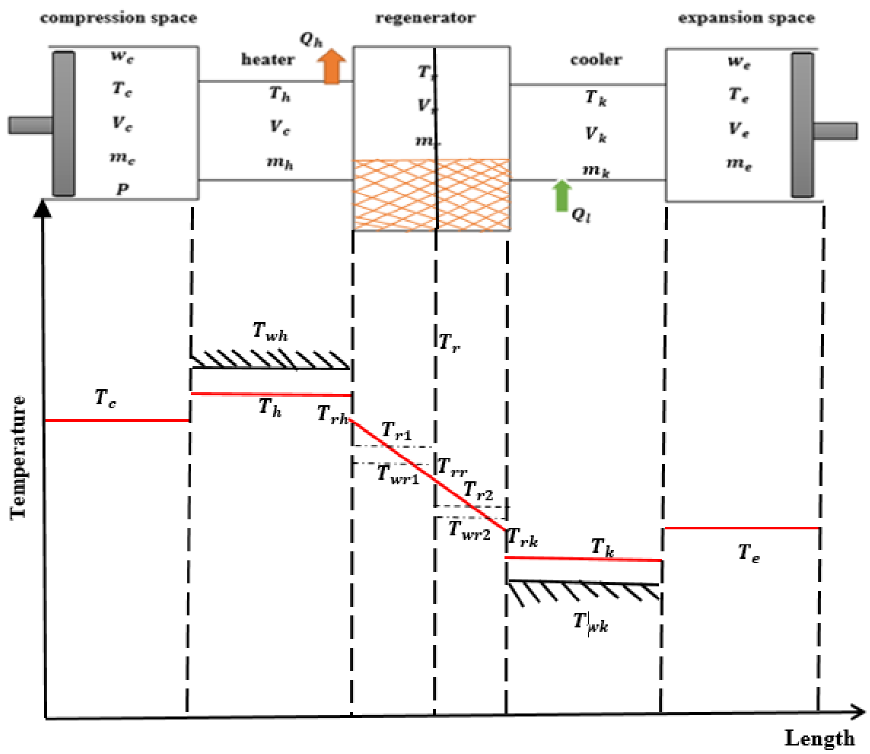

Fig. 5. Simple adiabatic model of Stirling heat pump

The energy equation for simple ideal analysis is the same with ideal analysis, except including the dissipation flow loss in the energy equation:

$$DQ - Diss + (dm_i C_p T_i - dm_o C_p T_o) = -dW + C_v d(mT) \tag{13}$$

Where flow dissipation $Diss$ = the internal heat generation which occurs when the frictional drag force forces the gas to flow [20] and is given by:

$$Diss_i = \Delta\rho\dot{m}/\rho \tag{14}$$

where ρ and $\dot{m}$ are the density and mass flow rate of the gas respectively.

4.3.3. Simple II Adiabatic model: The simple adiabatic analysis is that simple and deviates from the realistic model because it only considers the imperfect or non ideal heat exchanger and ignores the thermal and power loss see

Fig.6.Thus,to solve this problem, Abaelahi and Sayyaadi [4] studies on Stirling engine incorporates the shuttle effect and mass leakage loss in the ordinary differential equation of simple analysis.

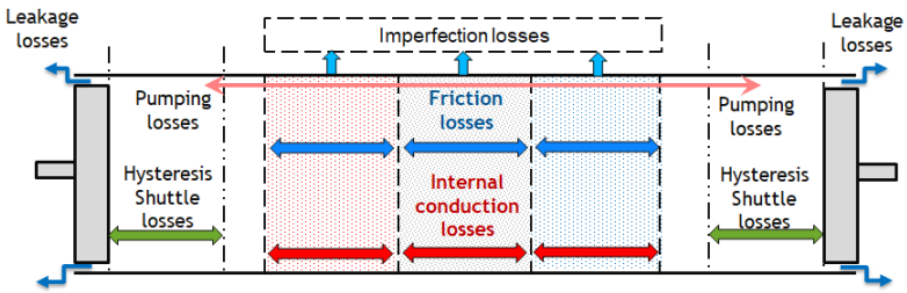

Fig. 6. various energy losses in the system [10]

The basic energy equation for simple II is given by [4]:

$$dQ + (m_i C_p T_i - m_o C_p T_o) - DQ_{shuttle} = -dW + C_v d(mT) \tag{15}$$

Where $DQ_{shuttle}$ is the heat loss resulting from the displacer's shuttle effect, which can be determined as [20]:

$$DQ_{shuttle} = (\pi S^2 k_g D_d)/8JL_d(T_C - T_e) \tag{16}$$

where $K_g, S, L_d, J, D_d, T_c, and, T_e$ are working fluid thermal conductivity , stroke of the displacer , length of displacer, gap between displacer, and wall of the cylinder , and displacer diameter, compression space working fluid temperature and expansion space working fluid temperature respectively. The working fluid's net mass is assumed to be constant in the adiabatic model and is calculated by [20]:

$$M = (m_c + m_h + m_r + m_k + m_e) - m_{leak} \tag{17}$$

Where m_{leak} is the leakage of working gas to the crankcase is given by [20]

$$m_{leak} = \Pi D(P + P_{buffer})/(4RT_g)(u_P J - J^3/6\mu(P - P_{buffer})/L) \tag{18}$$

Where $T_g, u, J, \mu, L, P, P_{buffer}, and, D$ are temperature of working fluid, piston linear velocity , piston-cylinder circular gap , viscosity of working fluid , length of piston , pressure working space , pressure in the buffer space and piston diameter respectively. The shuttle and gas leakage loss added to the basic differential equations in the simple adiabatic analysis, and obtains a new basic differential equations. The set equation considers the effects of pressure loss in heat exchangers, pressure loss due to the piston's finite speed, power loss from mechanical friction, conduction heat loss from the regenerator wall, and non-ideal heat transfer in the heat exchangers. The input power requirement $(W_{II}))$ for

Table 3. Simple II adiabatic set of equation [4].

Parameter	Equation
Pressure	$P = R(M - m_{leak})/(V_c/T_c + V_h/T_h + V_r/T_r + V_k/T_k + V/T_e)$
pressure variation	$dP = (-P\gamma(DV_c/T_{Ch} + DV_e/T_{ke}) + \gamma R(DQ_{shut})/C_p((T_{Ch} - T_{ke})/(T_{Ch}T_{ke})) - 2\gamma R Dm_{leak})/(V_c/T_{ch} + \gamma(V_h/T_h + V_r/T_r + V_k/T_k) + V_e/T_{ke})$
Mass	$m_i = (PV_i)/RT_i$,where, i =c,h,r,k,e
Change of mass	$Dm_i = (m_i DP)/P = (DP_i/R)(V_i/T_i)$,where, i=h,r,k
	$Dm_c = (PDV_c + (V_c DP)/\gamma)/(RT_{ch}) + (DQ_{shut})/(C_p T_{ch}) + Dm_{leak}$
	$Dm_e = (PDV_e + (V_e DP)/\gamma)/(RT_{he}) - (DQ_{shut})/(C_p T_{he})$.
Mass flow	$\dot{m}_{ch} = (Dm_c - DQ_{shut}/(C_p T_{ch}) - Dm_{leak}$
	$\dot{m}_{hr} = \dot{m}_{hr} - Dm_h, \dot{m}_{rk} = \dot{m}_{ke} + Dm_K$
	$\dot{m}_{ke} = D\dot{m}_e - (DQ_{shut)/(C_p T_{kc})}$
Conditional temperature	If $\dot{m}_{ch} > 0, then T_{ch} = T_c, else, T_{ch} = T_h$
	$\dot{m}_{ke} > 0, then T_{ke} = T_k, else T_{ke} = T_e$
Temperature variation	$dT_e = T_e(dP/P) + (dV_e/V_e) - (dm_e/m_e)$
	$dT_c = T_c(dP/P) + (dV_c/V_c) - (dm_c/m_c)$
Energy power	$DQ_{h,IIl} = (V_h DPC_v)/R - C_p(T_{ch}\dot{m}_{ch} - T_{hr}\dot{m}_{hr})$
	$DQ_{r,IIl} = (V_r DPC_v)/R - C_p(T_{hr}\dot{m}_{hr} - T_{rk}\dot{m}_{rk})$
	$DQ_{k,II} = (V_k DPC_v)/R - C_p(T_{rk}\dot{m}_{rk} - T_{ke}\dot{m}_{ke})$
	$DW_{in,II} = -(DW_C + DW_{e)}$
	where $,DW_C = PDV_C, and, DW_e = PDV_e$

the heat pump is evaluated as the sum of actual work input in the simple analysis and all the losses and calculated as:

$$W_{II} = DW_{in} + W_{mec,f} + W_{FST} + \Delta P_{trott}(dV) \tag{19}$$

Where $W_{mech,f}, P_{trott}(dV), and W_{FST}$ are work loss by mechanical friction , work loss by pressure loss in the heat exchanger ,and work loss by finite speed of the piston respectively.

The heat losses cause the heater to reject less heat and the heating power decreases due to power losses. Then, the heating power produced from the heat pump system Q_{II} is calculated based on difference between simple adiabatic analysis heating power and the heat power losses from the system [4]:

$$Q_{a,II} = DQ_{in} - (DQ_{shutt} + Q_{rl} + Q_{cond}) \tag{20}$$

Where $Q_{rl}, Q_{cond}, and Q_{shutt}$ are regenerator imperfection loss, conduction loss, and shuttle loss, respectively.

$$COP_{hp} = Q_{II}/W_{II} \tag{21}$$

4.3.3. Combined Adiabatic Finite speed (CAF) model: This approach relies on the combination of finite speed thermodynamics and adiabatic analysis [11].This model incorporates pressure drop in the the three heat exchangers(heater, regenerator, and cooler) into the simple adiabatic analysis of Urieli, the impact of the piston's finite speed mechanical rubbing between the cylinder and piston. The influence of finite speed of the piston and mechanical rubbing incorporated in to the equation of simple adiabatic analysis equation [11]:

$$W_{a,CAF} = DW_{in} + W_{mec,f} + W_{FST} + \Delta P_{trott}(dV) \tag{22}$$

$$\Delta P_{trott} = \Delta P_h + \Delta P_r + \Delta p_k$$

The performance of the heat pump is calculated based on the equation (21) by including the mechanical rubbing between the cylinder and piston and finite speed loss in the actual work input, and it does not include the shuttle loss and leakage in the ordinary differential equation of (20) and the working flow diagram of this model is shown in Fig 7.

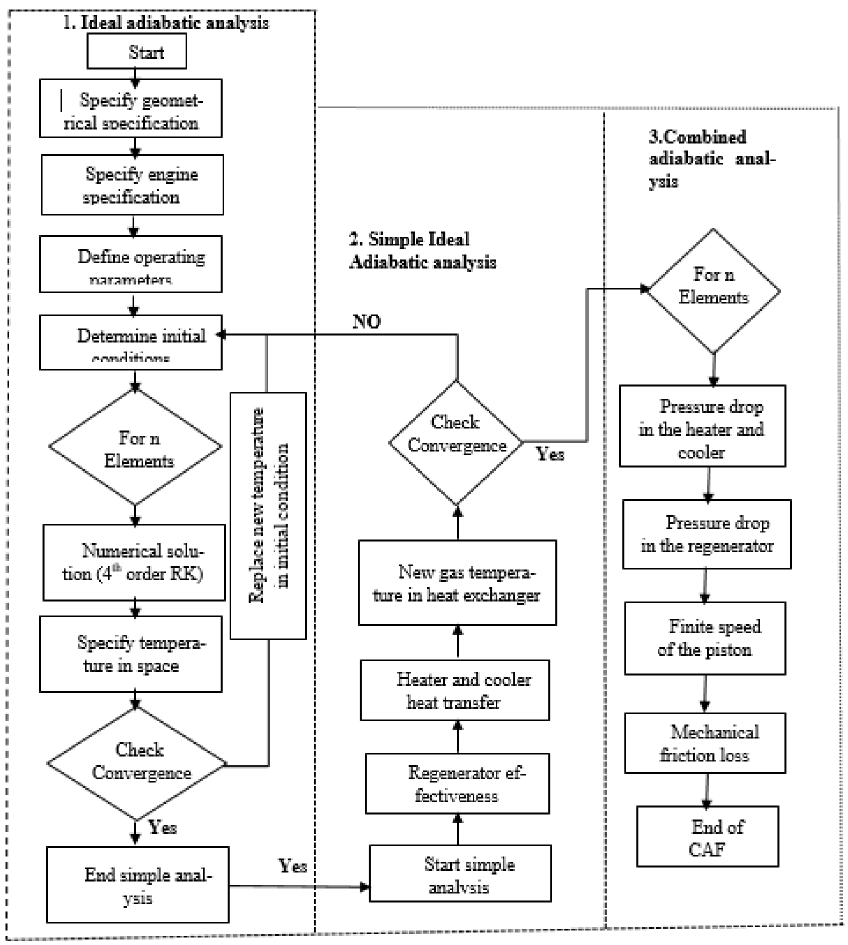

Fig. 7. Working flow diagram of CAFS mo[11]

4.4. Polytropic analysis: On the basis of the modification of the adiabatic analysis model, the polytropic analysis was proposed and initially Babaelahi and Sayyaadi [5] recommended the polytropic analysis of Stirling engine with various loss mechanisms (PSVL). In the adiabatic analysis, the compression and exanision process is assumed to be adiabatic but in the polytropic analysis,the compression and expansion space is replaced by polytropic process. In this analysis the basic differential equation of adiabatic analysis adjusted to include the polytropic compression and expansion process in order to forecast the performance of Stirling heat pump.

The entire engine's volume is divided into five cells similar to the adiabatic analysis that is compression space(c), heater (h), regenerator(r), cooler (k) and expansion space (e) and the four interfaces that is compression space/heater(ch), heater/regenerator(hr),regenerator/cooler(rk) and cooler/expansion space(ke) see in Fig.4. The basic differential equation changed to include the effect of shuttle loss and polytropic heat transfer given by [2]

$$DQ - DQ_{Polytropic} - DQ_{Shuttle} + (dm_i C_p T_i - dm_o C_p T_o) = -dW + C_v d(mT) \tag{23}$$

where $D_{Polytropic}$ is the polytropic heat loss in the compression , and expansion space and is given by [22].

$$DQ_{polytropic} = C_n(T_a - T)dm - mC_n dT \tag{24}$$

Where T and T_a are the section temperature and ambient temperature, correspondingly. C_n stands is the specific heat capacity of the polytropic and is given by [16]:

$$C_n = C_v(\gamma - k/n - 1) \tag{25}$$

where n is the polytropic index and is obtained by [5]

$$n = -Vdp/pdV : \tag{26}$$

4.4.1.Polytropic with Various Loss (PSVL):This modeling is based on the polytropic expanision and compression space of the engine.The following losses are considered in this thermodynamics model to enhance its accuracy of the analysis:

- Impact of mass leakage from the working space to buffer space,
- Heat loss(shuttle heat transfer effect),
- Non ideal regenerator,
- Heat exchangers pressure drop,
- Mechanical friction,
- Heat exchangers longitudinal heat conduction loss.

The governing differential equations of this model is same as equation (23).
The resulting model was applied to the GPU-3 Stirling engine prototype, and the results were compared to those of other developed models and experimental findings. It was found that 24.44% thermal efficiency and helium gas as a refrigerant at heater temperature of 977K and cooler temperatures of 286K [5].

4.4.2.Polytropic Finite Speed Thermodynamics (PFST):This model has been developed through an integration of finite speed thermodynamics and polytropic analysis. The work input for this model includes:

- The pressure drop in the heat exchangers,
- Mechanical friction,
- Finite speed of the piston.

The governing differential equation of this model is a modification of equation(23) which don,t include the shuttele heatloss:

$$DQ - D_{Polytropic} + (dm_i C_p T_i - dm_o C_p T_o) = -dW + C_v d(mT) \tag{27}$$

The resulting model was applied to the GPU-3 Stirling engine prototype, and the results were compared to those of other developed models and experimental findings. It was found that 23.3% thermal efficiency and helium gas as a refrigerant at heater temperature of 977K and cooler temperatures of 286K[12].
4.4.2.Compressive Polytropic model (CPMS):This modeling technique is a modification of polytropic with Various Loss (PSVL) analysis.The heater and cooler is considered as non isothermal instead of isothermal process see Fig 8.

4.5.New Non ideal second order thermal model:An enhanced non ideal thermodynamic model of the Stirling heat pump with a variety of losses has been created in order to improve the previous second order models used for the thermodynamic analysis of Stirling devices.In this model, the basic differential equations for the heater and parts of the devices have been coupled with the shuttle heat loss[20]. In addition, the mass leakage into the crankcase and the cooler part were added into the conservation of mass equations [39],by taking into account the mass leakages across the system's boundaries.Generally, the following losses are considered in this non-ideal second order model [19, 8]:

- Dissipation losses,
- Conduction losses,
- Buffer space heat leakage,
- Non ideal heat exchanger losses,
- Pressure drop work loss,
- Loss of frictional work,
- Gas spring hysteresis Work loss.

Therefore, governing differential equation for this model is given by [19]:

$$\text{DQ-DQ}_{shuttle} - DQ_{disp} - DQ_{cond} - DQ_{r,nonideal} - DQ_{leak} + (dm_i C_p T_i - dm_o C_p T_o)$$
$$= -DW_{ideal} - DW_{mech,frc} - DW_{FST} - DW_{hys} - DW_{p,drop} + C_v d(mT) \tag{28}$$

The resulting model was applied to the GPU-3 Stirling engine prototype, and the results were compared to those of other developed models and experimental findings. It was found that 20.4% thermal efficiency and helium gas as a refrigerant at heater temperatures of 977K and cooler temperatures of 288K [19]. .

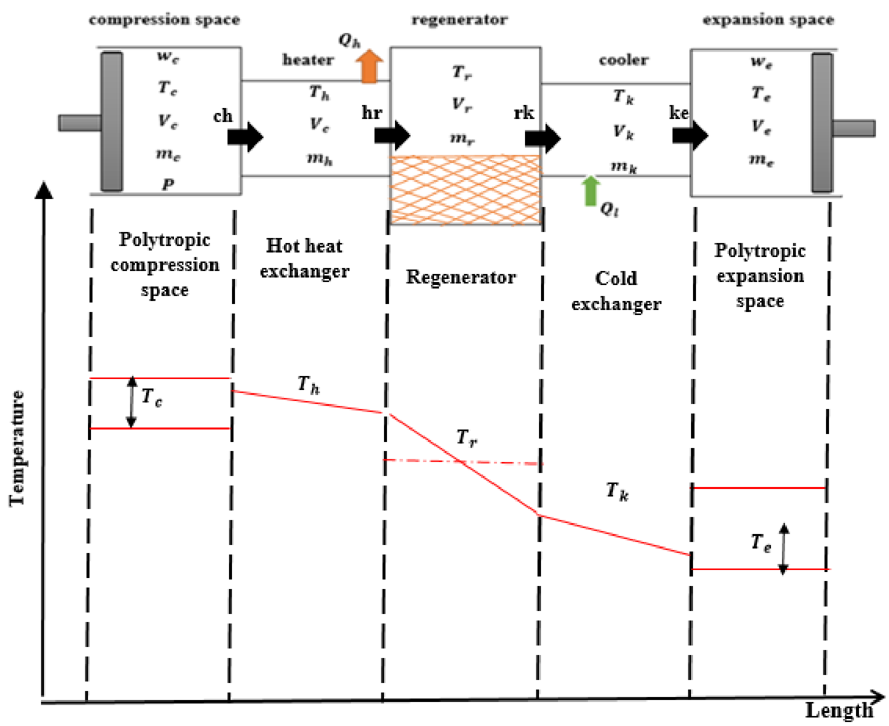

Fig. 8. CPMS model of Stirling heat pump

4.4 Third and Fourth order model

Based on the mass, momentum, and energy balances of the various control volumes, third order modeling involves developing partial differential equations which govern the functioning of the devices.Feurer [6] had previously developed a Stirling engine using the third-order concept.To create an effective regenerator for the Stirling engine, Anderson [3] extensively examined the impact of regenerator matrix temperature oscillations using the third order model.

Toghyani [18]developed a third order analysis to evaluate the GPU-3 Stirling's suitable heat source temperature, frequency, engine stroke, and effective mean pressure in to obtain the best output power and performance.However, this model did not produce results that were superior compared to those of the adiabatic models already in use due to simplification of the process in order to increase the computational speed.

The fourth order analysis use three dimensional computational fluid dynamics to solving complex flow equations at each node of the created mesh of the modeling. [19].This model is very useful for analyzing the flow field and heat transfer of the working fluid inside the heat pump.Ahmed et al. [1] conducted an extensive investigation to assess the energy effectiveness of the different types of Stirling engines .

Marek and Jan [13] study shows that in three-dimensional computational fluid dynamics modeling, ,a dynamic mesh was used to map the Stirling engine's different volumes.The researchers compared the adiabatic model findings with the results produced using a fourth order modeling strategy. According to their findings, second-order analysis is better for design and optimization due to the shorter computational time.

5 Model Validation

The thermal methods have been studied with a standard Stirling engine that has been implemented as a subject of investigation for several studies.A General Power Unit-3 (GPU-3) Stirling engine is used in order to validate each thermodynamic analysis of a Stirling engine, heat pump, and refrigerator. Comparisons between the thermal efficiency of Stirling engines with different thermal approaches are illustrated through Fig 9 .
The efficiency of various second-order thermodynamic modeling techniques of the Stirling engine is shown in Fig. 9. Therefore, it can be concluded that the non-ideal second order model is more accurate than all current second-order thermal models to forecast the performance of the GPU-3 engine at the design average effective pressure of 4.14 MPa, heater temperature of 977 K, and cooler temperature of 288 K.

6 Conclusion

In this paper, the thermodynamic analysis of the Stirling heat pump, which is adapted from the Stirling engine model, are extensively reviewed and compared with the experimental value of GPU-3 Stirling engine. Those models start from the simple zero-order model up to the fourth-order model.The zero order models are empirical approaches that use experimental parameters for predicting the performance of the engine, whereas the first-order models concentrate on closed-form mathematical models.On the other hand, the second, third, and fourth-order models are numerical models with increasing degrees of accuracy.

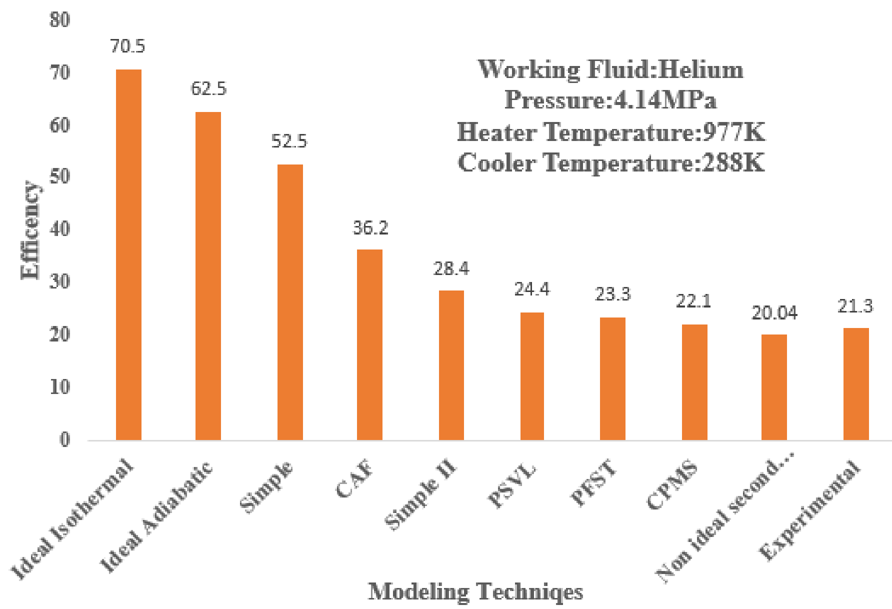

Fig. 9. Comparison of various second order engine models

The distribution of losses and the flow fields in the engine had been enhanced by the fourth-order model compared to the other numerical models; however, the outcomes of this method were not considerably superior to those of second order models.Furthermore, compared to second order numerical analyses, fourth order studies need a lot of computing time.

As a result, the review highlights considerable value in the second-order model, which is better than the other model for thermal analysis of Stirling heat pumps in terms of predicting precise and consistent findings in less computational time. A comparison between the thermal efficiency of each second-order model and the experimental approaches is illustrated.The result of this comparison showed that the non-ideal second order with additional loss effects is better than the other methods.

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