Adaptive Neural Network-Based Higher-Order Sliding Mode Control for Floating Offshore Wind Turbines

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Abstract— **This paper introduces a novel adaptive feedback control approach for disturbed chains of integrators with smooth disturbances with unknown upper bound. The proposed approach combines adaptive neural network with higher-order sliding mode control to achieve the convergence of system states towards a vicinity of the origin. Notably, this approach does not rely on any prior information about the disturbance. The adaptive neural network term compensates the disturbance with an error, while the higher-order sliding mode control term effectively addresses this error and ensures the stabilization of the system state. Compared with existing neural network-based sliding mode control approaches, our proposed method does not require reducing the system order and utilizes only two terms for control. These characteristics contribute to its simplicity and lead to improved closed-loop performance. The effectiveness of the adaptive feedback control is specifically assessed for semi-submersible floating offshore wind turbines operating above rated speed. Simulation results demonstrate superior performance in rotor speed regulation and platform pitch reduction compared to the baseline gain-scheduling proportional integral controller.**

I. INTRODUCTION

Sliding Mode control has emerged as an effective technique for handling systems with matched disturbances [1]. Indeed, it has proven its high efficiency due to its insensitivity to the disturbances and its ability to guarantee the finite-time convergence. The first-order sliding mode control requires that the sliding variable has a relative degree equal to one. To address relative degree equal to r, Higher Order Sliding Mode Controllers (HOSMCs) have been introduced [2], [3], [4], [5], [6]. However, implementing HOSMCs requires knowledge of the upper bound of the rth derivatives, which is often unknown or subject to variation in practical systems.

To tackle this challenge, two families of approaches can be employed. The first approach involves utilizing adaptive schemes that dynamically adjust the control gain of the HOSMC in order to be as small as possible whereas sufficient to counteract the disturbances and ensures the stabilisation of the system state [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19].

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The second approach entails integrating sliding mode control with other techniques capable of approximating the disturbance, such as neural networks [20] or fuzzy logic systems [21]. In this context, the Radial Basis Function Neural Networks (RBFNN) algorithm has demonstrated exceptional capability in approximating nonlinear functions, making it a compelling choice to address this problem [22]. Indeed, RBFNN has been effectively combined with various nonlinear control techniques, including backstepping [23], Model Predictive Control (MPC) [24], and sliding mode control [25].

In the case of sliding mode control, the approach consists in utilizing the RBFNN for disturbance estimation, while the sliding mode control handles estimation errors and ensures the convergence of the system state to a vicinity of the origin [26]. In [27], a combination of the RBFNN with the Super-twisting sliding mode control has been proposed. This approach is suitable for first-order disturbed systems but may not be generalized for disturbed chains of integrators of order r.

To address this limitation, researchers have proposed various approaches in the literature, which can be classified into two main categories: (a)

1) Creating a sliding surface [26], [28], [29].
2) Utilizing the concept of integral sliding mode control [30].

Approach (a) suggests creating an auxiliary sliding surface to reduce the system's order to one on the sliding surface. Subsequently, first-order sliding mode control is applied in conjunction with the RBFNN to ensure the convergence of the system state towards the origin vicinity. However, the drawback of this approach is that it does not allow for the utilization of a sliding variable with arbitrary relative degree.

To overcome this drawback, approach (b) proposes employing the concept of integral sliding mode control. Integral sliding mode control combines first-order sliding mode control with a nominal control to robustify the pre-designed nominal control while maintaining its dimension. The use of the RBFNN is essential to compensate the disturbance. By combining the RBFNN and integral sliding mode control, the convergence of the system state towards a vicinity of the origin can be ensured. However, it is worth noting that this approach introduces three control terms, which can pose challenges during real-time implementation.

This paper presents a novel approach to address the mentioned limitations by utilizing a family of homogeneous HOSMCs in combination with RBFNN. The proposed approach offers a solution for disturbed chains of integrators of order r while ensuring system state convergence to a vicinity of the origin. Remarkably, this is achieved using only two control terms, without reducing the system's order. As a result, the proposed approach offers simplicity and ease of implementation in real-world applications.

This paper is organized as follows. In Section II, the problem formulation with some preliminary results is given. In Section III, the proposed approach is presented. In Section IV, the proposed approach is applied to FOWT and its performance are illustrated through OpenFAST simulation. Finally, some conclusions are drawn in section V.

II. Problem formulation

Consider the case of disturbed chain of integrators of order r given by

$$\begin{cases} \dot{\sigma}_i = & \sigma_{i+1},\ i = 1, \cdots, r-1, \\ \dot{\sigma}_r = & u(t) + \phi(\sigma), \end{cases} \tag{1}$$

where $\sigma = [\sigma_1\ \sigma_2\ \cdots\ \sigma_r]^\top$ is the system state, $u(t) \in \mathbb{R}$ is the control input and $\phi(\sigma) \in \mathbb{R}$ is an unknown smooth function.

The control objective is to design an adaptive feedback control law that can drive σ asymptotically to zero with non-overestimated control and without requiring any information on the disturbance.

A. Preliminairis

The proposed adaptive control relies on the following two algorithms: the discontinuous higher order sliding mode control algorithm and the RBFNN algorithm. These two algorithms will be summarized in this subsection. For simplicity, the notation $\lfloor x \rceil^\gamma$ is used to represent $|x|^\gamma \operatorname{sign}(x)$, e.g. $\lfloor x \rceil^{\frac{1}{2}} = |x|^{\frac{1}{2}} \operatorname{sign}(x)$.

Proposition 1: [31] Consider system (1). Let us suppose that there exists a feedback law $u = u_0(\sigma)$ that ensures the convergence of σ to the origin in a finite-time and the following conditions hold true:

(i) there exists a continuous positive definite function $V_0(\sigma)$ such that there exists $c > 0$ and $\alpha \in (0, 1)$ for which the time derivative of $V(\sigma)$ satisfies

$$\dot{V}_0(\sigma) \le -cV_0^\alpha(\sigma); \tag{2}$$

(ii) the function $\sigma \mapsto u_0(\sigma)\frac{\partial V_0(\sigma)}{\partial \sigma_r}$ is non positive over $\mathbb{R}^r$ and, for every non zero $\sigma \in \mathbb{R}^r$ verifying $u_0(\sigma) = 0$, one has $\frac{\partial V_0(\sigma)}{\partial \sigma_r} = 0$. As a consequence function $\sigma \mapsto \operatorname{sgn}(u_0(\sigma))\frac{\partial V_0(\sigma)}{\partial \sigma_r}$ is well-defined over $\mathbb{R}^r \setminus \{0\}$ and non positive.

Remark 1: Among different controllers that can fulfill Proposition 1, Hong's controller [32] can be used. This controller is defined as follows:
Let $\kappa < 0$ and $l_1, \cdots, l_r$ positive real numbers. Define $u_0 = v_r$ for $i = 0, ..., r-1$:

$$v_0 = 0,\ v_{i+1} = -l_{i+1}\lfloor \lfloor \sigma_{i+1} \rceil^{\beta_i} - \lfloor v_i \rceil^{\beta_i} \rceil^{(\alpha_{i+1}/\beta_i)}, \tag{3}$$

where $p_i = 1 + (i-1)\kappa,\ \beta_0 = p_2,\ (\beta_i + 1)p_{i+1} = \beta_0 + 1 > 0$ and $\alpha_i = \frac{p_{i+1}}{p_i}$.

Now, let $\psi_r(\sigma_1, \cdots, \sigma_r) = \lfloor \sigma_r \rceil^{\beta_{r-1}} - \lfloor v_{r-1} \rceil^{\beta_{r-1}}$. Then according to [31], if $\kappa = -1/r$, one can obtain the following homogeneous HOSMC

$$u_0 = -l_r \operatorname{sgn}\Big(\psi_r(\sigma_1, \cdots, \sigma_r)\Big). \tag{4}$$

Here are HOSMC examples for $r = 1, \ldots, 4$:

1) $u_0 = -l_1 \operatorname{sgn}(\sigma_1)$.
2) $u_0 = -l_2 \operatorname{sgn}(\lfloor \sigma_2 \rceil^2 + l_1^2 \lfloor \sigma_1 \rceil)$.
3) $u_0 = -l_3 \operatorname{sgn}\big(\lfloor \sigma_3 \rceil^4 + l_2^4 \big\lfloor \lfloor \sigma_2 \rceil^{\frac{3}{2}} + l_1^{\frac{3}{2}} \lfloor \sigma_1 . \rceil \big\rceil^{\frac{4}{3}}\big)$
4) $u_0 = -l_4 \operatorname{sgn}\Big(\lfloor \sigma_4 \rceil^6 + l_3^6 \Big\lfloor \lfloor \sigma_3 \rceil^{\frac{5}{2}} + l_2^{\frac{5}{2}} \big\lfloor \lfloor \sigma_2 \rceil^{\frac{4}{3}} + l_1^{\frac{4}{3}} \lfloor \sigma_1 \rceil \big\rceil^{\frac{15}{16}} \Big\rceil^{\frac{12}{15}}\Big)$.

Proposition 2: [33], [34] Consider a continuous function $f(\sigma)$ over a compact set $\Omega_\sigma \subset \mathbb{R}^r$. Consider the following RBFNN given by

$$f_{nn}(\sigma) = w^\top h(\sigma), \tag{5}$$

where $\sigma \in \Omega_\sigma$ is the input, $w = [w_1\ w_2\ \cdots\ w_m]^\top \in \mathbb{R}^m$ is the weight vector, m is the node number, and $h(\sigma) = [h_1(\sigma)\ h_2(\sigma)\ \cdots\ h_m(\sigma)]^\top$ is the basis function vector, where $h_i(\sigma)$ satisfies

$$h_i(\sigma) = \frac{1}{\sqrt{2\pi} r_i} \exp\Big(-\frac{(\sigma - c_i)^\top (\sigma - c_i)}{2r_i^2}\Big),\ \ i = 1, 2, \cdots, m \tag{6}$$

with c_i is the center of the receptive field, and r_i is the width of the Gaussian function.
Then, the RBFNN given in (5) can approximate the continuous function $f(\sigma)$ over the compact set $\Omega_\sigma \subset \mathbb{R}^r$ to any arbitrary accuracy as

$$f(\sigma) = w^{*\top} h(\sigma) + \varepsilon(\sigma), \quad \forall \sigma \in \Omega_z, \tag{7}$$

where w^* is the ideal constant weights, and $\varepsilon(\sigma)$ is the approximation error.

Assumption 1: [33] There exist ideal constant weights w^* such that $|\varepsilon(\sigma)| \le \varepsilon^*$ with unknown constant $\varepsilon^* > 0$ for all $\sigma \in \Omega_\sigma$.

III. Adaptive Neural-networks-based Higher order sliding mode control design

To implement the proposed algorithm, the control input is chosen as follows

$$u = u_0 - u_{NN}, \tag{8}$$

where u_0 is the HOSMC introduced in [31] and u_{NN} is the adaptive RBFNN given by

$$u_{NN} = w^\top h(\sigma), \tag{9}$$

with

$$\frac{dw}{dt} \triangleq \eta \frac{\partial V}{\partial \sigma_r} h(\sigma). \tag{10}$$

Here, η is an arbitrary positive constant.

Theorem 1: Consider system (1) with smooth nonlinear function $\phi(\sigma)$. Let the control input u is chosen as (8)-(9). Then, there exists a small positive constant $l_r > \varepsilon^*$ that ensures the asymptotic convergence of the system state σ to the origin.

Proof: Substituting (8) and (9) into system (1) yields

$$\begin{cases}\dot{\sigma}_i = & \sigma_{i+1},\ i=1,\cdots,r-1,\\ \dot{\sigma}_r = & -l_r\, sign\Big(\psi_r(\sigma)\Big) - w^\top h(\sigma) + \phi(\sigma).\end{cases} \tag{11}$$

Lemma 1: [35] The RBFNN $w^\top h(\sigma)$ can be applied to approximate $\phi(\sigma)$, where $\sigma \in \Omega_\sigma \subset \mathbb{R}$. In this case, $\phi(\sigma)$ can be expressed as

$$\phi(\sigma,t) = w^{*\top} h(\sigma,t) + \varepsilon(\sigma), \tag{12}$$

where w^* denotes the ideal constant weights, and $|\varepsilon(\sigma)| \leq \varepsilon^*$ is the approximation error with constant $\varepsilon^* > 0$.

Define $\Delta w = w - w^*$, then system (11) can be rewritten as

$$\begin{cases}\dot{\sigma}_i = & \sigma_{i+1},\ i=1,\cdots,r-1,\\ \dot{\sigma}_r = & -l_r\, sign\Big(\psi_r(\sigma)\Big) - \Delta w^\top h(\sigma) + \varepsilon(\sigma).\end{cases} \tag{13}$$

Define the Lyapunov function as

$$V = V_0 + \frac{1}{2\eta}||\Delta w||^2. \tag{14}$$

The time derivative of V along the trajectory of (13) is

$$\dot{V} = \dot{V}_0 + \frac{1}{\eta}\Delta w^\top \frac{d\Delta w}{dt}. \tag{15}$$

this yields

$$\begin{aligned}\dot{V} =& \frac{\partial V}{\partial \sigma_1}\sigma_2 + ... + \frac{\partial V}{\partial \sigma_r}\left(-l_r\,\mathrm{sgn}(\psi_r) - \Delta w^\top h(\sigma) + \varepsilon(\sigma)\right)\\ &+ \frac{1}{\eta}\Delta w^\top \frac{d\Delta w}{dt} \leq -cV_0^\alpha - |\frac{\partial V}{\partial \sigma_r}|(l_r - \varepsilon^*)\\ &+ \Delta w^\top\Big(\frac{1}{\eta}\frac{d\Delta w}{dt} - \frac{\partial V}{\partial \sigma_r}h(\sigma)\Big).\end{aligned} \tag{16}$$

Substituting (10) into (16), and suppose that there exists a positive constant $l_r > \varepsilon^*$, then one can obtain the following inequality

$$\dot{V} \leq -cV_0^\alpha, \tag{17}$$

which ensures the asymptotic convergence of the state variables σ to zero. ■

Remark 2: It is important to highlight that in this paper, the RBFNN is specifically utilized for disturbance estimation. Nevertheless, it is worth noting that the theoretical framework presented in this study can be extended to accommodate alternative algorithms, such as the Hermite neural network or other viable options.

Remark 3: In real-world applications, it is possible for the initial state variables to be located outside the compect set Ω_σ. In such cases, one can employ a barrier function-based adaptive sliding mode control approach to drive the states towards a neighborhood of zero [31]. Once the states approach this vicinity, the algorithm seamlessly transitions to the proposed adaptive neural network-based higher-order sliding mode control strategy.

IV. Application to Floating Offshore Wind Turbine

In this section, the proposed adaptive control method is applied to regulate the operation of a FOWT above its rated speed. This approach is particularly well-suited to FOWT systems due to their high-order nonlinear nature, uncertainties, and susceptibility to significant wind and wave disturbances. The FOWT system studied in this paper is the NREL OC4-DeepCwind 5 MW semi-submersible FOWT [36]. The nonlinear Control-Oriented Model (COM) developed by Homer in [37] specifically for this FOWT serves as the foundation for designing the proposed adaptive controller law.

A. FOWT Modeling

FOWT models comprise three components: the mechanical structure model, the wind turbine model, and the drive-train model. In this paper, the semi-submersible FOWT under consideration, consisting of the floating platform and the wind turbine, is treated as a single rigid body. The mechanical structure is described using Newton-Euler's equation of motion, while the interaction between the wind and the wind turbine is represented by a single thrust force F_A:

$$F_A = \frac{1}{2}\rho_a \pi R_r^2 C_t(\lambda,\beta)\,\|v_n\|_2\, v_n \tag{18}$$

where $||\cdot||_2$ is the Euclidean norm of a vector, ρ_a is the air density, R_r is the effective rotor radius, v_n is the equivalent velocity vector, C_t is the thrust coefficient, a highly nonlinear function in terms of the tip speed ratio λ and the blade pitch angle β.

For the drive-train dynamic model, a one-mass rigid shaft model is adopted in (19) depicting the relationship between rotor speed ω_r, generator torque T_g, gearbox ratio η_g, low-speed shaft equivalent inertia J_l, and aerodynamic power P_A:

$$\dot{\omega}_r = \frac{1}{J_l}(\frac{P_A}{\omega_r} - n_g T_g), \tag{19}$$

$$P_A = \frac{1}{2}\rho_a \pi R_r^2 C_p(\lambda,\beta)\,\|v_n\|_2^3 \tag{20}$$

where, the aerodynamic power is influenced by the nonlinear power coefficient C_p, which varies with λ and β.

The states vector given in (21) comprises the position vector $x_m = [x_m, y_m, z_m]$ and the orientation vector $\theta = [\theta_x, \theta_y, \theta_z]$, along with their respective time derivatives, the rotor azimuth angle θ_r, and the rotor speed ω_r.

$$x = [x_m, \theta, \theta_r, \dot{x}_m, \dot{\theta}, \omega_r]^\top \tag{21}$$

Considering the generator torque as fixed, the control input u corresponds to the blade pitch angle β:

$$u = \beta \tag{22}$$

Based on (18)-(20) and using the nonlinear COM proposed in [37], the model of the FOWT system can be represented as follows:

$$\begin{bmatrix} \ddot{x}_m \\ \ddot{\theta} \\ \dot{\omega}_r \end{bmatrix} = \begin{bmatrix} (m_g I_3 + m_a)^{-1} (F_A) \\ R(\theta) I_\theta^{-1} R(\theta)^\top (T_A + T_B + T_C + T_D) \\ \frac{1}{J_l}\left(\frac{P_A}{\omega_r} - n_g T_g\right) \end{bmatrix} \\ = f(x, u, v, w) \tag{23}$$

where $f = (x, u, v, w)$ describes the nonlinear function vector governing the equations of motion, with v and w, the disturbances caused by the wind and wave, respectively. m_g denotes the total mass of the FOWT, while m_a represents the hydrodynamic added mass vector. T_A, T_B, T_C, T_D are the aerodynamic torque vector, the buoyancy torque vector, the mooring line torque vector, the hydrodynamic drag and inertial torque vector, respectively. I_θ is the considered inertia tensor of the FOWT and $R(\theta)$ is the simplified rotation matrix.

The coupling between the state variables and the control input in (23) poses a significant challenge for designing the adaptive nonlinear controller. Overcoming this mathematical complexity requires reformulating and adapting the system equations. Specifically, the nonlinear coefficients $C_t(\lambda, \beta)$ and $C_p(\lambda, \beta)$, typically obtained from lookup tables, pose difficulty in isolating the command β. To address this limitation, both coefficients are expressed as polynomial functions of the form:

$$C_x(\lambda, \beta) = \left(\sum_{i=0}^{4} c_{0i}^{C_x} \lambda^i\right) \beta + \sum_{i=0}^{5} c_{1i}^{C_x} \lambda^i + \Delta C_x \tag{24}$$

where C_x corresponds to either C_t or C_p, and ΔC_x represents the fitting error. The coefficients $c_{0i}^{C_x}$ and $c_{1i}^{C_x}$ are determined through polynomial regression using MATLAB's curve fitting toolbox. Consequently, the polynomial expressions for C_t and C_p are rewritten as:

$$C_t(\lambda, \beta) = g_{ct}\beta + f_{ct} \tag{25}$$

$$C_p(\lambda, \beta) = g_{cp}\beta + f_{cp} \tag{26}$$

where g_{ct}, g_{cp}, f_{ct}, and f_{cp} are polynomial functions of λ.

Substituting (25) and (26) into (18) and (20), respectively, and considering external disturbances, parametric uncertainties, and unmodeled dynamics, the first time derivative of $\dot{\theta}_y$ and ω_r can be expressed as:

$$\begin{aligned} \dot{\omega}_y &= \theta_{\ddot{y}} = g_y\beta + h_y = (g_{yn} + \Delta g_y)\beta + h_y = g_{yn}\beta + H_y, \\ \dot{\omega}_r &= \theta_{\ddot{r}} = g_r\beta + h_r = (g_{rn} + \Delta g_r)\beta + h_r = g_{rn}\beta + H_r \end{aligned} \tag{27}$$

where ω_y represents the platform pitch rate, g_y and g_r are nonlinear terms of ω_y and ω_r, respectively, g_{yn} and g_{rn} are the rated values of g_y and g_r, respectively, Δg_y and Δg_r are time-varying parametric uncertainties for g_y and g_r, respectively, h_y and h_r are the lumped disturbances, and H_y and H_r represent lumped uncertainties and external disturbances. They are expressed as follows:

$$\begin{cases} g_y = d_\theta g_{ct} \left\| v_n \right\|_2^2 (\theta_x^2 + \theta_y^2 + \theta_z^2 + 1) \\ h_y = d_\theta f_{ct} \left\| v_n \right\|_2^2 (\theta_x^2 + \theta_y^2 + \theta_z^2 + 1) + D_{\theta_y} \end{cases} \tag{28}$$

$$\begin{cases} g_r = \frac{\rho_a \pi R_r^2}{2 J_l \omega_r} g_{cp} \left\| v_n \right\|_2^3 \\ h_r = \frac{\rho_a \pi R_r^2}{2 J_l \omega_r} f_{cp} \left\| v_n \right\|_2^3 - \frac{n_g}{J_l} T_g \end{cases} \tag{29}$$

with the expression of D_{θ_y} and d_θ given as in [38].

Assumption 2: H_y and its time derivative $\dot{H}_y$ are bounded, satisfying $|H_y| \leq \rho_{y1}$ and $|\dot{H}_y| \leq \rho_{y2}$, where ρ_{y1} and ρ_{y2} are two positive constants.

Assumption 3: H_r and its time derivative $\dot{H}_r$ are bounded, satisfying $|H_r| \leq \rho_{r1}$ and $|\dot{H}_r| \leq \rho_{r2}$, where ρ_{r1} and ρ_{r2} are two positive constants.

The blade actuator dynamics have been incorporated into the nonlinear model of the FOWT by the following second-order differential equation:

$$\ddot{\beta} = \omega^2 \beta^* - 2\xi\omega_0\dot{\beta} - \omega_0^2\beta \tag{30}$$

where ω_0 and ξ are the natural pulsation and the damping rate of the actuator, respectively, and β^* is the blade pitch angle produced by the actuator. Thus, the state vector and the control input can be rewritten as follows:

$$x = [x_m, \theta, \theta_r, \dot{x_m}, \dot{\theta}, \omega_r, \beta]^\top, \quad u = \beta^* \tag{31}$$

Based on (27), (30), and (31), the model of the FOWT system expressed in (23) is rewritten as:

$$\begin{bmatrix} \ddot{x}_m \\ \ddot{\theta} \\ \dot{\omega}_r \\ \ddot{\beta} \end{bmatrix} = \begin{bmatrix} (m_g I_3 + m_a)^{-1} (F_A) \\ R(\theta) I_\theta^{-1} R(\theta)^\top (T_A + T_B + T_C + T_D) \\ \frac{1}{J_l}\left(\frac{P_A}{\omega_r} - n_g T_g\right) \\ -2\xi\omega_0\dot{\beta} - \omega_0^2\beta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_0^2 \end{bmatrix} \beta^* \tag{32}$$

B. Control Design

Above the rated speed, the primary control objectives involve regulating the rotor speed to its rated value $\omega_{rd} =$ 12.1 rpm and ensuring stability in platform pitching motion. This is expressed through the following tracking errors e_r and e_y:

$$e_r = \omega_r - \omega_{rd} \tag{33}$$

$$e_y = \omega_y - 0 = \omega_y \tag{34}$$

Considering the refined dynamics of the platform pitch rate ω_y and the rotor speed ω_r derived previously, the control input β influences both dynamics. A practical approach to address the under-actuated control issue—arising from the shared control input β in both the platform pitch rate and rotor speed dynamics—is to modify the reference rotor speed from a fixed value ω_{rd} to a linear function related to the platform pitch rate: $\omega_{rd}^* = \omega_{rd}(1 - k_y\omega_y)$, where ω_{rd}^* is the adjusted reference rotor speed and k_y is a positive constant determined empirically.

Thus, the primary control objective for the FOWT in Region III is to drive the composite tracking error e to zero:

$$e = \omega_r - \omega_{rd}^* = \omega_r - \omega_{rd}(1 - k_y\omega_y) = e_r + k_s e_y \tag{35}$$

where $e_r = \omega_r - \omega_{rd}$, $e_y = \omega_y - 0$ and $k_s = k_y\omega_{rd}$, a positive scalar constant with units of $\mathrm{rpm} \cdot \mathrm{s/deg}$.

The first time derivative of e can be expressed as:

$$\dot{e} = \dot{e_r} + k_s\dot{e_y} = \dot{\omega_r} + k_s\dot{\omega_y} \tag{36}$$

Substituting (27) into (36), the first time derivative is expressed as:

$$\dot{e} = (g_{rn}\beta + H_r) + k_s(g_{yn}\beta + H_y) = g_{sn}\beta + H_s \quad (37)$$

where $g_{sn} = g_{rn} + k_s g_{yn}$ and $H_s = H_r + k_s H_y$, with H_s the lumped uncertainties and external disturbances.

Assumption 4: According to Assumption 2 and Assumption 3, $|H_s| \leq \rho_{s1}$ and $|\dot{H}_s| \leq \rho_{s2}$, with $\rho_{s1} = \rho_{r1} + k_s\rho_{y1}$ and $\rho_{s2} = \rho_{r2} + k_s\rho_{y2}$.

The control objective is for e converge to the origin in finite time in the presence of lumped disturbances.

Based on (32) and defining the sliding variable as $\sigma_1 = e$, the following third-order system is considered:

$$\begin{cases} \dot{\sigma_1} = \sigma_2 \\ \dot{\sigma_2} = \sigma_3 \\ \dot{\sigma_3} = g_{sn}\omega_0^2\beta^* + g_{sn}\omega_0^2(\frac{-2\xi}{\omega_0}\dot{\beta} - \beta) + \ddot{H}_s = g_{sn}\omega_0^2\beta^* + \phi \end{cases} \quad (38)$$

where ϕ represents the unknown disturbance.

The control law $u(t)$ is designed according to (8) where u_0 is the HOSMC chosen for $r = 3$. The adaptive RBFNN term u_{NN} satisfies (9)-(10), with $\frac{\partial V}{\partial \sigma_3}$ given by:

$$\frac{\partial V}{\partial \sigma_3} = \lfloor \sigma_3 \rceil^4 + l_2^4 \lfloor \lfloor \sigma_2 \rceil^{\frac{3}{2}} + l_1^{\frac{3}{2}} \lfloor \sigma_1 \rceil \rceil^{\frac{4}{3}}. \quad (39)$$

Here add expression V with Vo from Hong(2002) Then partial derivative of Vo/sigma3

C. Simulation Results

The effectiveness of the proposed adaptive controller for the presented third-order system (38), is validated through simulation studies using the Matlab/Simulink interface. The high-fidelity modeling software OpenFAST is employed with the 5 MW semi-submersible NREL OC4-DeepCwind model. The disturbances, comprising the wind and waves, are given in Fig.1 , where the mean wind speed is 18 m/s with a turbulence rate of 15%, and the wave height is close to 5.18 m with a peak period of 12 s.

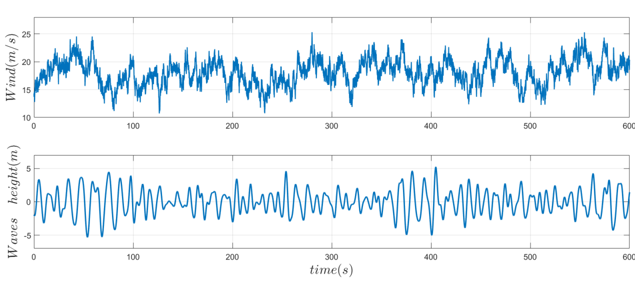

Fig. 1. Operating conditions for simulation

The HOSMC gains are $l_1 = 1.5$, $l_2 = 1.5$ and $l_3 = 0.5$. The RBFNN structure comprise 5 neurons in its hidden layer ($m = 5$). The parameters of the Gaussian function are $c_1 = [-1 \quad -1 \quad -1]^\top$, $c_2 = [-0.5 \quad -0.5 \quad -0.5]^\top$, $c_3 = [0 \quad 0 \quad 0]^\top$, $c_4 = [0.5 \quad 0.5 \quad 0.5]^\top$, $c_5 = [1 \quad 1 \quad 1]^\top$, $r_i = 1$ for $i = 1, \cdots, 5$, with $\eta = 0.01$.

TABLE I
CONTROL PERFORMANCE: MEAN ERRORS AND STANDARD DEVIATION

Controllers	**Mean ω_r** [rpm]	**STD ω_r** [rpm]
Baseline	12.2808	1.3930
RBFNN-HOSMC	11.9771	0.4340
	Mean θ_y [deg]	**STD θ_y** [deg]
Baseline	1.9693	1.1694
RBFNN-HOSMC	1.9952	0.8875
	Mean ω_y [deg/s]	**STD ω_y** [deg/s]
Baseline	$-6.32e-04$	0.4088
RBFNN-HOSMC	$3.1236e-05$	0.3473

The results of the comparative analysis conducted against the baseline controller [39] are depicted in fig.2, while the mean errors and standard deviations of the control performance are provided in Table I.

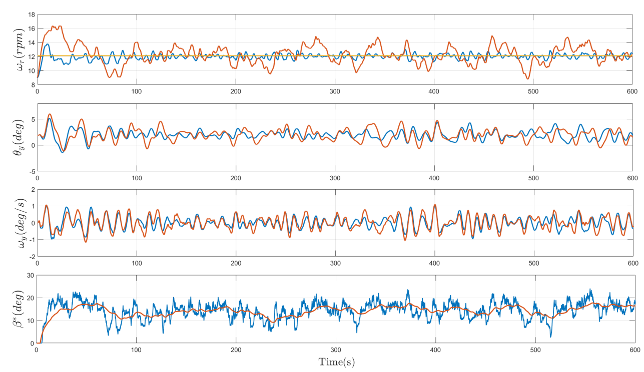

Fig. 2. Simulation Results: Proposed adaptive controller RBFNN-HOSMC (blue) and baseline (red)

The simulation results demonstrate the effectiveness of the proposed adaptive approach, RBFNN-HOSMC, in achieving the control objectives. The controller successfully regulates the rotor speed ω_r to its rated value (12.1 rpm), while ensuring stabilization of the platform pitch. Notably, the RBFNN-HOSMC exhibits better tracking performances compared to the baseline controller, as evidenced by the standard deviation being reduced by a factor of more than three. Additionally, the standard deviation of the platform pitch angle θ_y, as well as the platform pitch rate ω_y, are lower than those of the baseline controller, further highlighting the superiority of the proposed approach in term of achieving control objectives.

V. CONCLUSIONS

In this paper, we have presented a new adaptive feedback control approach for disturbed chains of integrators with smooth disturbances. By combining adaptive neural networks with higher-order sliding mode control, we have achieved the convergence of system states to a vicinity of the origin without requiring any prior knowledge of the disturbance upper bound. The adaptive neural network term effectively compensates for the disturbance with an error, while the higher-order sliding mode control term handles this error and ensures the stabilization of the system state. The simplicity of implementation and the absence of the need to reduce the system order make our proposed method well-suited for practical applications.

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