Fibre Supercontinuum Generation: Progress and Perspectives

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Abstract

Broadband supercontinuum generation in optical fibre has revolutionized multiple fields of physics including frequency metrology, spectroscopy, and imaging. Although the last twenty five years have seen many remarkable results, the field remains as active as ever, and this perspective aims to provide insights into current and emerging developments. Particular topics of review include recent advances in new fibre materials and designs, the use of gas-filled hollow core fibres, multimode and spatio-temporal effects, improved understanding of the underlying nonlinear dynamics, and techniques for spectral optimisation using machine learning.

INTRODUCTION

The study of broadband spectra has had major influence on physics for centuries. This includes for example Newton's work on the separation of colours in white light in the 1660s, Fraunhofer's study of absorption lines in solar spectra around 1814, measurements of blackbody radiation in the 1880's, and the development of the Planck radiation formula and quantum mechanics in the early 20th century. The invention of the laser in 1960 and the development of nonlinear optics led to a fundamentally new class of spatially coherent "supercontinuum" spectrum, first reported in 1970 by Alfano and Shapiro in glass [1], and then in 1976 by Lin and Stolen in optical fibre [2]. Whilst early experiments in fibres were carried out in the normal dispersion regime such that spectral broadening was dominated by Raman effects, the 1980s saw the development of sources where spectral broadening arose from soliton effects in the anomalous dispersion regime, particularly the process of soliton fission [3]. Numerical and theoretical work was then carried out to significantly improve understanding of the underlying supercontinuum generation processes. For a discussion of early developments, the reader is referred to refs. [4, 5].

With the basic physics understood, there was reduced interest in supercontinuum generation in the 1990s aside for some specific applications in telecommunications and soliton-based pulse compression [6]. However, a major milestone 25 years ago was the generation of octave-spanning supercontinuum in photonic crystal fibre (PCF), where dispersion engineering allowed soliton dynamics to be exploited using near-infrared femtosecond pump sources [7]. Amongst other key results, this rapidly led to the fibre-based stabilized frequency comb (recognized with the 2005 Nobel Prize in Physics) and completely reinvigorated the study of supercontinuum sources, with extensive work studying the underlying soliton dynamics, noise properties, and many new applications [8].

Research since has seen significant progress in both theory and experiment, with impact touching on nearly all areas of optical physics. Despite this remarkable progress to date, however, the study of fibre supercontinuum generation remains as active as ever, and continues to open unexpected new research directions. In this perspective, we highlight key areas where we believe future research is poised to be especially fruitful. In what follows, the organisation of topics is thematic rather that chronological. A general overview is provided in Figure 1.

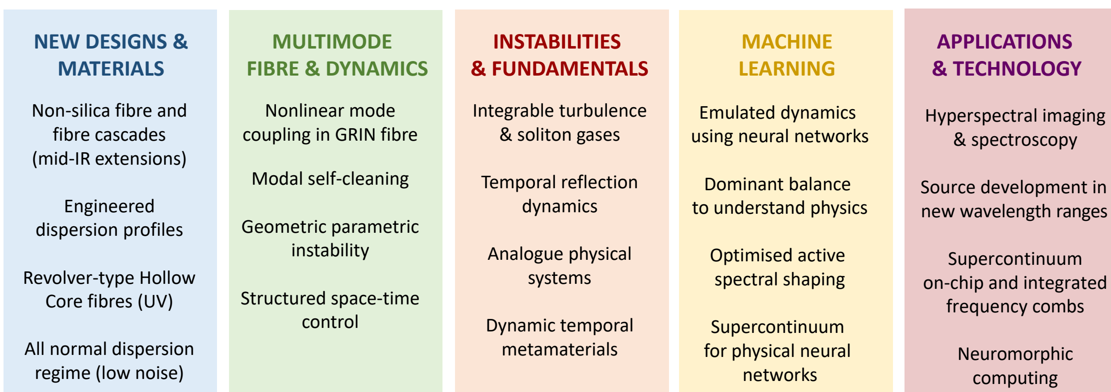

FIG. 1. An overview of broad areas of supercontinuum research, and likely specific topics of future interest.

EMERGING MATERIALS AND NEW WAVELENGTHS

Probably the first thing that comes to mind when assessing future directions of supercontinuum generation research is to exploit new fibre materials and designs to extend the available spectral coverage. Specifically, the idea here is to go beyond the conventional 0.4–2.4 μm range possible using silica-glass fibres into new wavelength regimes in the ultraviolet (UV) and mid-infrared (MIR). To this end, we expect greatly expanded interest in developing new supercontinuum sources based on specialty solid-core fibre fabricated from materials such as soft infrared glasses, doped-silica, and semiconductors, as well as systems based on liquid-core and gas-filled hollow-core fibres [9].

Concerning soft-glass fibres for MIR spectra, chalcogenide will continue to play a pivotal role due to its broad transmission (up to 20 μm) and high nonlinear coefficient [10, 11]. Indeed, advances in material composition, such as As_2Se_3 and Ge-Se-Te, have already demonstrated spectra spanning 1.7–18 μm with power levels of several hundred milliwatts [12]. The development of new telluride-based glasses with even higher nonlinearity and broader transmission could potentially extend to 25 μm.

When optimising such MIR spectral extension, an important related area of work is the study of cascaded fibre systems employing different fibre materials. In this case, filtering after an initial stage of spectral broadening in one fibre can act as a pump for additional broadening in a second fibre (and so on), and this approach has already been demonstrated for a silica-ZBLAN-chalcogenide fibre cascade leading to 2–10 μm spectra [13]. However, there remains a great deal of further experimental work needed to fully explore the physics of such cascaded pumping, especially with regard to understanding how best to exploit different spectral broadening mechanisms in each segment. This is an area where machine-learning aided design (see below) could be very productive.

An additional material suitable for future MIR applications is the crystalline-core semiconductor fibre which offers a unique combination of robustness, high nonlinearity, and strong mode confinement [14]. Whilst materials such as Ge, SiGe, ZnSe, and InSb have all been explored, silicon-core fibres are likely the only viable option for practical supercontinuum generation due to their favourable transmission. The recent demonstration of a 1.6-5.3 μm MIR supercontinuum in a tapered Si-core fibre highlights their potential, yet reducing loss and optimizing fibre tapering techniques will be crucial for further development [14]. Silicon-core fibres operating in the normal dispersion regime and exhibiting nonlinearities ten thousand times higher than conventional fibres are also being explored for ultra-compact, low-noise nonlinear mixers for frequency combs at repetition rates exceeding 25 GHz [15].

Doping silica fibres with highly nonlinear nanoparticles is another emerging strategy [16]. Because nanoparticles can exhibit a strongly wavelength-dependent nonlinear refractive index that can be both positive or negative, this technique potentially enables the design of fibres with enhanced, reduced, or even negative nonlinearity. This provides new degrees of freedom for tailoring nonlinear properties, minimizing noise, and enabling novel physical phenomena. Soliton formation in the normal dispersion regime is just one example of novel effects that can be observed as a result of such “nonlinearity-engineering” [17], and this concept may well have a transformative effect comparable to the introduction of dispersion-engineering in PCF 25 years ago.

Another novel material platform for MIR applications is based on the liquid-core optical fibre which exhibits broad MIR transparency. However, unlike solid-core fibres, liquids exhibit non-instantaneous nonlinear responses, which leads to more complex spectral broadening mechanisms which have not yet been fully studied or exploited. Future research should focus on developing novel liquid compositions with enhanced nonlinear properties and exploring tunable supercontinuum generation through dynamic liquid-core modifications [18, 19].

Looking towards spectral extension to shorter wavelengths in solid-core fibres, UV-resistant high-OH silica PCF have been developed supporting supercontinuum generation in the 300–400 nm black-light range [20]. Additionally, ZBLAN-glass fibres have demonstrated spectral generation down to 200 nm (and extending to the MIR) when pumped in their cladding. Although further work on the use of solid-core fibre for UV applications is expected to continue [21], it is likely that most interest in fibre-generated UV will focus on hollow-core gas filled platforms as we discuss below. As a general concluding comment in this section, we also note that accurately understanding the nonlinear properties of new materials is essential in the context of future developments in supercontinuum sources, and we refer the reader to an excellent recent review of this area of research [22].

ALL NORMAL-DISPERSION FIBRE

The initial applications of supercontinuum generation in frequency metrology naturally led to intense interest into its noise properties. It was quickly realised that with anomalous dispersion regime pumping, spectral broadening was dominated by soliton dynamics, and studies showed how this could be extremely sensitive to input pulse noise (even at the quantum shot-noise limit) depending on pulse duration, power, and the fibre parameters [23]. Yet even when generated under stable conditions, supercontinuum spectra generated from soliton dynamics often possess significant non-uniform structure, and are not always suitable for applications.

To this end, a substantial body of research has emerged studying broadband spectral generation in fibres or other waveguides with All-Normal group velocity Dispersion (ANDi).

In these systems, nonlinear dynamics are governed by self-phase modulation and optical wavebreaking [6], enabling low-noise spectral broadening, exceptional spectral flatness, and high spectral power densities. ANDi-based spectral broadening has already shown its value in enabling the first shot-noise limited dual-comb supercontinuum source at GHz repetition rates [24], and the combination of soliton-self compression with optical wavebreaking has led to octave-spanning spectra with excellent noise properties, even when pumped with low-energy pulses [25, 26]. Additionally, the broadband compressible chirped pulses generated from ANDi fibres facilitate the production of low-noise single-cycle pulses, which are ideal for further amplification, and of relevance to attosecond science [4, 9, 27]. There are also possibilities for further noise reduction using normal dispersion chiral fibres with circular birefringence such that potential polarisation instabilities are suppressed [28].

HOLLOW CORE FIBRE

Fibre waveguides that can guide light in a gas-filled hollow core are a remarkable platform for generating radiation at wavelengths well-beyond what is feasible in solid-core fibres, most notably in the deep and vacuum ultraviolet regions. The nonlinear interaction between optical pulses and the filling gas in such fibres provides new mechanisms for frequency conversion, while still maintaining excellent spatial coherence and beam quality [29, 30].

It is important to note, however, that first-generation photonic band-gap hollow core structures are not well-adapted for supercontinuum generation due to their narrowband guidance characteristics. Rather, mode confinement over the large wavelength ranges associated with broadband frequency conversion requires a different physical mechanism of "antiresonant" guidance. Initial examples of such fibre used a Kagome-lattice structure, but the recently-optimised ring or "revolver" structure of circular resonators around a larger core is now very clearly the preferred choice [31, 32]. Such a structure offers broadband guidance well-beyond that of glass, and with low bend-loss such that optimised designs constitute the lowest loss optical waveguides ever fabricated [33]. They also possess much higher damage thresholds, with one recent experiment demonstrating guidance at an intensity of 3 PWcm^{-2} [34].

In the context of nonlinear optics, gas-filled hollow core fibre offers a design flexibility to be found in no other waveguide platform, because both dispersion and nonlinearity can be readily tuned through suitable selection of gas species and pressure. More specifically, an evacuated hollow fibre naturally provides anomalous dispersion, and this can be balanced by increasing the density of normally dispersive (and nonlinear) gas inside the core - most commonly noble gases such as argon, or simple molecular gases such as hydrogen. As might be expected, the spectral broadening mechanisms can be more complex than in solid-core fibres. Although fibre dispersion and nonlinearity can combine to yield well-understood soliton dynamics [35], there can be notable differences: (i) in noble-gas-filled hollow fibres there is no Raman scattering, reducing spectral expansion to longer wavelengths; (ii) the intensities used are often sufficient to ionise the filling-gas, creating new soliton-ionisation interactions which lead to a soliton blue-shift (in comparison to a soliton red-shift due to Raman in solid-core fibre) [36, 37]; (iii) due to the overall flatter dispersion profile and broader guidance band, the supercontinuum can extend to much shorter wavelengths.

For example, Raman-enhanced soliton self-compression and resonant dispersive-wave emission yielded vacuum ultraviolet to below 200 nm in a hydrogen-filled fibre [38], with these results further extended through soliton-ionisation dynamics in a helium-filled fibre, to 113 nm [39]. While vacuum ultraviolet supercontinuum extension is a unique capability of gas-filled hollow-core fibres, they can also extend into the mid-infrared [40, 41], and cascading with other nonlinear processes has already been shown to yield ultra-broadband spectral generation. For example, a 7-octave (-70 dB level) supercontinuum was obtained by combining a mid-infrared pumped antiresonant fibre with difference-frequency generation in a crystal [42], where the fibre-generated spectrum spanned from 340-5000 nm (-40 dB level).

As with solid-core fibre, pumping with longer and more energetic pulses in the anomalous dispersion region leads to modulation instability dynamics and spectra that are often flatter and with higher spectral power density, at the cost of coherence and shot-to-shot noise [43]. Building on work on soliton self-compression deep-ultraviolet supercontinuum in thin-walled antiresonant fibres [44], the short-wavelength limit of modulation-instability based supercontinuum was recently extended down to 260 nm by making use of ultra-thin core-wall antiresonant fibres ($\sim$90 nm) and pumping at 515 nm [45]. This is clearly an area of much interest for applications where spectral coherence is not needed.

A further future area of work is the use of molecular gases to enhance supercontinuum dynamics beyond the soliton self-compression regime. One approach is to start from vibrational Raman frequency-comb formation in the normal dispersion region. If driven sufficiently strongly, the comb lines begin to broaden and eventually form an exceedingly flat and broadband supercontinuum [46]; as a result this technique has been coined "comb-to-continuum," and it is closely related to previous work in solid-core fibres [47], liquid-core fibres [18], and air-filled fibres [48]. Initial experiments used nitrogen, but this can suffer from a universal Raman gain suppression [49], motivating the search for other suitable gases, such as methane [50]. The above works were concentrated in the ultraviolet to near-infrared spectral range, however, the use of Raman-enhanced supercontinuum in gas-filled fibres can also lead to efficient extension to the mid-infrared [51].

MULTIMODE AND SPATIO-TEMPORAL EFFECTS

Unlike single-mode fibres, multimode fibres support the propagation of multiple spatial guided modes, which interact nonlinearly in both space and time. This spatio-temporal coupling yields a rich landscape of nonlinear phenomena that can be exploited for generating controllable broadband spectra, and opens new opportunities for structuring light in multiple dimensions simultaneously [52]. In this context, one of the key developments has been the observation in graded-index (GRIN) multimode fibres of a modal "self-cleaning" effect where an injected high-power field that initially excites multiple spatial modes evolves during propagation into a beam that is spatially more localized, typically in the fundamental mode [53, 54]. These dynamics, favoured by the particular parabolic GRIN refractive index profile [55], provide a new approach to high power supercontinuum generation with a spatially coherent and near-Gaussian output [56]. This self-organization behaviour has also been linked to statistical mechanics and thermodynamic concepts describing mode energy redistribution and the emergence of coherent structures in nonlinear multimode systems [57–59].

Another class of nonlinear effects unique to multimode GRIN fibres is the "geometric parametric instability" phase-matched by periodic focusing and defocusing of multimode beams. This can significantly contribute to spectral expansion, and the generation of multiple discrete sidebands far detuned from the pump wavelength [60].

The formation of multimode solitons from the balance between dispersion and nonlinearity across multiple modes is also unique to multimode environments [61]. These solitons can experience spatio-temporal fission and radiate multiple dispersive waves, significantly broadening the supercontinuum spectrum [62]. More generally, recent studies have further highlighted the growing role of fibre spatio-temporal effects in nonlinear physics. For example, it has been shown how discretized X-shaped wavepackets can emerge from intermodal dispersive wave emission [63] or how azimuthal modulation instability can lead to breather formation and azimuthal solitons in ring-core fibres [64]. A recent study also demonstrated how GRIN fibres can exploited for the generation of space-time wave packets with tunable group velocities and transverse motion [65, 66].

Together, these findings demonstrate that multimode fibre platforms, far from being a complication, offer new degrees of freedom for shaping supercontinuum light through the interplay of spatial modes, temporal dynamics, and nonlinear interactions. Spatio-temporal nonlinear interactions have recently been extended to non-silica MMFs with engineered dispersion profiles and enhanced nonlinear response, enabling octave-spanning, high-power supercontinuum generation in the mid-infrared [67]. These results open the way for further developments in this spectral region, where tailored multimode platforms are expected to play an increasingly important role. More generally, the convergence of material engineering, advanced fibre design, and data-driven control methods is expected to rapidly transform multimode supercontinuum sources into versatile tools [68, 69] for on-demand structured light with applications ranging from high resolution imaging [70] to ultrafast spectroscopy [71] and even optical computing [72].

INSTABILITIES AND FUNDAMENTAL DYNAMICS

As discussed above, the noise properties of supercontinuum generation have always been a subject of major interest. Future work will continue in this field, motivated both by the need to develop coherent sources for applications, as well as by continued interest in the underlying nonlinear dynamics. In this context it is important to recall how significant the study of real-time fluctuations in supercontinuum spectra was in developing the analogy between nonlinear noise amplification and the emergence of giant ocean rogue waves [73]. This stimulated much interdisciplinary work spanning optics and hydrodynamics, and in clarifying the the key role of modulation instability as the key mechanism in driving the generation of extreme events [74]. The study of noise in nonlinear fibre propagation remains an important current area of research, and a general interpretation has recently developed in terms of integrable turbulence and a soliton gas model. This is likely to become a very influential area of work in the coming years [75]. These developments are related to earlier studies of nonlinear fibre propagation in terms of thermodynamic concepts [76, 77], and these ideas deserve continued attention.

Interestingly, a perhaps-unexpected outcome of these studies into supercontinuum instabilities was the use of optical fibres to generate different families of stable NLSE soliton and breather structures, excited by suitable injected initial conditions. Theory and experimental work in this area actively continues in optics (with much accompanying support in applied mathematics), and continues to stimulate related work in other fields of physics [78]. As a general remark, however, it is probably safe to say that research into the role of nonlinearity in ocean wave dynamics has now moved primarily to controlled experiments in hydrodynamic wave tanks or to analysis of real-world ocean wave data. Still, the role of nonlinear fibre optics in renewing interest in the topic should be widely acknowledged.

Shortly after the supercontinuum-rogue wave analogy was proposed, an intriguing link was proposed between the dynamics of two-pulse interactions in fibre via cross-phase modulation and the physics of a gravitational event horizon [79]. Although developing ideal fibre-based "analogue gravity" experiments has proven problematic, this work has shone new light on the study of nonlinear and dispersive mediated temporal interactions. For example, theories of cascaded wave mixing [80] and temporal reflection and refraction [81] have been developed that describe the same physics as the optical event horizon, highlighting to the nonlinear fibre optics community that "pump-probe" dynamics involving interacting ultrafast pulses can actually be interpreted in terms of the modern picture of nonlinear dynamics in time-varying media [82]. With advances in experimental pulse-shaping and control, it may well be that developing soliton-based "dynamic temporal metamaterials" and ideas of temporal topology and synthetic dimension emerge as important future areas of research. It is also possible that many of the soliton interaction processes in supercontinuum generation will be able to be interpreted in the same way. In fact, controlling temporal pulse interactions in supercontinuum pumping has already been demonstrated as an effective means for supercontinuum spectral shaping [83] and optimising temporal reflection dynamics may become a useful practical means of bandwidth enhancement.

Finally in this section, we note that supercontinuum spectral broadening dynamics and instability properties are now also being studied in the context of fibre laser oscillation [84, 85]. Although the control of ultra broad supercontinuum bandwidths in a recirculating cavity is certainly challenging experimentally, the dissipative soliton concept is a powerful approach to design and optimise cavities even in the presence of significant variation in intracavity field properties [86].

MACHINE LEARNING

Machine learning is having a dramatic transformative impact in all areas of science, including in ultrafast photonics and nonlinear fibre optics [87]. Its impact on the field of supercontinuum generation research is at a relatively early stage, but there is no doubt that the coming years will see the methods of machine learning become widespread in both numerical and experimental research.

From the point of view of modelling, work has already showed that neural networks can successfully predict supercontinuum temporal characteristics based only on spectral data [88], but perhaps more significant is the use of neural networks to actually emulate the complex nonlinear dynamics of soliton fission and supercontinuum broadening. The idea here is to replace the numerical integration of the governing generalised nonlinear Schrödinger equation describing supercontinuum generation by direct emulation using a suitably-trained neural network. Of course, direct numerical integration is needed to generate the training data for the network, but once trained, predicting output temporal and spectral characteristics for new parameters (within the range of training data) can be essentially instantaneous [89]. Work in this area is however at a very early stage, and future research will be expected to greatly expand the parameter range over which such emulation can be performed.

Another important application of machine learning from a modelling perspective has been to apply clustering and combinatoric methods to analyse the complex spatio-temporal supercontinuum dynamics obtained from simulations, and develop an automated algorithmic approach to interpret the underlying physics [90]. The key concept here is that of "dominant balance" where an algorithm quantitatively analyses the contribution of different terms in the governing partial differential equation at each spatio-temporal point, and then identifies regions where subsets of the different terms "dominate" to satisfy the differential equation equality. This method is generally applicable across many different areas of physics [91], and is likely to significantly add to future work studying the physics of supercontinuum and nonlinear fibre propagation more broadly.

From an experimental perspective, the most evident application of machine learning is in the active control of supercontinuum spectral characteristics, aiming for example to enhance spectral intensity at specific wavelengths for particular applications. This has indeed been successfully demonstrated using genetic algorithms to optimise intensity in target wavelength bands through feedback to temporal pulse sequence control [83] or Fourier-domain spectral shaping to modify the injected initial conditions [92]. Other experimental studies have shown the optimization of supercontinuum-like broadband spectra in a fibre laser cavity using a genetic algorithm for intracavity polarisation control [93], and numerical studies have shown that a genetic algorithm can also optimise intracavity gain and loss parameters in a fibre laser to optimise spectral broadening from coherent soliton propagation dynamics [85]. This field of work is clearly at an early stage, and much wider use of optimization approaches to supercontinuum generation are expected for targeted applications. Indeed in this context, we note that numerical modelling has also demonstrated the feasibility of optimised spectro-temporal properties of supercontinuum for the specific application of multi-photon microscopy [94].

Nonlinearity is key to the operation of neural networks, playing an essential role in their ability to learn and model complex patterns. In this context, a very exciting recent development has been to exploit the complex nonlinear evolution of supercontinuum generation to act as the nonlinear element in physical neural network architectures. In other words, the idea here is to exploit the supercontinuum to develop laboratory machine learning hardware. Several experiments have already reported successful proof of principle results for supercontinuum-based classification [95–97] and a recent numerical work has studied potential limits to this approach, including due to the effect of quantum noise [98].

FURTHER PERSPECTIVES AND CONCLUSIONS

The study of optical fibre supercontinuum generation continues to be a very active research field from both fundamental and applied perspectives. Developments in material science and fibre design have led to major achievements in reaching new wavelength ranges and increasing spectral power density, and there is every reason to expect that research in theory, numerical modelling, and experiment will continue to develop in the coming decade. It is also important to stress that the supercontinuum light source has now become a very important commercial product, and some of the key research papers in the field are becoming increasingly cited in the patent literature. Indeed, many important practical advances are being made in an industry context.

The development of the fibre supercontinuum has enabled new applications in what can be termed hyperspectral optics, for example imaging or sensing techniques that exploit detection over broad wavelength ranges. In this context we note that research in this field is contributing to pushing spectral detection limits to accommodate the ultrabroad bandwidths. Moreover, understanding the dynamics of fibre supercontinuum generation has resulted in new (and rediscovered) insights into the dynamics of temporal solitons [99], with important extensions to understanding propagation effects in non-fibre systems, particularly planar waveguides and microresonators, both candidates for genuine on-chip photonic devices [100, 101]. More generally, there is now no doubt that the study of optical fibre supercontinuum generation has become a cornerstone of research cutting across broad areas of nonlinear optical physics, and driving key applications in next-generation photonics.

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