A distributed controller for the rotor speed synchronous control problem of multiple PMSMs with port-Hamiltonian dynamics

Jingyi Zhao, Yongxin Wu, and Yuhu Wu

***Abstract*—In this paper, the synchronous rotor speed control problem of multiple permanent magnet synchronous motors (PMSMs) with port-Hamiltonian (PH) dynamics is considered. Firstly, we convert the synchronous rotor speed problem to an optimization problem, and based on this, a distributed controller is proposed for each PMSM using the information exchange between PMSMs. However, the exchanged information is the average rotor speed estimation value of each PMSM rather than the direct rotor speed, which may facilitate the sensitive state protection. Then, the stability of the closed-loop system is analyzed and we prove that the equilibrium of the closed-loop system is consistent with the optimum of the value function of the optimization problem. Finally, a simulation example is provided to validate the effectiveness of the proposed distributed controller.**

***Index Terms*—port-Hamiltonian system, distributed control, permanent magnet synchronous motors.**

I. Introduction

Permanent magnet synchronous motors (PMSMs), which encompass both surface-mounted permanent magnet synchronous motors and internal permanent magnet synchronous motors, exhibit distinctively preponderant characteristics within the realm of variable speed drives. These remarkable traits comprise high power density, favorable dynamics, remarkable efficiency, as well as an extensive operating speed range, as illustrated in [1, 2]. Consequently, PMSMs find extensive utilization in a multitude of industrial applications, spanning servo drives, high speed trains, electric vehicles, and household appliances, as indicated in [3].

However, in the context of certain practical engineering applications, it has been observed that a solitary PMSM falls short of fulfilling the requisite demands. In such scenarios, it becomes necessary for multiple PMSMs to operate in a coordinated manner so as to meet the engineering necessities. For example, in the manufacturing-oriented market environment, driven by the pursuit of fabricating commodities of superior manufacturing quality at a diminished production related cost, the manufacturing sector has proposed manufacturing-operational strategies centring on synchronizing two or more rotational shafts within mechanical assemblies. However, these established approaches are encumbered with a multiplicity of drawbacks (as mentioned in [4]), including the prolonged part-replacement time, uncertainties caused by wear-related degradation, unrefined speed trajectories, and relatively low mechanical-transmission reliability. To circumvent these obstacles, the rotor speed synchronous control of multiple motors has emerged as a highly effective countermeasure (as demonstrated in [5]). Consequently, the rotor speed synchronous control of multiple motors is of great significance as it enables each shaft, which is driven and controlled by an individual motor without physical connection to others, to maintain the synchronous operation during acceleration, deceleration, and under varying load conditions. This ensures the stability and consistency of the manufacturing process. As demonstrated in the research presented in [6], the implementation of multi-motor rotor speed synchronous control is conducive to significant improvements in production output volume, product manufacturing quality, and energy utilization efficiency.

There are some related researches about the speed synchronous control of multiple motors. For example, Zhao et al. [7] investigated a real-time speed synchronous control approach for multiple induction motors with speed acceleration and load changing based on the sliding mode control theory. Wang et al. [8] proposed a novel tracking and synchronization control strategy for multi-motor driving systems with unknown parameters including two subcontrollers: the adaptive immersion and invariance tacking and the robust integral of the sign of the error synchronization controller. Hu et al. [9] put forward a robust adaptive synchronization and tracking control strategy which was based on neural network for multi-motor driving servo systems. More details can be found in the review paper [10].

Since the concept of cyber-physical systems was introduced, research in this field has advanced rapidly with increasing attention from scholars toward distributed strategies combined with physical systems. In fact, since systems in multi-physical domains can be modeled using port-Hamiltonian (PH) frameworks, PH systems offer a broader range of applications compared to EL systems, and inherently benefit from their interconnected structure, making them well-suited for networking (see [11] and [12]). Due to its powerful

Manuscript received: Day Month Year; revised: Day Month Year; accepted: Day Month Year. *(Corresponding author: Yuhu Wu.)*

Citation: J. Zhao, Y. Wu, and Y. Wu, A distributed controller for the synchronous rotor speed control problem of multiple PMSMs with port-Hamiltonian dynamics, *IJICS*, 2024, vol(no), ppCpp.

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Digital Object Identifier

functions, some scholars have investigated the PMSM model under the PH framework and based on which designed the controller. Petrovic et al. [13] developed an energy-shaping controller for the speed regulation of PMSMs. Yaghmaei et al. [14] designed a full-order observer by using the notion of contractive PH systems and applied it into PMSMs. However, these works focus on the control of a single PMSM rather than the rotor speed synchronous control of multiple PMSMs.

This paper design a distributed controller for each PMSM to achieve the rotor speed synchronous of multiple PMSMs modeled under PH framework. The main contributions are summarized as follows:

1) Within the realm of networked communication control for multiple agents, the PH framework exhibits distinct advantages over alternative frameworks, owing to its inherent inter-connectivity and extensive application scope. Hence, this paper designs a distributed speed rotor synchronous controller for multiple PMSMs with PH dynamics. Under the action of the designed controller, the closed-loop system exponentially converges to the equilibrium where all PMSM achieve the rotor speed synchronization.
2) The proposed distributed controller empowers PMSMs to conduct information exchange through estimated values and auxiliary variables, rather than relying on exact state values. This approach effectively mitigates the risk of sensitive data disclosure. Concurrently, when contrasted with the central ized controller, the proposed distributed controller substantially alleviates the burden of network communication.

The paper is organized as follows. In Section 2, some preliminaries are introduced and in Section 3, the problem is formulated. Then, a distributed controller is proposed in Section 4 and the convergence is analyzed. Section 5 gives an example to verify the proposed algorithm. Finally, Section 6 conclude this paper.

Notations. $\mathbb{R}^n$ denotes n-dimensional Euclidean space. $1_n \in \mathbb{R}^n$ and $0_n \in \mathbb{R}^n$ represent the vector of all ones and all zeros, respectively. $x^\top$ is the transpose of x. The set $\{i, i+1, \cdots, j-1, j\}$ described by $[i{:}j]$ where $i, j \in \mathbb{R}$ and $i < j$. $\text{col}(x_1, \cdots, x_N) = (x_1^\top, \cdots, x_N^\top)^\top \in \mathbb{R}^{Nn}$ with $x_i \in \mathbb{R}^n$, $i \in [1{:}N]$. I_n is the identity matrix in $\mathbb{R}^{n\times n}$. $\text{diag}(\lambda_1, \cdots, \lambda_n)$ is the diagonal matrices of elements $\lambda_1, \cdots, \lambda_n \in \mathbb{R}$. For a function $f(x,y): \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}$, the partial derivative with respect to x is $\nabla_x f(x,y) = \frac{\partial f(x,y)}{\partial x}$, the gradient of $f(x,y) = \nabla f(x,y) = \text{col}(\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y})$. The symbol $\ker(L)$ represents the zero space of L while $\text{range}(L)$ represents the range space of L.

II. Preliminaries

This section mainly introduces some basic preliminary knowledge used below and describes the problem investigated in this work.

A. Some basic concepts

Consider an undirected graph $\mathscr{G} := \{\mathscr{V}, \mathscr{E}, \mathscr{A}\}$, where $\mathscr{V} = [1:N]$ and $\mathscr{E}$ denote the node set and the edge set, respectively, and $\mathscr{A} := (a_{ij})_{N\times N}$ denotes the adjacency matrix. If i is a neighbor of j, then the pair $(i,j) \subset \mathscr{E}$ is an edge of $\mathscr{G}$ as well as $a_{ij} = 1$. Moreover, for all $i \in \mathscr{V}$, $a_{ii} = 0$. If $a_{ij} = 1$, we said j is the neighbor of i, and the set of all such j is the neighbor set of i, denoted by $\mathscr{N}(i)$. With the degree matrix $\mathscr{D} = \text{diag}(\text{deg}_1, \cdots, \text{deg}_N)$ where $\text{deg}_i = \sum_{j=1}^N a_{ij}$, the Laplacian matrix of $\mathscr{G}$ is defined by $L = \mathscr{D} - \mathscr{A}$. If there is a connection path between any pair of nodes, then $\mathscr{G}$ is called connected. If the eigenvalues of L are denoted by $\lambda_1 \le \cdots \le \lambda_N$, then $\mathscr{G}$ is connected if and only if $\lambda_2 > 0$ by [15]. In this paper, we use the undirected connected graph to describe the communication topology between multiple players.

A function $f: \mathbb{R}^n \to \mathbb{R}$ is said to be convex on $\mathbb{R}^n$ when $f(\alpha x + (1-\alpha)x') \le \alpha f(x) + (1-\alpha) f(x')$ is satisfied for all $x, x'\mathbb{R}^n$ and for all $\alpha \in [0,1]$.

A function $f: \mathbb{R}^n \to \mathbb{R}^n$ is L_f-Lipschitz ($L_f > 0$) on $\mathbb{R}^n$ if $\|f(x) - f(x')\| \le L_f \|x - x'\|$ is satisfied for all $x, x' \in \mathbb{R}^n$.

If there exists a $w > 0$ such that $(x-x')^\top (f(x) - f(x')) \ge w\|x - x'\|^2$ holds for all $x, x' \in \mathbb{R}^n$ and $x \ne x'$, the function $f: \mathbb{R}^n \to \mathbb{R}^n$ is w-strongly monotone. More details about the above definitions can be found in [16].

B. The PH model of PMSMs

If an undirected graph $\mathscr{G}$ is used to describe the communication topology among multiple PMSMs which is shown as Fig. 1 [17], and the node set is denoted by $\mathscr{V} = \{1, \cdots, N\}$, then by basic principles of electromagnetics such as voltage equation, the kth PMSM ($k \in \mathscr{V}$) is modeled with the standard d-q model given as follows [13]:

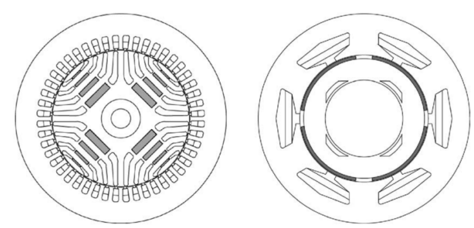

Fig. 1. The structure diagram of interior permanent magnet (IPM) and surface-mounted permanent magnet (SPM) synchronous motor.

$$\begin{cases} L_{dk} \frac{di_{dk}}{dt} = -R_{sk} i_{dk} + w_k L_{qk} i_{qk} + V_{dk}, \\ L_{qk} \frac{di_{qk}}{dt} = -R_{sk} i_{qk} - w_k L_{dk} i_{dk} - w_k \phi_k + V_{qk}, \\ \tilde{J}_k \frac{dw_k}{dt} = n_k ((L_{dk} - L_{qk}) i_{dk} i_{qk} + \phi_k i_{qk}) - \tau_{lk}, \end{cases} \tag{1}$$

where the meanings of different symbols is listed:

n_k is the number of pole pairs;

L_{dk} and L_{qk} are stator inductances in d-q frame, mH;

R_{sk} is stator winding resistance, Ω;

τ_{lk} is a known constant load torque, Nm;

ϕ_k is the d-q back emf constant, Vs;

$\tilde{J}_k$ is the moment of inertia, $\text{kg}\cdot\text{m}^2$;

w_k is the angular velocity, rad/s.

The viscous friction is neglected in this model, because it is usually small. By define the state vector as $x_k = \text{col}(x_{1k}, x_{2k}, x_{3k})$, where $x_{1k} = L_{dk} i_{dk}$, $x_{2k} = L_{qk} i_{qk}$, $x_{3k} =$

$(\tilde{J}_k/n_k)w_k)$, the energy function of system (1) can be given as

$$H(x_k) = \frac{1}{2}(\frac{1}{L_{dk}}x_{1k}^2 + \frac{1}{L_{qk}}x_{2k}^2 + \frac{n_k}{\tilde{J}_k}x_{3k}^2). \quad (2)$$

Based on the energy function (2), the system (1) can be rewritten in PCH form as

$$\dot{x}_k = (J_k(x_k) - R_k)\frac{\partial H(x_k)}{\partial x_k} + g_k u_k + \xi_k. \quad (3)$$

with

$$g_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad u_k = \begin{bmatrix} V_{dk} \\ V_{qk} \end{bmatrix}, \quad \xi_k = \begin{bmatrix} 0 \\ 0 \\ -\frac{\tau_{lk}}{n_k} \end{bmatrix},$$

the structure matrix $J_k(x_k)$ and the dissipation matrix R_k are

$$J_k(x_k) = \begin{bmatrix} 0 & 0 & x_{2k} \\ 0 & 0 & -(x_{1k}+\phi_k) \\ -x_{2k} & x_{1k}+\phi_k & 0 \end{bmatrix}, R_k = \begin{bmatrix} R_{sk} & 0 & 0 \\ 0 & R_{sk} & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

respectively.

III. Problem formulation

In this paper, we consider the speed synchronization problem of multiple PMSMs with the dynamic described by (3). The problem is presented as follows:

Problem 1 *Design a controller u_k for the kth PMSM ($k \in \mathscr{V}$) such that the rotor speed w_k of the kth PMSM asymptotically converge to the minimum point w_k^* of the value function*

$$V_k(w) = \frac{\alpha N}{N-1}(w_k - \sigma(w(t)))^2 + \beta(w_k - w_k(0))^2, \quad (4)$$

where $w = col(w_1, \cdots, w_N)$, the average rotor speed of all PMSMs is denoted by

$$\sigma(w(t)) = \frac{1}{N}\sum_{k=1}^{N} w_k(t),$$

and the initial speed of the kth PMSM is denoted by $w_k(0)$, the constant parameter $\alpha >> \beta > 0$.

The value function (4) is consistent with two parts:

- Each PMSM aims to achieve the speed synchronization with the control objective

$$\lim_{t\to\infty} w_k(t) - \sigma(t) = 0, \quad k \in \mathscr{V}. \quad (5)$$

- Every PMSM does not want to change its own initial speed, that is,

$$\min_{u_k} \|w_k(t) - w_k(0)\|, \quad k \in \mathscr{V}.$$

Since the rotor speed synchronization objective is more important, we define $\alpha >> \beta$ to enlarge the weight of the first term in the value function (4). Hence, the rotor speed synchronous control problem is converted to the optimization problem 1.

It should be noticed that, in this optimization problem, only the rotor speed of the kth PMSM ($k \in \mathscr{V}$) and the average rotor speed $\sigma(w(t))$ is required for the kth PMSM. Since the rotor speed of other PMSMs $j \in \mathscr{V}/\{k\}$ is unavailable for the kth PMSM with sensitive state concerns, we define $\eta_k(t)$ as the estimation of $\sigma(w(t))$ for the kth PMSM to design a controller solve Problem 1.

It's easy to find that the value function $V_k(w)$ is continuously differentiable in w and convex in w_k when $w_j,\ j \in \mathscr{V}/\{k\}$ is fixed, the following Lemma is given.

Lemma 1 *[18] The minimum point of the value function* (4) *is w^* if and only if*

$$\nabla_{w_k} V_i(w^*) = 0, \quad k \in \mathscr{V}.$$

Hence, our task is to find a distributed controller for the kth PMSM ($k \in \mathscr{V}$) to solve Problem 1.

IV. Main results

Before given the distributed controller, the following mappings are defined as

$$\begin{aligned} &F_k(w_k, \sigma(t)) := \nabla_{w_k} V_k(w), \\ &G_k(w_k, \eta_k) := F_k(w_k, \sigma(t))|_{\sigma(t)=\eta_k}, \end{aligned} \quad (6)$$

where $\eta_k \in R$ is the estimation of the kth PMSM ($k \in \mathscr{V}$) to the average rotor speed $\sigma(t)$.

Furthermore, define

$$\begin{aligned} &F(w) := \text{col}(\nabla_{w_1} V_1(w), \cdots, \nabla_{w_N} V_N(w)), \\ &G(w, \eta) := \text{col}(G_1(w_1, \eta_1), \cdots, G_N(w_N, \eta_N)), \end{aligned}$$

where $\eta = \text{col}(\eta_1, \cdots, \eta_N) \in R^N$.

Based on these mappings, the distributed controller of the kth PMSM is defined as

$$\begin{cases} u_k = -A_k \begin{bmatrix} \frac{\partial H(x_k)}{\partial x_{1k}} \\ \frac{\partial H(x_k)}{\partial x_{2k}} \end{bmatrix} - B_k w_k + C_k, \\ \dot{\eta}_k = s_k, \\ \dot{s}_k = -k_2 s_k - \delta(\eta_k - w_k) - \sum_{j=1}^N a_{kj}((\eta_k - \eta_j) - (v_k - v_j)), \\ \dot{v}_k = \sum_{j=1}^N a_{kj}(\eta_k - \eta_j) + \sum_{j=1}^N a_{kj}(s_k - s_j), \end{cases} \quad (7)$$

where the matrix $A_k = \text{diag}(-R_{sk}, -R_{sk})$, the matrix $B_k = \text{col}(x_{2k}, -(x_{1k}+\phi_k))$, the matrix$C_k = \text{col}(0, k_1(-k_2\dot{w}_k - G_k(w_k, \eta_k)))$, the constant $k_1 = \frac{L_{qk}\tilde{J}_k}{((L_{dk}-L_{qk})i_{dk}(0)+\phi_k)n_k}$, the constant $k_2 > 0$ is given in Theorem 1, and we assume the initial value of each agent satisfies $i_{dk}(0) \neq \frac{\phi_k}{L_{qk}-L_{dk}}$, $k \in \mathscr{V}$.

In the distributed controller (7), the first equation is designed relate to the coupled-state relationship of the PH system (3), and the estimation η_k of the kth PMSM ($k \in \mathscr{V}$) is updated by the estimation from its neighbours $j \in \mathscr{N}(k)$, and s_k, v_k are auxiliary variable to ensure the accuracy of the estimation.

By inserting x_k and A_k, B_k, C_k, we have

$$u_k = \begin{bmatrix} R_{sk} i_{dk} - L_{qk} i_{qk} w_k \\ R_{sk} i_{qk} + (L_{dk} i_{dk} + \phi_k) w_k + k_1(-k_2\dot{w}_k - G_k(w_k, \eta_k)) \end{bmatrix}.$$

Then, we are going to illustrate the convergence of the closed-loop system (the PH system (3) under the action of the distributed controller (7)), the following theorem is given.

Theorem 1 *The PH system* (3) *of the kth PMSM ($k \in \mathscr{V}$) is exponentially convergent to the equilibrium under the action of the distributed controller* (7) *with $k_2 > 1 + \frac{1}{4}\lambda_N + \frac{L_f^2}{\gamma}$*

where $L_f > 0$ and $\gamma > 0$ are the Lipschitz constant and the strongly monotone parameter of mapping Ξ defined in (11)*, respectively.*

Proof *Consider the multi-agent systems* (3)*, taking the derivative of x_{3k} with respect to time, we have*

$$\ddot{w}_k = \frac{n_k}{\tilde{J}_k}\big(-\dot{x}_{2k}i_{dk} - x_{2k}\dot{i}_{dk} + \dot{x}_{1k}i_{qk} + (x_{1k}+\phi_k)\dot{i}_{qk}\big). \tag{8}$$

By inserting the distributed controller (7) *into the PH model* (3) *of the kth PMSM ($k \in \mathscr{V}$), we obtain*

$$\begin{cases} \dot{i}_{dk} = 0, \\ \dot{i}_{qk} = \frac{k_1}{L_q}(-k_2\dot{w}_k - G_k(w_k,\eta_k)), \\ \dot{\eta}_k = s_k, \\ \dot{s}_k = -k_2 s_k - \delta(\eta_k - w_k) - \sum_{j=1}^N a_{kj}((\eta_k-\eta_j)-(v_k-v_j)), \\ \dot{v}_k = \sum_{j=1}^N a_{kj}(\eta_k-\eta_j) + \sum_{j=1}^N a_{kj}(s_k - s_j). \end{cases} \tag{9}$$

Combination (8) *and* (9)*, we have*

$$\begin{cases} \ddot{w}_k = -k_2\dot{w}_k - G_k(w_k,\eta_k), \\ \dot{\eta}_k = s_k, \\ \dot{s}_k = -k_2 s_k - \delta(\eta_k - w_k) - \sum_{j=1}^N a_{kj}((\eta_k-\eta_j)-(v_k-v_j)), \\ \dot{v}_k = \sum_{j=1}^N a_{kj}(\eta_k-\eta_j) + \sum_{j=1}^N a_{kj}(s_k - s_j), \qquad k \in \mathscr{V}. \end{cases} \tag{10}$$

Define a map as

$$\Xi = \begin{bmatrix} G(w,\eta) \\ \delta(\eta - w) \end{bmatrix}, \tag{11}$$

where $\eta = col(\eta_1,\cdots,\eta_N)$.

Let $\tilde{w} = col(w,\eta)$, $\tilde{s} = col(\dot{w},s)$, $v = col(v_1,\cdots,v_N)$, $\dot{w} = col(\dot{w}_1,\cdots,\dot{w}_N)$ and $s = col(s_1,\cdots,s_N)$. By virtue of the definition of (11) and write the closed-loop system (10) *in a compact form, we have*

$$\begin{cases} \dot{\tilde{w}} = \tilde{s}, \\ \dot{\tilde{s}} = -k_2\tilde{s} - \Xi(\tilde{w}) - \Phi_1\tilde{w} - \Phi_2 v, \\ \dot{v} = \Phi_3(\tilde{w}+\tilde{s}). \end{cases} \tag{12}$$

where

$$\Phi_1 = \begin{bmatrix} 0 & 0 \\ 0 & L\otimes I_n \end{bmatrix}, \Phi_2 = \begin{bmatrix} 0 \\ L\otimes I_n \end{bmatrix}, \Phi_2 = \begin{bmatrix} 0 & L\otimes I_n \end{bmatrix}.$$

With the following coordinate transformation, we have

$$\begin{cases} \bar{w} = col(\bar{w}_q,\bar{w}_y) = \tilde{w} - \tilde{w}^*, \\ \bar{s} = col(\bar{s}_q,\bar{s}_y) = \tilde{s} - \tilde{s}^*, \quad \bar{v} = v - v^*, \end{cases}$$

where $\tilde{w}^* = col(w^*,0)$, $\bar{w}_q$, $\bar{w}_y$, $\bar{s}_q$, $\bar{s}_y \in \mathbb{R}^N$. Then the following system is obtained by (12)*,*

$$\begin{cases} \dot{\bar{w}} = \bar{s}, \\ \dot{\bar{s}} = -k\bar{s} - h - \Phi_1\bar{w} - \Phi_2\bar{s}, \\ \dot{\bar{v}} = \Phi_3(\bar{w}+\bar{s}), \end{cases} \tag{13}$$

where $h = \Xi(\tilde{w},\eta) - \Xi(\tilde{w}^*,\eta)$.

Since $\tilde{w} \to \tilde{w}^*$ is equivalent to $\bar{w} \to 0$, we are going to analyze the convergence of $\bar{w} \to 0$.

With the eigenvalue decomposition of Laplace matrices,

$$\begin{cases} col(w_{qr},w_{qR}) = [r \quad R]^\top \bar{w}_q, \\ col(w_{yr},w_{yR}) = [r \quad R]^\top \bar{w}_y, \\ col(s_{qr},s_{yR}) = [r \quad R]^\top \bar{s}_q, \\ col(s_{yr},w_{sR}) = [r \quad R]^\top \bar{s}_y, \\ col(\delta_1,\delta_2) = [r \quad R]^\top \bar{v}, \end{cases} \tag{14}$$

where w_{qr}, w_{yr}, s_{qr}, s_{yr}, $\delta_1 \in \mathbb{R}$, w_{qR}, w_{yR}, s_{qR}, s_{yR}, $\delta_2 \in \mathbb{R}^{N-1}$, $r = \frac{1}{\sqrt{N}}1_N$, $RR^\top = I_N - \frac{1}{N}1_N 1_N^\top$ and $r^\top R = 0_{N-1}^\top$, $R^\top R = I_{N-1}$.

Let $w_r = col(w_{qr},w_{yr})$, $w_R = col(w_{qR},w_{yR})$, $s_r = col(s_{qr},s_{yr})$, $s_R = col(s_{qR},s_{yR})$, and then we rewritten (13) *as*

$$\begin{cases} \dot{w}_r = s_r, \\ \dot{s}_r = -k_2 s_r - \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}^\top h, \\ \dot{\delta}_1 = 0_{N-1}, \end{cases} \tag{15}$$

$$\begin{cases} \dot{w}_R = s_R, \\ \dot{s}_R = -k_2 s_R - \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}^\top h - \begin{bmatrix} 0 \\ R^\top LR \end{bmatrix}(w_R - \delta_2), \\ \dot{\delta}_2 = \begin{bmatrix} 0 & R^\top LR \end{bmatrix} w_R + \begin{bmatrix} 0 \\ R^\top LR \end{bmatrix} s_R. \end{cases} \tag{16}$$

Take the following candidate Lyapunov function as

$$L = \frac{1}{2}\big(\|w_r+s_r\|^2 + (k_2-1)\|w_r\|^2 + \|w_R+s_R\|^2 + (k_2-1)\|w_R\|^2 + \|\delta_2\|^2 + \theta\|s_{yR}+\delta_2\|^2\big), \tag{17}$$

where $0 < \theta < \min\{\frac{2\lambda_2 k_2\gamma}{2k_2+\lambda_2}, \frac{2\lambda_2(k_2 - \frac{L_y^2}{\gamma} - \frac{1}{4}\lambda_N - 1)}{k_2(k_2+1)+2\lambda_2\lambda_N}\}$ and $k_2 > \frac{L_y^2}{\gamma} + \frac{1}{4}\lambda_N + 1$.

Let $\tilde{r} = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix}$ and $\tilde{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$, then the derivative of L_y along (15) *and* (16) *is*

$$\begin{aligned} \dot{L}_y = & -w_r^\top\tilde{r}^\top h - w_R^\top\tilde{R}^\top h - (k_2-1)\|s_r\|^2 - (k_2-1)\|s_R\|^2 \\ & - s_r^\top\tilde{r}^\top h - s_R^\top\tilde{R}^\top h - w_{yR}^\top R^\top LRw_{yR} - s_{yR}^\top R^\top LRw_{yR} \\ & + \theta(-k_2\delta_2^\top s_{yR} - \delta_2^\top[0 \quad R^\top]h - \delta_2^\top R^\top LR\delta_2 \\ & + s_{yR}^\top R^\top LRs_{yR} - k_2\|s_{yR}\|^2 - s_{yR}^\top[0 \quad R^\top]h). \end{aligned} \tag{18}$$

With the definition of $\Xi(w,\eta)$ in (11) *and F_k, G_k in* (6)*, it is easy to find that the mapping $\Xi(w,\eta)$ is γ-strongly monotone and L_f-Lipschitz for some $\delta > 0$ on $\mathbb{R}^2$, and $G(w,\eta)$ is l_g-Lipschitz. Hence, we have*

$$w_r^\top\tilde{r}^\top h + w_R^\top\tilde{R}^\top h \geq \gamma(\|w_r\|^2 + \|w_R\|^2), \tag{19}$$

and

$$s_r^\top\tilde{r}^\top h + s_R^\top\tilde{R}^\top h \leq \frac{1}{2}\big(\frac{L_f^2}{\gamma}(\|s_r\|^2+\|s_R\|^2) + \gamma(\|w_r\|^2+\|w_R\|^2)\big), \tag{20}$$

With the schur complement lemma, the matrix

$$\begin{bmatrix} R^\top LR & \frac{1}{2}R^\top LR \\ \frac{1}{2}R^\top LR & \frac{1}{4}\lambda_N I \end{bmatrix} \geq 0$$

implies that

$$\begin{aligned}
&w_{yR}^{\top}R^{\top}LRw_{yR}+s_{yR}^{\top}R^{\top}LRw_{yR}\\
&=[w_{yR}^{\top}\quad s_{yR}^{\top}]\begin{bmatrix}R^{\top}LR & \frac{1}{2}R^{\top}LR\\ \frac{1}{2}R^{\top}LR & \frac{1}{4}\lambda_N I\end{bmatrix}\begin{bmatrix}w_{yR}\\ s_{yR}\end{bmatrix}-\frac{1}{4}\lambda_N\|s_{yR}\|^2 \qquad (21)\\
&\geq -\frac{1}{4}\lambda_N\|s_R\|^2.
\end{aligned}$$

In addition, with $ab \leq \frac{c}{2}a^2+\frac{1}{2c}b^2$, $c>0$, *we have*

$$-k_2\delta_2^{\top}s_{yR} \leq \frac{1}{2}k_2\Big(\frac{\lambda_2}{k_2+1}\|\delta_2\|^2+\frac{k_2+1}{\lambda_2}\|s_{yR}\|^2\Big), \qquad (22)$$

$$-\delta_2^{\top}[0 \quad R^{\top}]h \leq \frac{1}{2}\Big(\lambda_2\|\delta_2\|^2+\frac{1}{\lambda_2}(\|w_r\|^2+\|w_R\|^2)\Big), \qquad (23)$$

$$-s_{yR}^{\top}[0 \quad R^{\top}]h \leq \frac{1}{2}\Big(2k_2\|s_{yR}\|^2+\frac{1}{2k_2}(\|w_r\|^2+\|w_R\|^2)\Big), \qquad (24)$$

With (19)-(24), *the derivative of* L_y *satisfies*

$$\begin{aligned}
\dot{L}_y \leq &-\Big(\frac{1}{2}\gamma-\theta\Big(\frac{1}{2\lambda_2}+\frac{1}{4k_2}\Big)\Big)(\|w_r\|^2+\|w_R\|^2)\\
&-\Big(k_2-\frac{l_f^2}{\gamma}-1\Big)\|s_r\|^2-\Big(k_2-\frac{l_f^2}{\gamma}-1-\frac{\lambda_N}{4} \qquad (25)\\
&-\theta\Big(\lambda_N+\frac{k_2(k_2+1)}{2\lambda_2}\Big)\Big)\|s_R\|^2-\frac{\theta\lambda_2}{2(k_2+1)}\|\delta_2\|^2.
\end{aligned}$$

Since L_y *and its derivative* $\dot{L}_y$ *are quadratic, and all states occur in* L_y *also in* $\dot{L}_y$, *the rotor speed* w *exponentially converges to* w^*.

Next, we are going to prove that the equilibrium w^* where the closed-loop system (12) converges to is the minimum point of the value function $V_k(w)$ in Problem 1.

Theorem 2 *If* w^* *is a minimum point of the value function* $V_k(w)$ *that is defined in Problem 1, then there exist* $\eta^*, s^*, v^* \in \mathbb{R}^N$ *such that* (w^*,η^*,s^*,v^*) *is an equilibrium of multi-agent systems with PH dynamic (3) under the action of the distributed controller (7). Conversely, if* (w^*,η^*,s^*,v^*) *is an equilibrium of multi-agent systems with PH dynamic (3) under the control of the distributed controller (7), then* w^* *is a minimum point of the value function* $V_k(w)$ *which is defined in Problem 1.*

Proof *If (12) is at its equilibrium, we have*

$$\begin{cases}\tilde{s}^*=0_{2N},\\ -k\tilde{s}^*-\Xi(\tilde{w}^*)-\Phi_1\tilde{w}^*-\Phi_2v^*=0_{2N},\\ \Phi_3(\tilde{w}^*+\tilde{s}^*)=0_{2N}.\end{cases} \qquad (26)$$

which indicated that

$$\begin{cases}L\eta^*=0_N,\\ G(w^*,\eta^*)=0_N,\\ \delta(\eta^*-w^*)+Lv^*=0_N.\end{cases} \qquad (27)$$

Since the undirected connected graph $\mathcal{G}$ *has a character that its Laplace matrix* L *satisfies* $1_N^{\top}L=0_N^{\top}$, *if we multiply both sides by* $1_N^{\top}$ *to the left of the third equation of (27), then we have*

$$\eta_k^*=\frac{1}{N}\sum_{j=1}^{N}w_j^*, \qquad (28)$$

and at this time

$$G_k(w_k^*,\eta_k^*)=F_k(w_k^*,\sigma)=0, \qquad (29)$$

for any $k\in\mathcal{V}$.

Combing with the definition of $F_k(w_k^*,\sigma)$ *in* (6), *and with* (24) *and* (28), *it can be obtained that*

$$\nabla_{w_k}V_k(w^*)=0, \quad \forall k\in\mathcal{V}$$

and this indicates w^* *is the minimum point of the value function defined in Problem 1.*

On the other hand, if w^* *is a minimum point of the value function defined in Problem 1, then* $F_k(w_k^*,\sigma)=0_m$. *By choosing*

$$\eta_k^*=\sigma(w^*), \quad k\in\mathcal{V},$$

we have

$$G_k(w_k^*,\eta_k^*)=0.$$

In addition, take any $\hat{r}\in\mathbb{R}$ *and let* $r'=1_N\hat{r}$, *we have*

$$(\eta^*-w^*)^{\top}r'=0.$$

And by $1_N^{\top}L=0_N^{\top}$, *we have* $r'\in ker(L)$. *With the orthogonal decomposition of zero space and range space, it can be obtained that* $\delta(\eta^*-\psi(w^*))\in range(L)$, *where* $\delta>0$. *Therefore, there exists* v^* *such that*

$$\delta(\eta^*-\psi(w^*))+Lv^*=0_N.$$

Finally, when we choose

$$\tilde{s}^*=0_{2N},$$

the proof is completed. □

With Theorem 1 and Theorem 2, we can find that with the distributed controller (7), the PH system (3) exponentially converges to the equilibrium w^* which is the minimum point of the value function defined in Problem 1. And at this time, the rotor speed of N-PMSMs will reach a consensus.

V. SIMULATION

In this section, a simulation example is presented to illustrate the effectiveness of the proposed distributed controller (7).

Consider 6-PMSMs (modeled as (3)) communicate with a undirected connected graph $\mathcal{G}$ as in Fig. 2.

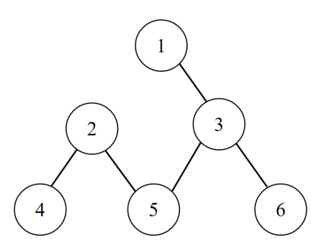

Fig. 2. The communicate topology of 6-PMSMs.

As described in Section III Problem Formulation, each PMSM has two goals

- To synchronize the rotor speed of multiple PMSMs, that is $t\to\infty, w_k\to\sigma(w)=\frac{1}{6}\sum_{k=1}^{6}w_k$, $k\in[1:6]$;

TABLE I
THE PARAMETER TABLE OF EXAMPLE 2

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$
n_k	2	4	2	4	2	4
ϕ_k	0.25	0.14	0.15	0.20	0.24	0.18
L_{qk}	5.45	5.4	5.5	5.8	5.65	5.35
L_{dk}	4.48	5.24	5.15	5.18	4.65	4.45
$\tilde{J}_k$	0.2	0.15	0.24	0.18	0.22	0.26
$w_k(0)$	80	144	70	135	122	90
$i_{qk}(0)$	2.1	1.9	2.2	1.5	1.2	1.9
$i_{dk}(0)$	0	0	0	0	0	0

- Try to maintain the initial angular speed to reduce changes of its working state, that is $w_k \to w_k(0)$, $k \in [1{:}6]$.

Since the first goal is far more important than the second one, the value function for the kth PMSM ($k \in [1{:}6]$) is designed as

$$V_k(w) = 10(w_k - \sigma(w))^2 + 0.1(w_k - w_k(0))^2. \quad (30)$$

Let $R_{sk} = 1$, $\tau_{lk} = 2$, $k \in [1{:}6]$, and other parameters are assignment in Table I. Then the simulation results is shown as Fig. 3.

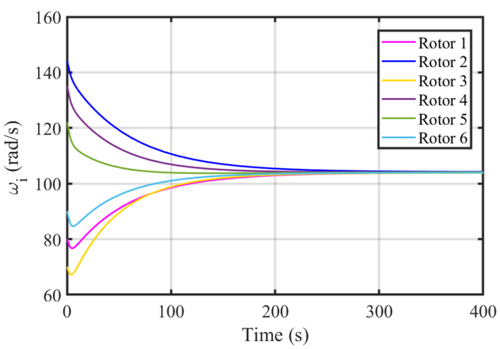

Fig. 3. The evolution of rotor speed of 6 PMSMs.

As shown in Fig. 3, the rotor speeds of the 6 PMSMs eventually converge to unity, achieving synchronised speed control of multiple PMSMs, which illustrate the effectiveness of the designed distributed controller (7).

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This work investigate the rotor speed synchronous problem of multiple PMSMs with dynamics modeled under the port-Hamiltonian framework. By converting the rotor speed synchronous problem into an optimization problem and designed a properly value function, we proposed a distributed controller for each PMSM. The proposed distributed controller only requires PMSMs to exchange its estimation of the average rotor speed rather than the exact rotor speed of each PMSM which may benefit the sensitive state protection. Under the action of the distributed controller, the closed-loop system exponentially convergent to the equilibrium which is also the minimum of the value function. Finally, a simulation example is given to demonstrate the effectiveness of the proposed distributed controller.

B. Future Works

This paper considers simple connected undirected graphs, which can be considered next to relax some assumptions and broader topologies. The unknown torque and the uncertainty of model may also be considered in the future.

ACKNOWLEDGMENT

The authors gratefully acknowledge the contribution of the National Natural Science Foundation of China under Grants 62173602 and the EIPHI Graduate School (contract ANR-17-EURE-0002).

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