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Analysis of barrier function based adaptive sliding mode control in the presence of deterministic noise[☆]

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ABSTRACT

Barrier function-based adaptive sliding mode control (BFASMC) is analyzed in presence of deterministic measurement noise. It is shown that, considering only boundedness of the measurement noise, it is impossible to select the controller parameters to track some perturbation with unknown bound. Nonetheless, under the assumption of continuity of the noise, the tracking of such a perturbation is possible; however, the barrier function width depends on the bound of the noise. If Lipschitz continuity of the noise is assumed, then it follows that the width of the barrier function can be chosen arbitrarily.

1. Introduction

Sliding-mode control (SMC) is well known by its robustness against coupled disturbances. To achieve exact compensation of such disturbances, the gains of SMCs must dominate the upper bound of the perturbations. This need for *a priori* knowledge of the bound of the perturbation makes the controller gains to be overestimated most of the time, which aggravates the problems of chattering and energy consumption. In real systems, this upper bound exists but it could be unknown or overestimated. Thus, adaptive sliding mode controllers (ASMCs) are considered as a solution to the problem of gain overestimation.

There are three classical methods for the design of ASMCs Shtessel, Fridman, and Plestan (2016):

- Increasing gains: The gain of the controller increases until the instantaneous bound on the perturbation is compensated. This scheme guarantees the establishment of the sliding mode. However, this approach has two main disadvantages: overestimation of the controller gain and uncertainty regarding the time at which the sliding mode will be finally established.
- Reconstruction of equivalent control: A filter is employed to estimate the value of the equivalent control, giving an approximation of the disturbance, but the filter introduces a delay in closed loop system meaning that the sliding mode can be lost. Moreover, for the design of the filter, an upper-bound of the derivative of the perturbation is needed; and so, continuous sliding-mode control can be employed, ensuring the existence of second order sliding mode with better asymptotic precision.
- Increasing and decreasing gains: The gains of the controller are increased and decreased to try and track the perturbation. This results in ultimate boundedness of the trajectories of the system. Nevertheless, the ultimate bound is contingent upon the unknown perturbation bound, making it impossible to ascertain the exact moment when this bound is achieved.

In Obeid, Fridman, Laghrouche, and Harmouche (2018) a Barrier function-based adaptive sliding mode control (BFASMC) was proposed, allowing the state to be constrained to a vicinity of the origin (barrier) with an *a priori* predefined size (barrier width) of the origin. This is achieved by choosing the gains of BFASMCs as a continuous concave function with vertical asymptotes in the boundary of the barrier. Recently in Cruz-Ancona, Fridman, Obeid, Laghrouche, and Pérez-Pinacho (2023), a modification of

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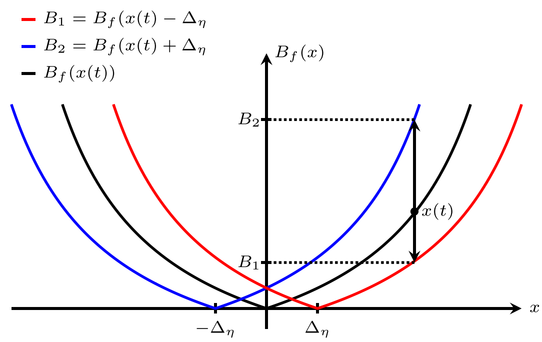

Fig. 1. Effects of η on the centering of the barrier function.

BFASMC is proposed, ensuring the system's solution will reach the prescribed barrier in a predefined time.

Some applications of BFASMCs have been presented for a variety of systems, such as robot manipulators (Mobayen, Alattas, & Assawinchaichote, 2021), surface vehicles (Yan, Zhao, Yu, & Wang, 2021), ABS systems (Rodrigues, Hsu, Oliveira, & Fridman, 2022), Duffing oscillator (Mousavi, Markazi, & Ferrara, 2023), linear motors (Shao et al., 2021), piezoelectric actuators (Ma et al., 2022), quad-rotors (Alattas et al., 2022), to name a few.

The theory of BFASMCs and its applications do not consider measurement noise which, in real applications, will always be present. When the state of the system is close to the boundary of the barrier, the gains of the controller tend to infinity and, consequently, the presence of the noise could force the trajectories of the system to leave the barrier.

This note will illustrate that:

- In the presence of general bounded deterministic noises, the barrier width cannot be chosen arbitrarily.
- In the case of continuous and bounded deterministic measurement noise, the barrier width can be chosen based only on the bound of the noise.
- In the case of globally Lipschitz continuous noise, the barrier width can be selected arbitrarily, in spite of an unknown Lipschitz constant.

2. Problem statement

Consider the following dynamical system

$$\begin{aligned} \dot{x} &= u + \delta(t, x) \\ y &= x + \eta(t) \end{aligned} \tag{1}$$

where $x, u, y \in \mathbb{R}$ are the state vector, control input and system output, respectively. $\eta : \mathbb{R}_+ \to \mathbb{R}$ represents the deterministic measurement noise and $\delta : \mathbb{R}_+ \times [-(\varepsilon + \Delta_\eta), \varepsilon + \Delta_\eta] \to \mathbb{R}$, for some $\Delta_\eta > 0$ to be defined in the sequel, is a Lebesgue measurable function representing uncertainties and disturbances.

Consider the following BFASMC

$$u = -B_f(y)\mathrm{sign}(y) \tag{2}$$

where $B_f(y)$ represents the barrier function $B_f(y) = \frac{|y|}{(\varepsilon - |y|)}$. The parameter $\varepsilon > 0$ will be referred to as the barrier width (BW).

Since $\eta(t)$ is not available, and $y(t) = 0$ implies $x(t) = -\eta(t)$, it is impossible to know the exact position of the center of the barrier, see Fig. 1.

Remark 1. Controller (2) requires $|y(t_0)| < \varepsilon$. Without loss of generality, we can assume that the initial condition of the output lies within the barrier, as it is always feasible to make modifications to reach the barrier within a predefined time (Cruz-Ancona et al., 2023). However, to guarantee that $|y(t_0)| < \varepsilon$ in (1), ε must be bigger than the value of $\eta(t_0)$. Since $\eta(t)$ is unknown, this imposes a restriction on ε in terms of the upper bound of the noise, whenever such a bound exists.

Assumption 1. There exists some positive constant $\bar{\delta} > 0$ such that, for all $t \in [t_0, \infty)$ and $x \in [-(\varepsilon + \Delta_\eta), \varepsilon + \Delta_\eta]$, for some $\Delta_\eta > 0$ the perturbation is bounded as $|\delta(t, x)| \le \bar{\delta}$.

The objective of this work is to analyze the restrictions on the selection of the BW for three classes of deterministic noises:

(1) Bounded
(2) Continuous bounded
(3) Globally Lipschitz continuous

3. Case 1: Bounded deterministic noises

Assumption 2. There exists some known positive constant $\Delta_\eta > 0$ such that, $|\eta(t)| \le \Delta_\eta$, for all $t \in [t_0, \infty)$.

Sufficient conditions for the boundedness of (1) controlled by (2) under Assumptions 1 and 2 are given in the following theorem

Theorem 1. *Let (1) be controlled by (2) and Assumptions 1 and 2 hold for some known $\bar{\delta}$. If $\varepsilon > 2\Delta_\eta(\bar{\delta} + 1)$ and $y(t_0) \in (-\gamma\varepsilon, \gamma\varepsilon)$, with $\gamma \in \left(0, 1 - \frac{2\Delta_\eta}{\varepsilon}\right)$ then the state will be constrained in the set $|x| < \varepsilon - \Delta_\eta$, which implies that $y(t) \in (-\varepsilon, \varepsilon)$ for all $t \in [t_0, \infty)$.*

Proof. The closed-loop system for (1)–(2) is given by

$$\dot{x} = \frac{x + \eta}{(\varepsilon - |x + \eta|)} + \delta \tag{3}$$

Consider the Lipschitz continuous barrier Lyapunov function candidate

$$V = \frac{1}{2}x^2 + \frac{|x|}{(\varepsilon - \Delta_\eta - |x|)}. \tag{4}$$

such that $\lim_{x \to \varepsilon - \Delta_\eta} V(x) = \infty$. For any $|x| < \varepsilon - \Delta_\eta$ (4) can be seen as the sum of a quadratic term in x and a rational function with positive numerator and positive denominator, thus V is positive definite for all $|x| < \varepsilon - \Delta_\eta$. Note that the proposed selection for y_0 implies that $|x_0| < \varepsilon - \Delta_\eta$. Its derivative, wherever it exists, along the trajectories of (3) satisfies the inequality $\dot{V} \le -F(x)\left(\frac{\mathrm{sign}(x)(x+\eta)}{(\varepsilon - |x+\eta|)} - \bar{\delta}\right)$ with $F(x) = \left(|x| + \frac{\varepsilon + \Delta_\eta}{(\varepsilon - \Delta_\eta - |x|)^2}\right) > 0$. Then, $\dot{V} \le 0$ if:

$$\mathrm{sign}(x + \eta)\mathrm{sign}(x) \ge \bar{\delta}\left(\frac{\varepsilon}{|x + \eta|} - 1\right). \tag{5}$$

Three different cases will be considered:

- Case a) $\mathrm{sign}(\eta) = \mathrm{sign}(x)$:

In this case, (5) is written as $1 \ge \bar{\delta}\left(\frac{\varepsilon}{|x| + |\eta|} - 1\right)$, which implies $|x| \ge \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}} - \Delta_\eta$. Then, V will be bounded for $|x| \ge \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}} - \Delta_\eta$.

- Case b) $\mathrm{sign}(\eta) \ne \mathrm{sign}(x)$ and $|x| > |\eta|$:

The conditions imply $|x + \eta| = |x| - |\eta|$ and $\mathrm{sign}(x + \eta) = \mathrm{sign}(x)$; then, (5) is written as $1 \ge \bar{\delta}\left(\frac{\varepsilon}{|x| - |\eta|} - 1\right)$, which implies $|x| \ge \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}} + \Delta_\eta$. Then, in this case, V will be bounded for $|x| > \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}} + \Delta_\eta$.

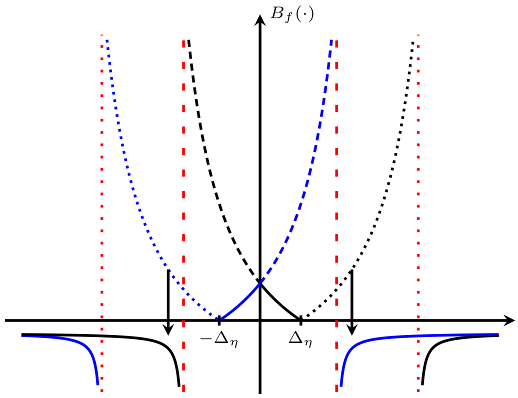

Fig. 2. Graphical behavior of the state in the three cases.

- Case c) $\operatorname{sign}(\eta) \neq \operatorname{sign}(x)$ and $|x| < |\eta|$:

The conditions imply $|x+\eta| = |\eta| - |x|$ and $\operatorname{sign}(x+\eta) = -\operatorname{sign}(x)$; then, (5) is written as $-1 \geq \bar{\delta}\left(\frac{\varepsilon}{|\eta|-|x|} - 1\right)$. From $\varepsilon > 2\Delta_\eta$, this is not feasible. Hence, $|x|$ might grow and reach case (a) or (b).

Since $\eta(t)$ is not assumed to be continuous, it is possible that the trajectories of the system jump between different sets where either case (a) or (b) occurs. Then, the conditions for cases (a) and (b) must be met simultaneously. This means that $\varepsilon - \Delta_\eta > \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}} + \Delta_\eta$ should hold, which implies $\varepsilon > 2\Delta_\eta(\bar{\delta}+1)$. This selection of ε implies that $\dot{V} < 0$ for $\varepsilon - \Delta_\eta > |x(t)| \geq \varepsilon - \Delta_\eta$. ■

Remark 2. If the condition $\varepsilon > 2\Delta_\eta(\bar{\delta}+1)$ is not fulfilled, the set where $\dot{V} \leq 0$ might be empty. Then, the system cannot remain within the barrier (see the simulation example).

Remark 3. Theorem 1 presents some **inherent limitations** of BFASMC: whenever the measurement noise is discontinuous, the barrier function width **must be chosen depending on the bound of the noise and the perturbation.** Fig. 5 shows that an improper selection of the barrier function width could lead BFASMCs to fail. Moreover, a linear stabilizing controller ensures at least ultimate boundedness of solution, even if the upper bound of the perturbation is unknown (Khalil, 2002).

The effects of the noise on the stability of the system are shown in Fig. 2. If the state lies in the discontinuous lines, $\dot{V} < 0$. The continuous line indicates the case where $\dot{V} > 0$. Furthermore, if the state is on the dotted line, it is impossible to avoid the jumps to the other side of the barrier. Some examples of these jumps is given by the vertical arrows. Nonetheless, the selection of ε implies the state is restricted to the set marked as vertical dashed lines. In this set all possible jumps will be between sets where $\dot{V} < 0$.

Remark 4. In the majority of applications the measurements for system (1) can be made by:

- A digital sensor discretizing the measured variable with very small sampling periods. In this case any measured signal can be assumed to be continuous.
- An analog sensor having its own dynamics. In this case the sensor output is at least Lipschitz continuous.

This motivates the study of the effects these kinds of noises would have on system (1).

4. Case 2: Continuous bounded noise

Assumption 3. The function $\eta(t)$ is a continuous function which satisfies Assumption 2.

Corollary 2. *Let (1) be controlled by (2) and Assumptions 1 and 3 hold. If the BW is chosen as $\varepsilon > 2\Delta_\eta$ and $y(t_0) \in (-\varepsilon, \varepsilon)$, then $y \in (-\varepsilon, \varepsilon)$ for all $t \geq t_0$, which implies that the state will be constrained in $|x| < \varepsilon + \Delta_\eta$.*

Proof. The proof follows directly from the proof of Theorem 1, with $V = \frac{1}{2}x^2 + \frac{|x|}{\varepsilon+\Delta_\eta-|x|}$, such that $\lim_{x\to\varepsilon+\Delta_\eta} = \infty$. Since $\dot{V}$ cannot be negative for Case (c), only Cases (a) and (b) are considered. Furthermore, since $\eta(t)$ is continuous, it is not possible for x to jump from one set to another, meaning that now that the conditions

$$\dot{V} \leq 0 \implies \begin{cases} \varepsilon - \Delta_\eta > |x| \geq \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}} - \Delta_\eta & \text{Case a)} \\ \varepsilon + \Delta_\eta > |x| \geq \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}} + \Delta_\eta & \text{Case b)} \end{cases}$$

do not have to be met simultaneously. From the fact that $|y| = |x| + |\eta|$ in Case (a) and $|y| = |x| - |\eta|$ in Case (b), one has:

$$\dot{V} \leq 0 \implies \begin{cases} \varepsilon > |x| + \Delta_\eta \geq |y| \geq \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}} & \text{Case a)} \\ \varepsilon > |y| \geq |x| - \Delta_\eta \geq \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}} & \text{Case b)} \end{cases}$$

meaning that $\dot{V} \leq 0 \implies \varepsilon > |y| \geq \varepsilon\frac{\bar{\delta}}{1+\bar{\delta}}$, which is feasible for any value of $\bar{\delta}$. Then, $\varepsilon > |y|$ implies $\varepsilon + \Delta_\eta > |x|$. ■

Remark 5. In the case of continuous noise, ε does not depend on $\bar{\delta}$. On the other hand, for any linear controller, the set where the state is ultimately bounded cannot be predefined if $\bar{\delta}$ is unknown.

Remark 6. In the case of continuous noise, the value of ε is restricted as $\varepsilon > 2\Delta_\eta$. In this case, the use of (2) ensures that the solutions of (1) will never leave the barrier.

5. Case 3: Lipschitz continuous noise

Assumption 4. The measurement noise is globally Lipschitz continuous, with some unknown Lipschitz constant $\bar{\Delta}_\eta$.

The following theorem shows that under Assumption 4, the size of the BW can be chosen arbitrarily.

Theorem 3. *Let (1) be controlled by (2) and Assumptions 1 and 4 hold. If the BW is chosen as $\varepsilon > 0$ and $y(t_0) \in (-\varepsilon, \varepsilon)$, then $y(t)$ will be constrained in the set $y(t) \in (-\varepsilon, \varepsilon)$, for all $t \geq t_0$.*

Proof. From the fact that $\eta(t)$ is globally Lipschitz continuous, it follows that it is differentiable almost everywhere and its derivative is essentially bounded. Thus, considering that $y(t) = x(t)+\eta(t)$, it is possible to rewrite (1) as $\dot{y} = -\frac{y}{\varepsilon-|y|} + \delta_1(t)$ almost everywhere, with $\delta_1(t) = \delta(t)+\dot{\eta}(t)$, and $|\delta_1(t)| \leq \bar{\delta}+\bar{\Delta}_\eta$, which is bounded but the bound is unknown. Then, one can directly apply the results of Obeid et al. (2018) to show that $y(t)$ is confined to $|y| \leq \varepsilon$. ■

Remark 7. The size of ε does not have a theoretical restriction. Nonetheless, since the initial condition of the noise is generally unknown, a reasonable selection for ε is $\varepsilon > 2\Delta_\eta$ if Δ_η exists and it is known.

Remark 8. For every single case, the output variable is constrained within a given set which restricts the values that the state can achieve. Then, the ultimate bound of the state depends linearly on the bound of the noise. These facts imply that, in practical terms, one cannot expect any filtering from the BFASMC.

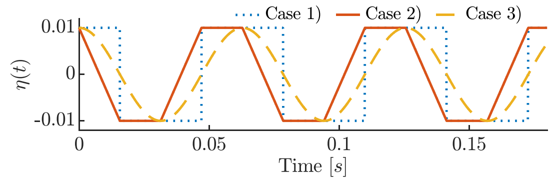

Fig. 3. Different noises to be considered.

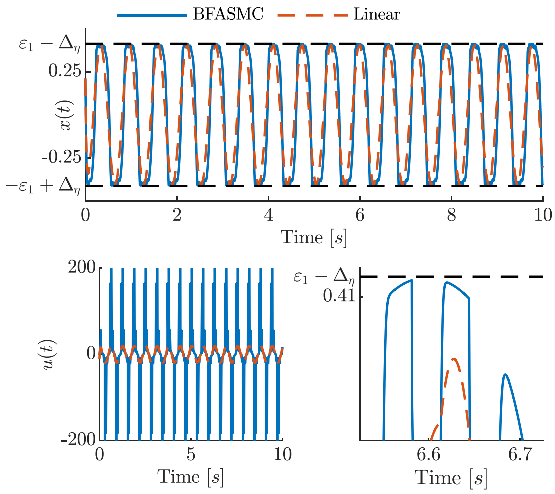

Fig. 4. Simulation results for Case (1). (Top) State trajectory constrained by the BFASMC and linear controller with ε_1; (Bottom-left) Control signal of the BFASMC and linear controller; (Bottom-right) zoom of the Top plot.

6. Simulations

To exemplify the practical implications of the presented results and to offer a deeper understanding of the discussed concepts, we will explore three specific cases, changing the definition of $\eta(t)$ in (1):

- Case (1): $\eta(t) = \Delta_\eta \text{sign}(\cos(\omega_d t))$
- Case (2): $\eta(t) = \begin{cases} \Delta_\eta\,(1-2T) & 0 < T \leq 1 \\ -\Delta_\eta & 1 < T \leq 2 \\ \Delta_\eta\,(2T-5) & 2 < T \leq 3 \\ \Delta_\eta & 3 < T \leq 4 \end{cases}$
- Case (3) $\eta(t) = \Delta_\eta(\cos(\omega_d t))$

with $\omega_d = 100$, $\Delta_\eta = 0.01$ and $T = \{\frac{2t\omega_d}{\pi} \mod (4)\}$. These noises are depicted in Fig. 3. Every simulation is performed using backward Euler discretization with a sample time of $\tau = 1\mu s$, considers a perturbation given by $\delta = 20\cos(10t)$ and $x(t_0) = 0.5\varepsilon$. the rest of this section will be divided in three subsections, each of them dealing with one type of deterministic noise. The first subsection will compare the BFASMC against a linear controller for Case (1), since $\bar{\delta}$ is needed *a priori* for the selection of ε (see Theorem 1). However, the next subsections will only deal with the BFASMC, since the dependence of $\bar{\delta}$ for the selection of ε is no longer needed in Case (2) and Case (3) (refer to Corollary 2 and Theorem 3). A linear controller ensuring that the BW is positively invariant for an unknown value of $\bar{\delta}$ cannot be designed.

6.1. Case (1): BFASMC in presence of discontinuous noise

In this case $\varepsilon > 2\Delta_\eta(\bar{\delta}+1)$ (from Theorem 1). Some knowledge of $\bar{\delta}$ is needed to properly apply BFASMC; furthermore, if $\bar{\delta}$ is known a linear controller of the form $u = -Ky$ can be applied to

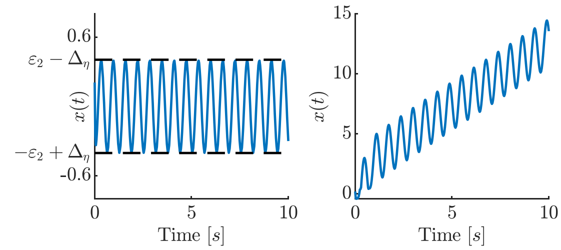

Fig. 5. Effects of underestimation of the bound of the disturbance for Case (1). (Left) Linear controller for an underestimation of $\bar{\delta}$; (Right) BFASMC for an underestimation of $\bar{\delta}$.

(1) to ensure that $|x(t)| < \varepsilon$ as stated in Remark 3, if $K > \frac{\bar{\delta}}{\varepsilon - \Delta_\eta}$[1]. Thus, a comparison between both controllers is shown.

To illustrate the tightness of the inequality $\varepsilon > 2\Delta_\eta(\bar{\delta}+1)$ for the case of discontinuous noises, two different values of ε are considered: $\varepsilon_1 = 2.01\Delta_\eta(\bar{\delta}+1) \implies \varepsilon_1 = 0.4221$ and $\varepsilon_2 = 1.97\Delta_\eta(\bar{\delta}+1) \implies \varepsilon_2 = 0.4137$.

Note that ε_1 is selected only 0.5% bigger, while ε_2 underestimates the value of ε by 1.5%. For the linear controller two values of K are considered, i.e. $K_i = \frac{\bar{\delta}}{\varepsilon_i - \Delta_\eta}$ for $i \in \{1, 2\}$ implies $K_1 = 48.532$ and $K_2 = 49.542$. The numerical simulations of the BFASMC against the linear controller for ε_1 and ε_2 are shown in Figs. 4 and 5 respectively.

The top of the first simulation (shown in Fig. 4) shows that both controllers are able to ensure that $|x(t)| < \varepsilon - \Delta_\eta$, which is made clearer by the zoom shown by the bottom-right plot. The bottom-left plot shows the control signal; $u(t)$ is more aggressive for the adaptive controller due to the jumps between the barrier functions (see Fig. 2) and the precision of both techniques is similar. Since the knowledge of $\bar{\delta}$ is needed for the selection of ε, no real advantage for the adaptive controller is achieved.

Fig. 5 shows the effect of a small underestimation of ε by considering ε_2. The left plot shows that under this situation, the linear controller is capable of achieving an ultimate bound, while the underestimation of ε leads to the destruction of the predefined behavior of the BFASMC; this is shown in the right plot of Fig. 5. This shows the advantage of the linear controller for a discontinuous $\eta(t)$ compared with the BFASMC.

6.2. Case (2): BFASMC in presence of continuous noise

Similar to the previous subsection in order to show the conservativeness of $\varepsilon > 2\Delta_\eta$ mentioned in Corollary 2 two ε are chosen: $\varepsilon_1 = 2.01\Delta_\eta \implies \varepsilon_1 = 0.0201$ and $\varepsilon_2 = 1.97\Delta_\eta \implies \varepsilon_2 = 0.0197$.

The top plot of Fig. 6 shows $x(t)$ and $y(t)$ when ε_1 is employed in the BFASMC, it should be clear by the figure that $|x(t)| < \varepsilon + \Delta_\eta$ and $|y(t)| < \varepsilon$. The control law is shown in the bottom-left of Fig. 6. However, as illustrated in the bottom-right of Fig. 6, when ε_2 (1.5% smaller than the theoretical result) is used in the BFASMC, the state cannot be longer constrained to the predefined set.

6.3. Case (3): BFASMC in presence of lipschitz continuous noise

The simulations considering Lipschitz continuous noises is shown if Fig. 7. The selection of ε in this case is, theoretically, arbitrarily; nonetheless, this parameter was selected according to the comments in Remark 7 as $\varepsilon = 2.01\Delta_\eta = 0.0201$.

[1] Consider $V = \frac{1}{2}x^2$; then, $\dot{V} \leq -|x|\left(K|x| - K\Delta_\eta - \bar{\delta}\right)$. Thus, $|x| \geq \frac{K\Delta_\eta + \bar{\delta}}{K} \implies \dot{V} \leq 0$. To ensure $\varepsilon > |x|$ one can chose $K > \frac{\bar{\delta}}{\varepsilon - \Delta_\eta}$.

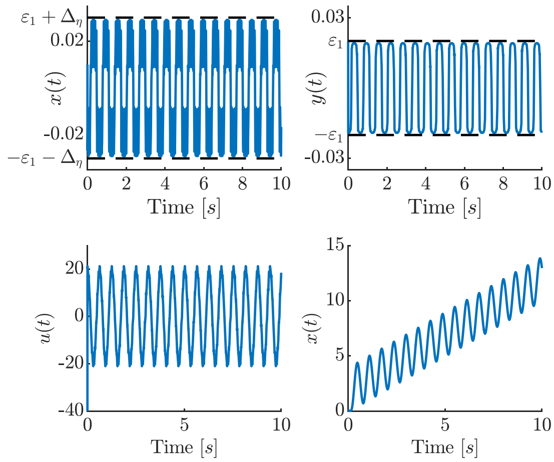

Fig. 6. Simulation results for Case (2). (Top and bottom-left) ε_1 employed in the BFASMC; (Bottom-right) ε_2 used in the BFASMC.

The simulation results of Lipschitz continuous noises using the selected ε are depicted in the top and middle of Fig. 7. The top-left and -right plots of Fig. 7 shows that both the state $x(t)$ and output $y(t)$ remain within the expected sets, while the middle of the plot shows $u(t)$. To assess that ε can be selected independently of Δ_η, a different noise, given by $\eta(t) = \Delta_\eta \cos(\omega_d t) + t$ is considered. This noise is Lipschitz continuous, but unbounded. Since the simulation is performed for $t \in [0, 10]$, one may consider $\Delta_\eta = 10.01$, which is 3 orders of magnitude larger than the rest of the simulations, while considering the same value for ε. Although, a noise with a large bound is difficult to find it in the practice, this extreme case of $\eta(t)$ helps to illustrate that selection of ε can be arbitrary. The bottom plots of Fig. 7 show both $x(t)$ and $y(t)$. In such plots, it is possible to see if the noise is Lipschitz continuous, the controller is capable of restricting the output of the system to the BW (even if $x(t)$ grows with the noise) for larger (or nonexistent) bounds of the noise. This is not the case for continuous noises, where it has been shown that the value of ε cannot be arbitrary.

7. Conclusion

The behavior of BFSMCs was analyzed under measurement noise and sufficient conditions for the selection of the BW are provided:

- For *bounded* noise, the BW can no longer be chosen arbitrarily and depends on the upper bounds of the noise and perturbation.
- For *bounded and continuous* noise, the BW does not depend on the bound of the perturbation.
- For *Lipschitz continuous* noise, the BW can be selected arbitrarily.

Simulations illustrate the results and show that values under 1.5% of the theoretical value cannot enforce the output to the predefined set.

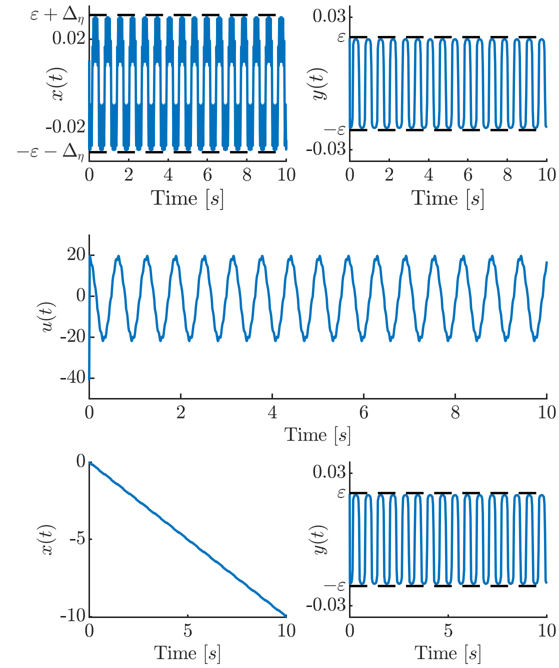

Fig. 7. (Top and middle) Simulation results for Case (3); (Bottom) Simulation results for $\eta(t) = \Delta_\eta \cos(\omega_d t) + t$.

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