A privacy preserving distributed controller for the general formation of multi-agent systems in port-Hamiltonian form

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Abstract

This paper considers the general formation control problem under the port-Hamiltonian framework and proposes a method to meet the requirements of general formation control while protecting agent privacy. First, the general formation problem is expressed as an optimization problem whose solution satisfies the requirements of the general formation. To protect the sensitive data of each agent, a distributed controller is designed through the desired general formation output dynamic, which still maintains a port-Hamiltonian form, making the Hamiltonian function the natural choice for the Lyapunov function candidate. It is then shown that the designed system converges exponentially to the global optimum of the optimization problem. Finally, simulations on an application case, namely underactuated unmanned surface vehicles with different parameters are provided to verify the effectiveness of the proposed method.

Key words: General formation control, port-Hamiltonian systems, distributed control, multi-agent systems, privacy preserving

1 Introduction

With the rapid development of sensors, industrial internet and other technologies, multi-agent collaborative control [1] has become a research hotspot. As one of the most actively studied topics of collaborative control, formation control [2–4] has also attracted remarkable attention in recent years.
The general formation control, which is summarized in [5], provides a broader framework than traditional formation control. In fact, under the framework of general formation control, many practical control problems in different physical domains can be described. For example, the formation shape control of mechanical agents [6] for mechanical systems, the speed synchronization of motors [7] for electrical systems, the power allocation of micro-grids [8] in power systems. Given the multi physics nature of the systems under consideration, it is crucial to use a suitable framework that leverages the intrinsic physical properties of these systems [9]. In this respect port-Hamiltonian (PH) system formulations highlight the physical properties of the considered systems through a well defined geometric structure and the definition of interconnected ports [10]. It is particularly well suited for control design.
There are few results about formation control of multiple PH agents. Interconnection and damping assignment passivity-based control (IDA-PBC) has been successfully applied to multi-spacecraft formation flying under the leader-followers framework in [11]. However, this work does not directly apply to other models or other formation scenarios. This remark also apply to the formation control and velocity tracking that have been developed for nonholonomic wheeled robots in [12] and [13].
The latest achievement in formation control of N-agent systems under PH framework can be found in [14, 15], where a generic methodology has been proposed for mechanical systems. In addition, the methods proposed in [14, 15] require that each agent exchanges and discloses its state with its neighbors, which is not desired in some practical applications [16]. To address the urgent need for privacy preserving in multi-agent systems, one may resort to homomorphic encryption [17, 18], differential privacy [19], multi-party secure computa-

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tion [20] or many other recent advances. However, these approaches present some thought-provoking challenges. For instance, to achieve the privacy protection, the differential-privacy mechanisms have to sacrifice provable convergence to the exact equilibrium [21], which is undesirable in the general formation problem. The authors in [22] proposed a fully distributed algorithm that achieves both privacy protection and guaranteed computational accuracy of the equilibrium, but it does not take into account the physical dynamics of each agent. In addition, if these methods are directly applied to PH systems, the structure cannot be preserved. To the authors knowledge, there is currently no results with privacy protection on the general formation with privacy protection of multi-agent systems with PH dynamics.

In this paper, a distributed controller for general formation control of multi-agent systems is proposed using the PH framework. The contributions are summarized as follows.

(1) A distributed controller is proposed to solve the general formation control problem for multiple agents with PH dynamics in different physical domains. In comparison with other works focussing on specific multi-agent models, such as spacecraft in [11,23], nonholonomic wheeled robots in [12,13], underwater vehicles in [24], DC micro-grids in [8], the proposed controller considers more general cases than different systems in multi-physical domains described as PH models. It considers a wider class of applications than other works focussing on a specific formation control problem such as the leader-followers formation control in [11, 25] and consensus in [8, 26].

(2) The designed system related to the general formation output adopts a PH form, allowing the use of the Hamiltonian function as a candidate Lyapunov function for stability analysis. This simplifies the task of selecting the Lyapunov function for stability analysis, which can be challenging in conventional distributed algorithms [27–30]. In addition, we can show that the multi-agent system under the proposed controller achieves exponential stability which is different from the asymptotic stability discussed in [11–15, 24, 31, 32].

(3) The proposed controller only requires exchanging each agent's self-estimated values of the global average information with its neighbors, avoiding the risk to some extent of directly exchanging explicit states as observed in numerous prior works such as [12, 14, 15, 31, 32], guaranteeing the privacy of each agent.

The remainder of this paper is organized as follows. Some preliminaries are provided in Section II and then the general formation control problem is formulated in Section III. Section IV designs the general formation output dynamic, based on which the distributed controller of the ith agent ($i \in \mathcal{V}$) is presented. In addition, the convergence and the privacy analysis is given. An application example and the parameter analysis are provided in Section V. Finally, Section VI conclude this work.

2 Notations

Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and I_n denotes the identity matrix in $\mathbb{R}^{n\times n}$. $\mathbb{Z}$ is the set of integer, $\mathbb{N}$ is the set of natural numbers. $1_n \in \mathbb{R}^n$ and $0_n \in \mathbb{R}^n$ represent the $n \times 1$ column vector of one values and zero values, respectively. The Euclidean norm is denoted by $\|\cdot\|$ while the Kronecker product is denoted by $\otimes$. The matrix $A = (a_{ij})_{m\times n}$ means that a_{ij} is the element of A in i row and j column and A is a $m \times n$ matrix. $x^\top$ is the transpose of x. The set $\{i, i+1, \cdots, j-1, j\}$ is described by $[i{:}j]$ where $i, j \in \mathbb{N}$ and $i < j$. $\mathrm{col}(x_1, \cdots, x_N) = (x_1^\top, \cdots, x_N^\top)^\top \in \mathbb{R}^{Nm}$ with $x_i \in \mathbb{R}^m$, $i \in [1{:}N]$. The vector $a_S = (a_j, j \in \mathcal{S})$ means that $a_S = (a_1, \cdots, a_s)$ if the set $S = \{1, \cdots, s\}$. $\mathrm{diag}(\lambda_1, \cdots, \lambda_n)$ is the diagonal matrice of elements $\lambda_1, \cdots, \lambda_n \in \mathbb{R}$. For a scalar function $H(x, y) \in \mathbb{R}$ of vectors $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$, the gradient with respect to x is denoted by $\nabla_x H(x, y) = \big(\frac{\partial H}{\partial x_1}, \cdots, \frac{\partial H}{\partial x_n}\big)^\top$ where $\frac{\partial H}{\partial x_i}$ is the partial derivative of $H(x, y)$ with respect to x_i. If $x(t) : \mathbb{R} \to \mathbb{R}^m$ is a vector function of t, then for a function $\psi(x(t)) : \mathbb{R}^m \to \mathbb{R}^n$, its derivative with respect to t is $\dot{\psi}(x) = \nabla_x \psi(x)\dot{x}$ where $\dot{x}$ is the first-order derivative of $x(t)$.

3 Preliminaries

In this section, some essential preliminaries on graph theory and PH framework are given.

Consider an undirected connected graph $\mathcal{G} := \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = [1{:}N]$ denotes the node set, $\mathcal{E}$ denotes the edge set, and $\mathcal{A} := (a_{ij})_{N\times N}$ denotes the adjacency matrix. If there exists an edge $(i, j) \subset \mathcal{E}$ between two nodes $i, j \in \mathcal{V}$, we say that i is a neighbour of j. Then, i belongs to j's neighbour set $\mathcal{N}(j)$. In an undirected graph $\mathcal{G}$, if i is a neighbour of j, then j is a neighbour of i.

The adjacent matrix $\mathcal{A}$ is symmetric with $a_{ij} = a_{ji}$ and $a_{ij} = 1$ when i is a neighbour of j, or otherwise, $a_{ij} = 0$. Moreover, for every $i \in \mathcal{V}$, $a_{ii} = 0$. The degree matrix $\mathcal{D} = \mathrm{diag}(\mathrm{deg}_1, \cdots, \mathrm{deg}_N)$ is a diagonal matrix, where each element $\mathrm{deg}_i = \sum_{j=1}^{N} a_{ij}$ describes the degree of each node. Based on the adjacency matrix and the degree matrix, the Laplacian matrix of $\mathcal{G}$ is defined by $L = \mathcal{D} - \mathcal{A}$, and the the eigenvalues of L are denoted by $\lambda_1 \leq \cdots \leq \lambda_N$. If there is a path from node i to node j, then node i and node j are connected. An undirected graph whose any two nodes are connected is called a connected graph. There is a criterion [33] that $\mathcal{G}$ is connected if and only if $\lambda_2 > 0$. For the convenience of discussion, this paper assumes that undirect connected graphs are used for communication between multi-agent systems.

Consider N-agents communicating through an undirect connected graph $\mathcal{G}$, the dynamics of the ith agent ($i \in \mathcal{V}$) is described by the following standard input-state-output PH system [1]:

$$\dot{x}_i = \big(J_i(x_i) - R_i(x_i)\big)\frac{\partial H_i(x_i)}{\partial x_i} + g_i(x_i)u_i, \qquad (1)$$

where $x_i \in \mathbb{R}^m$ represents the ith agent state, $u_i \in \mathbb{R}^n$ represents the control input of the ith agent, the smooth function $H_i(x_i) : \mathbb{R}^m \to \mathbb{R}$ is the total energy of the system called the *Hamiltonian* function, the structure matrix $J_i(x_i) : \mathbb{R}^m \to \mathbb{R}^{m\times m}$ is skew-symmetric, that is $J_i(x_i) = -J_i^\top(x_i)$, the dissipation matrix $R_i(x_i) : \mathbb{R}^m \to \mathbb{R}^{m\times m}$ is a symmetric positive-semidefinite matrix, that is, $R_i(x_i) = R_i^\top(x_i) \geq 0$. The input mapping $g_i(x_i) : \mathbb{R}^m \to \mathbb{R}^{m\times n}$ depends on x_i. More details about the properties of PH systems can be found in [10].
In most of the conventional formation problems, the formation objective is described by using the system state x_i directly [14,15,24]. In order to describe more general problems within a broader range of applications, we design the general formation output of the ith agent ($i \in \mathcal{V}$) as a second-order continuously differentiable function of the form

$$\psi_i(x_i) : \mathbb{R}^m \to \mathbb{R}^l, \quad l \leq m. \qquad (2)$$

By a classification of the internal variables appearing in (1) based on how the inputs act on the states in [34], and motivated by [5], the ith agent ($i \in \mathcal{V}$) with PH dynamics and general formation output is described as

$$\begin{cases} \begin{bmatrix} \dot{x}_i^f \\ \dot{x}_i^h \end{bmatrix} = \begin{bmatrix} J_i^f - R_i^f & J_i^{fh} - R_i^{fh} \\ J_i^{hf} - R_i^{hf} & J_i^h - R_i^h \end{bmatrix} \frac{\partial H_i}{\partial x_i} + \begin{bmatrix} g_i^f(x_i)\tilde{u}_i \\ 0 \end{bmatrix}, \\ z_i = \psi_i(x_i), \end{cases} \qquad (3)$$

where $x_i^f \in \mathbb{R}^f$, $x_i^h \in \mathbb{R}^h$ with $f, h \in \mathbb{Z}$, $f \geq 1$ and $h+f = m$, $g_i^f(x_i)$ has row-full rank and $\frac{\partial H_i}{\partial x_i} = \mathrm{col}(\frac{\partial H_i}{\partial x_i^f}, \frac{\partial H_i}{\partial x_i^h})$, $z_i \in \mathbb{R}^l$ denotes the general formation output of agent i.

4 Problem formulation

The goal of the general formation problem [5] considered in this paper is to design a controller such that

$$\lim_{t\to\infty} z_i - z_j = z_{ij}^*, \quad i, j \in \mathcal{V}, \qquad (4)$$

where the $z_{ij}^* \in \mathbb{R}^l$ ($i, j \in \mathcal{V}$) are predefined vectors related to the general formation problem and actual physical meanings. The desired general formation matrix $\tilde{z}^*$ is defined by $\tilde{z}^* = (z_{ij}^*)_{Nl\times Nl}$. From (4), $\tilde{z}^*$ is skew-symmetric and satisfies

$$z_{ij}^* = -z_{ji}^*, \quad {z_{ij}}^* = {z_{ik}}^* + {z_{kj}}^*, \quad i, j, k \in \mathcal{V}.$$

In conventional formation control problems, the function $z_i = \psi_i(x_i)$ of the ith agent ($i \in \mathcal{V}$) is simplified as x_i. For example, in [35], the multi-agent system (3) is said to achieve formation if for any given initial state,

$$\lim_{t\to\infty} \big(x_i - x_1\big) = d_{x_i}, \quad i \in \mathcal{V}, \quad i \neq 1, \qquad (5)$$

holds. This is a leader-followers formation problem and the agent with index 1 is the leader while other agents are followers. The constant vector $d_{x_i} \in \mathbb{R}^m$ ($i \in \mathcal{V}$) is the desired relative vector between agent i and agent 1. Obviously, using the general formation output $z_i = \psi_i(x_i)$ in the formation objective allows to describe a wider range of practical problems than the use of the state x_i directly. In fact, if agent 1 is defined as the leader, others are followers, then the goal (4) is equal to

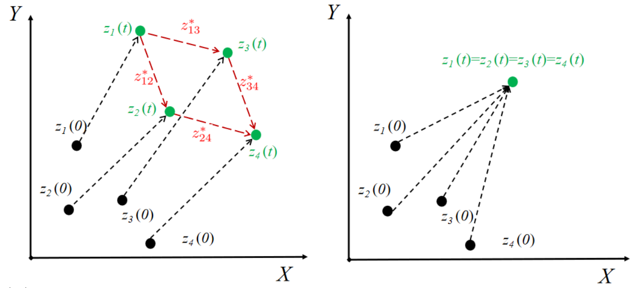
$$\lim_{t\to\infty} z_i - z_1 = z_{i1}^*, \quad i, j \in \mathcal{V},$$

while z_{i1}^* is predefined, and $z_{ij} = z_{i1} + z_{1j}$ can be calculated. Let $\psi_i(x_i) = x_i$, $z_{i1}^* = d_{x_i}$ and $z_{11}^* = 0$, $i, j \in \mathcal{V}$, then the objective of general formation control (4) degenerates into (5). Some illustrative examples of the general formation goal defined in (4) are given hereafter.

Example 1 *Consider the example of* 4 *underactuated unmanned surface vehicles (USVs) with PH dynamics trying to form a parallelogram in the X-Y plane, as shown in Fig. 1(a). In this case,* $x_i = (q_i, p_i)^\top \in \mathbb{R}^6$ *where* $q_i = col(q_{Xi}, q_{Yi}, \phi_i) \in \mathbb{R}^3$ *includes X-axis coordinate, Y-axis coordinate, and angle with respect to the X-axis of the ith USV, while each component of* $p_i = col(v_{si}, v_{wi}, w_i) \in \mathbb{R}^3$ *represents the ith USV's momentum in X-axis, Y-axis and angular respectively. The parallelogram formation goals can be described as* $\lim_{t\to\infty} z_i - z_j = z_{ij}^*$, *(*$i, j \in \mathcal{V}$*), where* z_{ij}^* *is predefined by the desired geometry and* $\psi_i(x_i) = Kx_i$ *with* $K = (I_{2\times 2}, 0_{2\times 4})$. *Furthermore, if we consider that* $z_{ij}^* = 0$, $\forall i, j \in \mathcal{V}$, *then goals (4) can be used to described a* rendezvous *problem, or a point formation, as shown in Fig. 1(b). Hence, the considered general formation framework includes the* rendezvous *problem as a special case.*

Example 2 *In this example we consider the synchronization of rotor speed of permanent magnet synchronous motors (PMSMs) with PH dynamics [36]. The state of the ith agent is* $x_i = (L_{di}i_{di}, L_{qi}i_{qi}, J_{ri}w_i)^T \in \mathbb{R}^3$, *where* i_{di} *is the current on d-axis,* i_{qi} *is the current on q-axis,* w_i *is the angular velocity of the ith agent, and* L_{di}, L_{qi}, J_{ri} *are positive parameters of ith agent,* $i \in \mathcal{V}$. *Design* $\psi_i(x_i) = (0, 0, \frac{1}{J_{ri}})x_i$ *and* $z_{ij}^* = 0$ *in (4), then the synchronization problem is described under the framework of the general formation control.*

[1] In this system, $x_i = x_i(t), u_i = u_i(t), J_i(x_i) = J_i(x_i(t)), R_i(x_i) = R_i(x_i(t)), H_i(x_i) = H_i(x_i(t)), g_i = g_i(x_i(t)), y_i = y_i(t)$. In the following text, for simplicity of description, t is omitted without causing ambiguity.



(a) Parallelogram formation (b) Point formation

Fig. 1. Formation examples of four agents moving in the X-Y plane: (a) Parallelogram formation and (b) Point formation.

In this paper, we consider a connected communication topology where each agent can only exchange information with its neighbours. Without global information, the centralized controllers cannot be designed. Hence, the distributed controller for the ith agent ($i \in \mathcal{V}$) is considered to accomplish the formation control goals by scholars, such as [12, 14, 15].

In the aforementioned approach for the PH multi-agent system's formation control problem, they require agents to exchange and disclose their states explicitly to the neighboring agents. This may bring serious privacy concerns in many practical applications. For example, in these cases, their initial location will be revealed to neighbors, which may not be desirable in the rendezvous problem [37].

With concerns on the sensitive information, the estimation idea is considered. Taking into account the following fact: if the relative values between agents are given, then the value of each agent relative to the average value of all agents is also determined, and vice versa. Therefore, an average function is introduced to characterize the average general formation output of all agents:

$$z_{ave} = \psi_{ave}(x) = \frac{1}{N}\sum_{i=1}^{N} \psi_i(x_i), \tag{6}$$

where $x = \text{col}(x_1, \cdots, x_N)$. The estimation of $\psi_{ave}(x)$ by the ith agent ($i \in \mathcal{V}$) is denoted by $\hat{\eta}_i(t)$ and is used for communication with its neighbours. This means that each agent only sends its estimation about the average general formation output to its neighbours, thus the states of individual is not needed in the communication. Hence, the ith agent can use its own state x_i and the information received from its neighbours namely $\hat{\eta}_j(t)$, $j \in \mathcal{N}(i)$ to update its estimation $\hat{\eta}_i$ and design the controller. Based on the above analysis, the general formation problem of the agents is summarized as follows.

Problem 3 *Design a controller $\tilde{u}_i$ for the ith agent ($i \in \mathcal{V}$) described in* (3)*, such that its general formation output $z_i(t)$ under the action of $\tilde{u}_i$ is asymptotically stable and converges to the set*

$$S_{z^*} = \{z : z_i - z_j = z_{ij}^*, i, j \in \mathcal{V}\}, \tag{7}$$

It should be noted that, to achieve the general formation goal, there must exist some $x(t)$ such that,

$$\lim_{t\to\infty} \psi_i(x_i(t)) - \psi_j(x_j(t)) = z_{ij}^*, \quad i, j \in \mathcal{V}. \tag{8}$$

Meanwhile, based on the definition of z_{ij}^*, S_{z^*} defined in (7) is not empty. Otherwise, the general formation goal is meaningless. In order to solve Problem 3, we use an optimization point of view based on Theorem 4.

Theorem 4 *Consider a network of N agents described under PH form as in* (3)*. If there exists a controller $\tilde{u}_i$ for the ith agent ($i \in \mathcal{V}$) such that its general formation output $z_i(t)$ asymptotically converges to the minimum of*

$$V_i(z_i, z_{ave}) = \frac{1}{2}\|z_i - z_{ave} - \bar{z}_i^*\|^2, \tag{9}$$

with $\bar{z}_i^* = \frac{1}{N}\sum_{j=1}^{N} z_{ij}^*$, then the controller $\tilde{u}_i$ is a solution of Problem 3.

PROOF. To prove this theorem, at first, we demonstrate that if $z^* = \text{col}(z_1^*, \cdots, z_N^*)$ is a minimum point of (9), then z^* satisfies (8) and vice versa. If z^* satisfies (8), then z_i satisfies

$$z_i^* - z_1^* = z_{i1}^*, \cdots, z_i^* - z_N^* = z_{iN}^*, \quad i \in \mathcal{V}.$$

After summation that implies that

$$z_i^* - z_{ave}^* = \bar{z}_i^*, \quad i \in \mathcal{V}, \tag{10}$$

then z^* is a minimum of (9) for $V_i(z_i^*, z_{ave}^*) = 0$. Conversely, if (10) is satisfied, then we have

$$z_j^* - z_{ave}^* = \bar{z}_j^*, \quad j \in \mathcal{V}. \tag{11}$$

Furthermore, if we make a difference between z_i^* and z_j^*, $\forall i, j \in \mathcal{V}$, then

$$z_i^* - z_j^* = \bar{z}_i^* - \bar{z}_j^* = z_{ij}^*. \tag{12}$$

Above all, $V_i(z_i^*, z_{ave}^*) = 0$ is equivalent to z^* satisfies (8). Then we try to show that the minimum value of $V_i(z_i), z_{ave})$ is 0. If (9) cannot get 0 as its minimum value, then for any z, we have $z_i - z_{ave} - \bar{z}_i^* \neq 0$, that is $z_i - z_j \neq z_{ij}^*, i, j \in \mathcal{V}$, which is contradict to the previous analysis that S_{z^*} should not be an empty set. Hence, with the help of the definition of asymptotic stability in [38], the proof is completed. □

Before designing a suitable distributed controller, we first discuss how to find the minimum of the value function (9), and then the following lemma is given.

Lemma 5 *[39] $z^* = col(z_1^*, \cdots, z_N^*)$ is the minimum point of (9) if and only if*

$$\nabla_{z_i} V_i(z_i, z_{ave})|_{z_i = z_i^*} = 0_m, \quad i \in \mathcal{V}. \tag{13}$$

Under Lemma 5, our objective is to design a distributed controller $\tilde{u}_i$ for the ith agent ($i \in \mathcal{V}$) such that the general formation output z_i^* of system (3) satisfies (13).

5 Main results

5.1 The general formation output dynamic design

In this subsection, the dynamic related to the general formation output $z_i(t)$ ($i \in \mathcal{V}$) and its convergence analysis is given.
Before designing the general formation output dynamic, some maps are defined. To establish a connection between the valued function $V_i(z_i, z_{ave})$ for the ith agent ($i \in \mathcal{V}$) and its estimation $\hat{\eta}_i$ of the average formation function z_{ave}, we define

$$C_i(z_i, \hat{\eta}_i) := V_i(z_i, z_{ave})|_{z_{ave} = \hat{\eta}_i}, \quad i \in \mathcal{V}. \tag{14}$$

Using the definition of $V_i(z_i, z_{ave})$ in (9) and z_i in (3), with the above equation, we get

$$C_i(z_i, \hat{\eta}_i) = \frac{1}{2}\|z_i - \hat{\eta}_i - \bar{z}_i^*\|^2. \tag{15}$$

According to the definition of $C_i(z_i, \hat{\eta}_i) : \mathbb{R}^l \times \mathbb{R}^l \to \mathbb{R}$, the gradients with respect to variables z_i and $\hat{\eta}_i$ ($i \in \mathcal{V}$) are defined by $G_i(z_i, \hat{\eta}_i) : \mathbb{R}^l \times \mathbb{R}^l \to \mathbb{R}^m$ and $\delta_i(z_i, \hat{\eta}_i) : \mathbb{R}^l \times \mathbb{R}^l \to \mathbb{R}^l$ with the following form:

$$\begin{aligned} G_i(z_i, \hat{\eta}_i) &:= \nabla_{z_i} C_i(z_i, \hat{\eta}_i) = z_i - \hat{\eta}_i - \bar{z}_i^*, \\ \delta_i(z_i, \hat{\eta}_i) &:= \nabla_{\hat{\eta}_i} C_i(z_i, \hat{\eta}_i) = -(z_i - \hat{\eta}_i - \bar{z}_i^*). \end{aligned} \tag{16}$$

In compact form, we have

$$C(z, \hat{\eta}) := \mathrm{col}(C_1(z_1, \hat{\eta}_1), \cdots, C_N(z_N, \hat{\eta}_N)),$$
$$G(z, \hat{\eta}) := \mathrm{col}(G_1(z_1, \hat{\eta}_1), \cdots, G_N(z_N, \hat{\eta}_N)),$$

where $\hat{\eta} = \mathrm{col}(\hat{\eta}_1, \cdots, \hat{\eta}_N) \in \mathbb{R}^{Nm}$. Under the above definitions, the dynamic of the general formation output $z_i(t)$ for the ith agent ($i \in \mathcal{V}$) is designed as:

$$\begin{cases} \dot{z}_i = -G_i(z_i, \hat{\eta}_i), \\ \dot{\hat{\eta}}_i = -\gamma \sum_{j \in \mathcal{N}(i)} (\hat{\eta}_i - \hat{\eta}_j) - \delta_i(z_i, \hat{\eta}_i) - k_1 s_i, \\ \dot{s}_i = k_1\big(\gamma \sum_{j \in \mathcal{N}(i)} (\hat{\eta}_i - \hat{\eta}_j) + \delta_i(z_i, \hat{\eta}_i)\big) - k_2 s_i, \end{cases} \tag{17}$$

where $z_i(0) = \psi_i(x_i(0))$, $\hat{\eta}_i$ is the ith agent's ($i \in \mathcal{V}$) estimation of $z_{ave} = \psi_{ave}(x)$, the initial values $s_i(0)$ and $\hat{\eta}_i(0)$ can be chosen arbitrary, the parameter $\gamma > 0$, $0 < k_1 < 1$ and $k_2 = 1 - k_1^2$ are constants. The designed dynamic (17) can be rewritten as

$$\dot{Z}_i = (J_d - R_d)\frac{\partial H_{cli}}{\partial Z_i}, \tag{18}$$

where $Z_i = \mathrm{col}(z_i, \hat{\eta}_i, s_i)$,

$$H_{cli} = C_i(z_i, \hat{\eta}_i) + \frac{\gamma}{2} \sum_{j \in \mathcal{N}(i)} (\hat{\eta}_i - \hat{\eta}_j)^\top (\hat{\eta}_i - \hat{\eta}_j) + \frac{1}{2} s_i^\top s_i,$$

the structure matrix J_d and the dissipative matrix R_d are respectively defined as

$$J_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -k_1 \otimes I_l \\ 0 & k_1 \otimes I_l & 0 \end{bmatrix}, R_d = \begin{bmatrix} 1 \otimes I_m & 0 & 0 \\ 0 & 1 \otimes I_l & 0 \\ 0 & 0 & k_2 \otimes I_l \end{bmatrix}.$$

and $J_d = -J_d^\top$, $R_d = R_d^\top \geq 0$. Let $\delta(z, \hat{\eta}) = \mathrm{col}(\delta_1(z_1, \hat{\eta}_1), \cdots, \delta_N(z_N, \hat{\eta}_N))$, $s = \mathrm{col}(s_1, \cdots, s_N)$, then the dynamic (18) for all agents can be rewritten in compact form as

$$\dot{Z} = ((J_d - R_d) \otimes I_N)\frac{\partial H_{cl}}{\partial Z}, \tag{19}$$

with $Z = \mathrm{col}(z, \hat{\eta}, s)$ and

$$H_{cl} = \sum_{i=1}^{N} \Big(C_i(z_i, \hat{\eta}_i) + \frac{1}{2} s_i^\top s_i + \frac{\gamma}{2} \hat{\eta}_i \sum_{j \in \mathcal{N}(i)} (\hat{\eta}_i - \hat{\eta}_j)\Big). \tag{20}$$

The total *Hamiltonian* H_{cl} including three parts:

• The first term represents the energy injected by the valued function. By (15), we have $\sum_{i=1}^N C_i(z_i, \hat{\eta}_i) = 0$ if and only if for every $i \in \mathcal{V}$, $z_i - \hat{\eta}_i = \bar{z}_i^*$ is satisfied.

• The term $\sum_{i=1}^N \frac{1}{2} s_i^\top s_i$ is designed to eliminate integrator errors.

• The last term $\frac{\gamma}{2} \sum_{i=1}^N \hat{\eta}_i \sum_{j \in \mathcal{N}(i)} (\hat{\eta}_i - \hat{\eta}_j)$ represents the difference between the estimation of agents and its neighbours. It is equal to 0 if and only if all agents have the same estimation of z_{ave}, i.e. $\hat{\eta}_i = \hat{\eta}_j, \forall i, j \in \mathcal{V}$. Hence, taking 0 as the minimum value of H_{cl} means $\hat{\eta}_i^* = z_{ave}^*$, that is $V_i(z_i^*, z_{ave}^*) = C_i(z_i^*, \hat{\eta}_i^*) = 0$ which satisfies the general formation objective. The control scheme for the

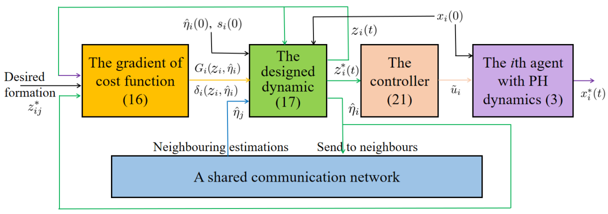


Fig. 2. The control scheme for the ith agent.

ith agent is shown in Fig.2 and the designed dynamic (17) consists of 4 parts described below:

• In the first equation, the term $G_i(z_i, \hat{\eta}_i)$ is an expected state changing injection of the value function (9).

• The estimation $\hat{\eta}_i$ of the ith agent ($i \in \mathcal{V}$) is influenced by the error between his own estimation $\hat{\eta}_i$ and his neighbor's estimation $\hat{\eta}_j$, $j \in \mathcal{N}(i)$. In addition, all agents' estimated values will eventually converge to the accurate average general formation output:

$$\lim_{t\to\infty} \hat{\eta}_i(t) \to z_{ave}(t), \quad i \in \mathcal{V}.$$

• The third one is designed as an integrator to improve the convergence characteristics of the algorithm.

• Parameters γ , k_1 and k_2 are predefined constants which may influences the convergence speed. More analysis of parameters will be mentioned in next section.

5.2 The distributed controller design

In this subsection, a distributed controller is proposed to solve Problem 3.

By the designed dynamic (19), we obtain the general formation output $z_i^*(t)$ ($i \in \mathcal{V}$). Motivated by [40], if there exists a partition of $x_i = \mathrm{col}(x_i^1, x_i^2)$, where $x_i^1 \in \mathbb{R}^l$ and $x_i^2 \in \mathbb{R}^{m-l}$, and a corresponding immersion $\pi_i = \mathrm{col}(\pi_i^1, \pi_i^2)$, where $\pi_i^1 : \mathbb{R}^l \to \mathbb{R}^l$, $\pi_i^2 : \mathbb{R}^l \to \mathbb{R}^{m-l}$ such that $x_i^1 = \pi_i^1(z)$, then we can calculate x_i^2 by the coupling relationship between different state components in actual physical systems. In fact, the mapping π_i is easy to find following physical and system theoretic considerations.

Based on $x_i^*(t)$, the controller $\tilde{u}_i$ is obtained by

$$\begin{aligned}\tilde{u}_i = g_i^f(x_i^*(t))^\dagger \big(\dot{x}_i^{*f}(t) - (J_i^f - R_i^f)\frac{\partial H_i}{\partial x_i^f} \\ - (J_i^{fh} - R_i^{fh})\frac{\partial H_i}{\partial x_i^h}\big),\end{aligned} \tag{21}$$

where $g_i^f(x_i^*(t))^\dagger$ is the pseudo-inverse of $g_i^f(x_i^*(t))$. By using controller (21), the trajectory of system (3) is consistent with the desired state trajectory $x_i^*(t) = \pi_i(z_i^*)$.

5.3 Convergence analysis

To analyze the convergence of the designed dynamic system (19) of the general formation output $z(t)$, the following theorem is given.

Theorem 6 *The the designed dynamic system* (19) *of the general formation output* $z(t)$ *is exponential stable.*

PROOF. Choosing H_{cl} in (20) as Lyapunov function,

$$L_y(z, \hat{\eta}, s) = 1_N^\top C(z, \hat{\eta}) + \frac{1}{2}s^\top s + \frac{\gamma}{2}\hat{\eta}^\top (L \otimes I_l)\hat{\eta}. \tag{22}$$

Taking the following orthogonal transformation,

$$\tilde{\eta} = \begin{bmatrix}\tilde{\eta}_1 \\ \tilde{\eta}_2\end{bmatrix} = \begin{bmatrix} r^\top \otimes I_l \\ R^\top \otimes I_l\end{bmatrix} \hat{\eta},$$

where $r = \frac{1}{\sqrt{N}}1_N$, $r^\top R = 0_{N-1}^\top$, $R^\top R = I_{N-1}$, $RR^\top = I_N - \frac{1}{N}1_N 1_N^\top$, $\tilde{\eta}_1 \in \mathbb{R}^l$ and $\tilde{\eta}_2 \in \mathbb{R}^{(N-1)l}$. With the orthogonal transformation,

$$\begin{aligned}\hat{\eta}^\top (L \otimes I_l)\hat{\eta} &= \hat{\eta}^\top (RR^\top \otimes I_l)(L \otimes I_l)(RR^\top \otimes I_l)\hat{\eta} \\ &= \tilde{\eta}_2^\top (R^\top LR \otimes I_l)\tilde{\eta}_2.\end{aligned} \tag{23}$$

Inserting (23) to (22), $L_y(z, \hat{\eta}, s)$ converts to

$$L_y(z, \hat{\eta}, s) = 1_N^\top C(z, \hat{\eta}) + \frac{1}{2}s^\top s + \frac{\gamma}{2}\tilde{\eta}_2^\top (R^\top LR \otimes I_l)\tilde{\eta}_2. \tag{24}$$

Considering that

$$\begin{aligned}\nabla_z C(z, \hat{\eta}) &= \mathrm{col}(\nabla_{z_1} C_1(z_1, \hat{\eta}_1), \cdots, \nabla_{z_N} C_N(z_N, \hat{\eta}_N)), \\ \nabla_{\hat{\eta}} C(z, \hat{\eta}) &= \mathrm{col}(\nabla_{\hat{\eta}_1} C_1(z_1, \hat{\eta}_1), \cdots, \nabla_{\hat{\eta}_N} C_N(z_N, \hat{\eta}_N)),\end{aligned}$$

we are going to show that $\dot{L}_y \leq 0$. The dynamic system (19) can be rewritten as follows

$$\begin{cases}\dot{z} = -G(z, \hat{\eta}), \\ \dot{\hat{\eta}} = -\delta(z, \hat{\eta}) - \gamma(LR \otimes I_l)\tilde{\eta}_2 - k_1 s, \\ \dot{\tilde{\eta}}_2 = -(R^\top \otimes I_l)\delta(z, \hat{\eta}) - \gamma(R^\top LR \otimes I_l)\tilde{\eta}_2 \\ \quad -k_1(R^\top \otimes I_l)s, \\ \dot{s} = k_1(\delta(z, \hat{\eta}) + \gamma(LR \otimes I_l)\tilde{\eta}_2) - k_2 s.\end{cases} \tag{25}$$

With the definition (16), by derivating (24) and inserting (19) and (25), we have

$$\begin{aligned}\dot{L}_y &= -G(z, \hat{\eta})^\top G(z, \hat{\eta}) - \delta(z, \hat{\eta})^\top \delta(z, \hat{\eta}) \\ &\quad - \gamma^2((R^\top LR \otimes I_l)\tilde{\eta}_2)^\top (R^\top LR \otimes I_l)\tilde{\eta}_2 \\ &\quad - 2\delta(z, \hat{\eta})^\top \gamma(LR \otimes I_l)\tilde{\eta}_2 - k_2 s^\top s.\end{aligned} \tag{26}$$

From the definition of $G_i(z_i, \hat{\eta}_i)$ and $\delta_i(z_i, \hat{\eta}_i)$ in (16),

$$\|G(z, \hat{\eta})\|^2 = \|\delta(z, \hat{\eta})\|^2 = \sum_{i=1}^N \|z_i - \hat{\eta}_i - \bar{z}_i^*\|^2, \tag{27}$$

Hence, (26) implies

$$\begin{aligned}\dot{L}_y &\leq -2\delta(z, \hat{\eta})^\top \delta(z, \hat{\eta}) - k_2 s^\top s \\ &\quad - \gamma^2((R^\top LR \otimes I_l)\tilde{\eta}_2)^\top (R^\top LR \otimes I_l)\tilde{\eta}_2 \\ &\quad - 2\delta(z, \hat{\eta})^\top \gamma(LR \otimes I_l)\tilde{\eta}_2.\end{aligned} \tag{28}$$

With the inequality $ab \leq \frac{1}{2c}a^2 + \frac{c}{2}b^2$, $c > 0$, we have

$$2\delta(z,\eta)^\top \gamma(LR \otimes I_l)\tilde{\eta}_2 \leq \frac{3}{2}\|\delta(z,\hat{\eta})\|^2 + \frac{2}{3}\gamma^2\|(R^\top LR \otimes I_l)\tilde{\eta}_2\|^2. \quad (29)$$

Inserting (29) into (28), one can get

$$\begin{aligned} \dot{L}_y &\leq -\frac{1}{2}\|\delta(z,\hat{\eta})\|^2 - \frac{\gamma^2}{3}\|\tilde{\eta}_2\|^2 - k_2\|s\|^2 \\ &= -1_N^\top C(z,\hat{\eta}) - \frac{2k_2}{2}s^\top s - \frac{2\gamma}{3}\frac{\gamma}{2}\|\tilde{\eta}_2\|^2 \\ &\leq -\min\{1, 2k_2, \frac{2\gamma}{3}\}L_y. \end{aligned} \quad (30)$$

Hence, the proof is completed with $\gamma > 0$ and $k_2 > 0$.□

Next, we are going to illustrate the relationship between the minimum point z^* of the valued function designed in Problem 3 and the equilibrium point $(z^*, \hat{\eta}^*, s^*)$ of system (19).

Theorem 7 *If $(z^*, \hat{\eta}^*, s^*)$ is an equilibrium of system* (19)*, then z^* is a minimum of V_i defined in (9).*

PROOF. According to the properties of the PH system, the total *Hamiltonian* is chosen as candidate Lyapunov function (22) in the proof of Theorem 6. Recalling the Lasalle Invariance Principle, system (19) converges to the maximum invariance set $S = \{Z^* | \frac{\partial L_y(Z^*)}{\partial Z} = 0\}$, because of $R_d > 0$.
Then we are going to analysis elements belong to the set S. With the definition of L_y in (22), $Z^* \in S$ is equivalent to $(z^*, \hat{\eta}^*, s^*)$ satisfies

$$G(z^*, \hat{\eta}^*) = 0, \quad (31a)$$
$$\gamma(L \otimes I_l)\hat{\eta}^* + \delta(z^*, \hat{\eta}^*) = 0, \quad (31b)$$
$$s^* = 0. \quad (31c)$$

For an undirected connected graph $\mathcal{G}$, its Laplacian matrix L has and only one zero eigenvalue, corresponding to an eigenvector of 1_N, that is $1_N^\top L = 0$. With (16), we find $G_i(z_i, \hat{\eta}_i) = -\delta_i(z_i, \hat{\eta}_i)$. Hence, by (31a) and (31b), we have $\gamma(L \otimes I_l)\hat{\eta}^* = 0$ which implies $\hat{\eta}_i^* = \hat{\eta}_j^*$ $(i, j \in \mathcal{V})$ with $\gamma > 0$. Since $G_i(z_i^*, \hat{\eta}_i^*) = z^* - \hat{\eta}^* - \bar{z}^* = 0$ by (31a), and $\sum_{i=1}^N z^* - \sum_{i=1}^N \hat{\eta}^* = \frac{1}{N}\sum_{i=1}^N \sum_{j=1}^N z_{ij}^* = 0$, by the predefined $z_{ij}^* = z_i^* - z_j^* = -z_{ji}^*$, we get $\hat{\eta}_i^* = \frac{1}{N}\sum_{i=1}^N z_i^* = z_{ave}^*$ $(i \in \mathcal{V})$. With the definition of $C_i(z_i, \hat{\eta}_i)$ in (15), when $\hat{\eta}_i^* = z_{ave}^*$, we have $V_i(z_i^*, z_{ave}^*) = C_i(z_i^*, \hat{\eta}_i^*) = 0$ $(i \in \mathcal{V})$. The gradient of $V_i(z_i, z_{ave})$ with respect to z_i in z_i^* is

$$\nabla_{z_i} V_i(z_i, z_{ave})|_{z_i = z_i^*} = \frac{N}{N-1}(z_i^* - z_{ave}^* - \bar{z}_i^*) = 0, i \in \mathcal{V}.$$

According to the Lemma 5, z^* is a minimum point of $V_i(z_i, z_{ave})$ $(i \in \mathcal{V})$ in (9). □

From Theorem 6 and Theorem 7, the designed dynamic (17) of the ith agent $(i \in \mathcal{V})$ converges to the minimum z_i^* of (9) which achieves the desired general formation.

5.4 Privacy analysis

In this paper, two types of adversaries are considered as in [41]: *an honest-but-curious adversary* is an internal adversary follows the communication topology who is curious and collects received data in an attempt to learn some information about other participating agents, we define the information accessible to the honest-but-curious agent j at time t in this paper as $E_j(t) = \{z_j(t), \hat{\eta}_j(t), s_j(t), \bar{z}_j^*, \hat{\eta}_k(t), k \in \mathcal{N}(j)\}$; *an eavesdropper* is an external attacker who knows the communication topology, and is able to wiretap communication links and access exchanged messages, the information wiretapped by the eavesdropper in this paper is $E(t) = \{\hat{\eta}_i(t), i \in \mathcal{V}\}$.
It should be noted that if an honest-but-curious agent connected to another honest-but-curious agent, they may collude, i.e. exchange their state directly to each other and estimate other agents' privacy together. And in this case, an eavesdropper will obtain the state of these two honest-but-curious agents.
In the problem considered in this paper, the general formation output $z_i(t)$ often contains the important information of the ith agent $(i \in \mathcal{V})$. For example, in *Example 1*, $z_i(t)$ denotes the position of the ith USV. In actual military missions, position exposure may result in operational failure. Hence, $z_i(t)$ is sensitive information of the ith agent $(i \in \mathcal{V})$. Based on this, the definition of privacy used throughout this paper is defined as follows.

Definition 8 *The general formation output $z_i(t)$ of the ith agent $(i \in \mathcal{V})$ is defined as its privacy. The privacy of the ith agent is preserved if an adversary cannot obtain the exactly value of $z_i(t)$ at any $t > 0$.*

Definition 8 requires that an adversary cannot find $z_i(t)$ and thus is more stringent than the privacy preservation definition considered in [42], which defines privacy of each agent as the initial value $z_i(0)$. Motivated by [41] and [42], the following theorem is given.

Theorem 9 *The privacy of any agent $i \in \mathcal{V}$, whose general formation output satisfies the designed dynamic* (17)*, is preserved even all of its neighbours are honest-but-curious adversaries and they are colluded.*

PROOF. Inserting $G_i(z_i, \hat{\eta}_i)$ and $\delta_i(z_i, \hat{\eta}_i)$ defined in

(16), the system (17) of the ith agent ($i \in \mathcal{V}$) becomes

$$\begin{cases} \dot{z}_i = -z_i + \hat{\eta}_i + \bar{z}_i^*, \\ \dot{\hat{\eta}}_i = -\gamma \sum_{j\in\mathcal{N}(i)}(\hat{\eta}_i - \hat{\eta}_j) + z_i - \hat{\eta}_i - \bar{z}_i^* - k_1 s_i, \\ \dot{s}_i = k_1\big(\gamma \sum_{j\in\mathcal{N}(i)}(\hat{\eta}_i - \hat{\eta}_j) - z_i + \hat{\eta}_i + \bar{z}_i^*\big) - k_2 s_i. \end{cases} \tag{32}$$

Define $\theta_i(t) = z_i(t) - k_i s_i(t)$, and by noting $k_2 = 1 - k_1^2$, (32) becomes

$$\begin{cases} \dot{\theta}_i = -k_2\theta_i + \hat{\eta}_i + \bar{z}_i^* + k_1^2\big(\gamma \sum_{j\in\mathcal{N}(i)}(\hat{\eta}_i - \hat{\eta}_j) + \hat{\eta}_i + \bar{z}_i^*\big) \\ \dot{\hat{\eta}}_i = -\gamma \sum_{j\in\mathcal{N}(i)}(\hat{\eta}_i - \hat{\eta}_j) + \theta_i - \hat{\eta}_i - \bar{z}_i^*, \quad i \in \mathcal{V}. \end{cases} \tag{33}$$

Let $\theta = \mathrm{col}(\theta_1, \cdots, \theta_N)$ and recalling $\hat{\eta} = \mathrm{col}(\hat{\eta}_1, \cdots, \hat{\eta}_N)$, then we rewrite (33) in compact form as

$$\begin{cases} \dot{\theta} = -k_2\theta + \hat{\eta} + \bar{z}^* + k_1^2\gamma(L \otimes I_l)\hat{\eta} + \hat{\eta} + \bar{z}^*, \\ \dot{\hat{\eta}} = -\gamma(L \otimes I_l)\hat{\eta} + \theta - \hat{\eta} - \bar{z}^*. \end{cases} \tag{34}$$

Without loss of generality, we prove the 1th agent's privacy can be preserved even all neighbour $j \in \mathcal{N}(1)$ are honest-but-curious adversaries and they are colluded, we show that any arbitrary variation of $z_1(t)$, $t \geq 0$ is indistinguishable to its any neighbour agent $j \in \mathcal{N}(1)$ if the 1th agent choosing the initial value of auxiliary variables $\hat{\eta}_1(0)$ and $s_1(0)$ properly.
Recalling the information $E_j(t)$ accessible to any agent $j \in \mathcal{N}(1)$ under the initial values $z_1(0)$, $\hat{\eta}_1(0)$, $s_1(0)$, $z_h(0)$, $\hat{\eta}_h(0)$, $s_h(0)$, $h \in \mathcal{V}/\{1\}$ is $E_j(t) = \{z_j(t), \hat{\eta}_j(t), s_j(t), \bar{z}_j^*, \hat{\eta}_k(t), k \in \mathcal{N}(j)\}$. Similarly, with the fixed initial values $z_h(0)$, $\hat{\eta}_h(0)$, $s_h(0)$, $h \in \mathcal{V}/\{1\}$, the information accessible to agent $j \in \mathcal{N}(1)$ under $z_1'(0)$, $\hat{\eta}_1'(0)$, $s_1'(0)$ is denoted by $E_j'(t)$. Then we are going to show that for any $z_1'(t) \neq z_1(t)$, the information accessible set of agent $j \in \mathcal{N}(1)$ is exactly the same, i.e., $E_j'(t) = E_j(t)$.
More specially, when $z_1(0)$ varying to $z_1'(0)$, we set $\hat{\eta}_1'(0) = \hat{\eta}_1(0)$, and

$$s_1'(0) = s_1(0) + \frac{1}{k_1}\big(z_1'(0) - z_1(0)\big).$$

Then we have $\theta'(0) = \theta(0)$ and $\hat{\eta}'(0) = \hat{\eta}(0)$. Hence, $\hat{\eta}(t) = \hat{\eta}'(t)$, where $\hat{\eta}(t)$ and $\hat{\eta}'(t)$ are the solutions of (34) with $\theta(0)$, $\hat{\eta}(0)$ and $\theta'(0)$, $\hat{\eta}'(0)$, respectively. Hence, $\hat{\eta}'(t) = \hat{\eta}(t)$ hold, which implies that $E_h'(t) = E_h(t)$, $t \geq 0$ with the fixed initial values $z_h(0)$, $\hat{\eta}_h(0)$, $s_h(0)$, $h \in \mathcal{V}/\{1\}$.
When all neighbours are colluded, they will share all information they obtained. It means that for any $j \in \mathcal{N}(1)$, the information accessible set becomes $\tilde{E}(t) = \cup_{j\in\mathcal{N}(1)} E_j(t)$. Since $E_j'(t) = E_j(t)$, we have $\tilde{E}(t) = \tilde{E}'(t) = \cup_{j\in\mathcal{N}(1)} E_j'(t)$, $t \geq 0$, i.e., the variation from $z_1(t)$ is distinguishable to its neighbours, which implies that the privacy of the 1th agent is preserved even all neighbours are colluded.

Theorem 10 *The privacy of any agent $i \in \mathcal{V}$, whose general formation output satisfies the designed dynamic* (17)*, is preserved against an eavesdropper.*

PROOF. Following the line of reasoning in Theorem 9, we can obtain that any change in $z_i(t)$ can be completely compensated by changes in $s_i(t)$ that are invisible to the eavesdropper. Therefore, the accessible information $E(t) = \{\hat{\eta}_i, i \in \mathcal{V}\}$ to the eavesdropper is exactly the same even when $z_i(t)$ were changed arbitrarily and hence the eavesdropper cannot infer $z_i(t)$ based on accessible information.

6 Simulation example

In this section, an example on underactuated unmanned surface vehicles (USVs) is given.

6.1 The hexagonal shape formation of USVs

We consider the formation control of a fleet of USVs, and the structure diagram of each USV is given as Fig. 3.

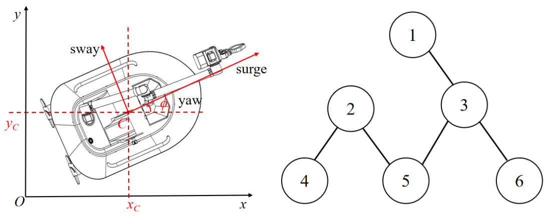


Fig. 3. The structure diagram and the communication topology diagram of USVs.

The system state $x_i = \mathrm{col}(q_i, p_i) \in \mathbb{R}^6$ of the ith USV ($i \in \mathcal{V}$) is designed as in *Example 1*, and its PH dynamic is described as follows [43]:

$$\begin{bmatrix} \dot{q}_i \\ \dot{p}_i \end{bmatrix} = (J_i - R_i)\begin{bmatrix} \partial H_i/\partial q_i \\ \partial H_i/\partial p_i \end{bmatrix} + \begin{bmatrix} 0 \\ I_2 \end{bmatrix}\tilde{u}_i, \tag{35}$$

where the structure matrix J_i and the dissipation matrix R_i are

$$J_i = \begin{bmatrix} 0 & K_i(q_i) \\ -K_i^T(q_i) & 0 \end{bmatrix}, \quad R_i = \begin{bmatrix} 0 & 0 \\ 0 & -D_i \end{bmatrix},$$

respectively, with the damping matrix D_i and the transformation matrix $K_i(q_i)$ defined by

$$D_i = \begin{bmatrix} -d_{i1} & 0 & p_{2i} \\ 0 & -d_{i2} & -p_{1i} \\ -p_{2i} & p_{1i} & -d_{i3} \end{bmatrix}, K_i(q_i) = \begin{bmatrix} \cos\phi_i & -\sin\phi_i & 0 \\ \sin\phi_i & \cos\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Table 1
Parameters of hexagonal formation

	M_i	d_{i1}	d_{i2}	d_{i3}	$z_i(0)$	$\bar{z}_i^*$
1	diag(25,26,2.5)	12	17	0.5	$(7,5)^\top$	$(2,0)^\top$
2	diag(26,25,2.7)	14	16	0.8	$(6,3)^\top$	$(1,\sqrt{3})^\top$
3	diag(24,27,3.0)	12	18	0.4	$(1,8)^\top$	$(-1,\sqrt{3})^\top$
4	diag(23,24,2.2)	15	17	0.7	$(3,4)^\top$	$(-2,0)^\top$
5	diag(28,26,2.7)	12	13	0.6	$(7,0)^\top$	$(-1,-\sqrt{3})^\top$
6	diag(30,28,3.2)	16	17	0.6	$(0,4)^\top$	$(1,-\sqrt{3})^\top$

where $d_{ij} > 0$, $(j \in [1{:}3])$ denote the hydrodynamic damping coefficients in conditions of surge, sway and yaw. The *Hamiltonian* of the system (35) is $H_i = \frac{1}{2}p_i^T(M_i)^{-1}p_i$, with $M_i = \mathrm{diag}(m_{i1}, m_{i2}, m_{i3})$, where $m_{ij} > 0$ $(j \in [1{:}3])$ represent the inertia coefficients of the ith USV $(i \in [1{:}6])$.
Consider 6-USVs communicating through an undirected graph depicted in Fig. 3. The control objective for the 6 USVs is to form a hexagonal shape starting from their initial positions. Noticing that only the positions q_{Xi} and q_{Yi} are required to achieve this formation. we design $z_i = \psi_i(x_i) = P_i x_i$, $i \in \mathcal{V}$ with $P_i = (I_{2\times2}, 0_{2\times2}, 0_{2\times2})$. Then the valued function of the ith agent $(i \in \mathcal{V})$ of the optimization function (9) is defined as

$$V_i(z_i, z_{ave}) = \frac{1}{2}\|z_i - \frac{1}{N}\sum_{j=1}^{N} z_j - \bar{z}_i^*\|^2. \quad (36)$$

The predefined $\bar{z}_i^*$, initial position $z_i(0)$ and other parameters of 6 USVs are given in Table 1. Choosing $\hat{\eta}_i(0) = (0,0)^\top$. Then the trajectory $z_i^*(t)$ of the ith agent $(i \in [1{:}6])$ is obtained by (17).
Then we are going to solve the distributed controller $\tilde{u}_i$. With (35), the ith USV system $(i \in [1{:}6])$ is rewritten as

$$\begin{cases} \dot{q}_{Xi} = v_{si}\cos\phi_i - v_{wi}\sin\phi_i, \\ \dot{q}_{Yi} = v_{si}\sin\phi_i + v_{wi}\cos\phi_i, \\ \dot{\phi}_i = w_i, \\ M_i^{-1}\dot{p}_i = D_i M_i^{-1} p_i + \tilde{u}_i. \end{cases} \quad (37)$$

Since the formation output of the ith USV $(i \in [1{:}6])$ is defined as $z_i = P_i x_i = \mathrm{col}(q_{iX}^*, q_{iY}^*)$, by designing $x_i^1 = \mathrm{col}(q_{iX}^*, q_{iY}^*) = \pi_i^1(z_i) = z_i$, we have $x_i^{1*}(t) = z_i^*(t)$ and the remaining state components are denoted as $x_i^2(t) = \mathrm{col}(\phi_i, p_i)$. The angular momentum w_i can be calculated by [44],

$$w_i = m_{i3}\frac{\ddot{q}_{Yi}\dot{q}_{Xi} - \dot{q}_{Yi}\ddot{q}_{Xi}}{\dot{q}_{Xi}^2 + \dot{q}_{Yi}^2}, \quad (38)$$

where $\dot{q}_{Xi} = \dot{q}_{Yi} = 0$ holds only at the initial point and the equilibrium. After that, ϕ_i is obtained by the integration of w_i. Furthermore, v_{si}, v_{wi} can be obtained by (37) with $q_{Xi}^*(t)$ and $q_{Yi}^*(t)$. With $p_i = M_i\dot{q}_i$, we have

$$\tilde{u}_i(t) = M_i^{-1}\dot{p}_i - D_i M_i^{-1} p_i. \quad (39)$$

With the distributed controller (39), the evolution of the system states are shown in Fig. 4 - Fig. 5. Fig.

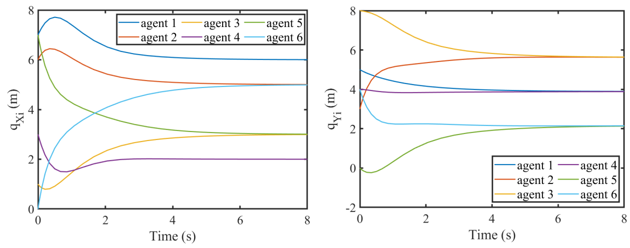


Fig. 4. The evolutions of q_{Xi} and q_{Yi} under algorithm (17).

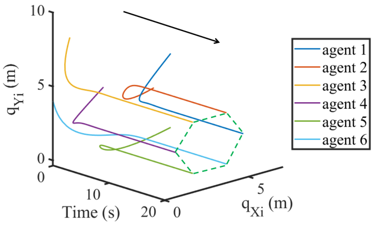


Fig. 5. The system trajectory under algorithm (17).

4 represents the evolution of q_{Xi} and q_{Yi}. Fig. 5 provides a three-dimensional diagram that shows the evolution of q_{Xi} and q_{Yi} over time. By Fig. 5, we find that the multi-agent system converges to an hexagon (green dotted line) from the initial points, satisfying the shape formation objective. Hence, by designing an appropriate valued function, the required formation mission of 6 USVs is accomplished and the control input is obtained.

6.2 Parameters analysis

In this subsection, the influence of the parameters in the designed dynamic (17) of general formation output $z_i(t)$ of the ith agent $(i \in \mathcal{V})$ are analyzed, and different parameters are given in Table 2.
Without loss of generality, we will use the 6 USVs system in (35) as an example for simulation explanation. Fig. 6(a) show that for the three different sets of parameters, 6 USVs are able to achieve the desired hexagonal formation. As shown in Fig. 6(b), the convergence speed I>II and III>II means that the increase of γ and k_1 speeds up the convergence of the designed dynamic (17), respectively. That is because in (17), γ is the coefficient of the difference between estimates $\hat{\eta}_i$ and $\hat{\eta}_j$. When γ increases, it accelerates the agents' achievement of consensus. And k_1 is the coefficient of the negative term of s_i, which accelerates the convergence speed when k_1 increases.

Table 2
Different parameters in Fig. 6

Parameter	I	II	III
$(\gamma,\ k_1)$	(1, 0.99)	(1, 0.1)	(10, 0.1)

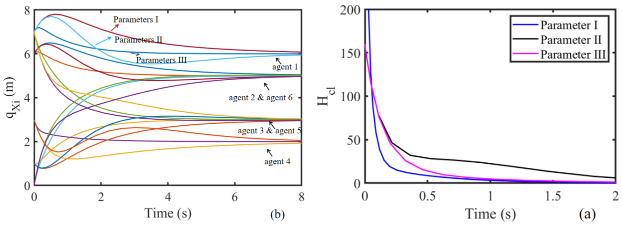


Fig. 6. (a) The evolutions of q_{Xi} under different parameters. (b) The evolutions of H_{cl} under different parameters.

7 Conclusion

This paper considers the general formation control of multi-agent systems within the PH framework. We first express the dynamics of multi-agents interacting through a connected graph as a PH system. We then propose a distributed controller allowing to drive the overall formation to a desired configuration. The main feature of the controller is that it operates without direct state exchange, which help to reduce the risk of leakage of sensitive information (such as position, velocity, value function...). From this, we show the exponential stability of it. Finally, simulations related to the control of a formation of USVs demonstrate the effectiveness of the proposed approach.

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