

Detecting bifurcations in 2D periodic metamaterials from images

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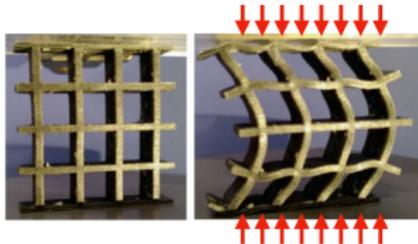
Buckling of architected materials



Global buckling

sudden softening

(periodic AM)

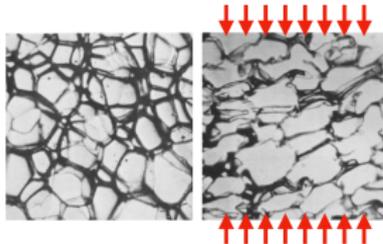


[He, Zhou et al. 2018]

Localization

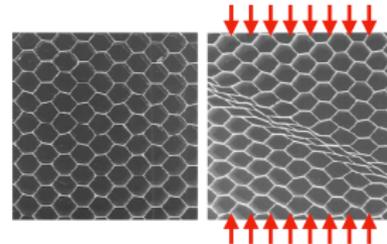
plateau force

(Aperiodic)



[Shaw & Sata 1966]

(periodic AM)



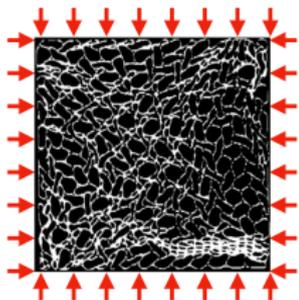
[Gibson 1981]



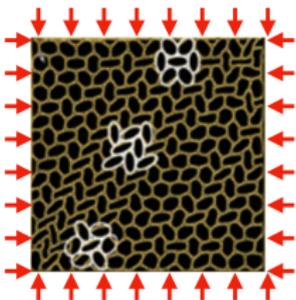
Buckling of architected materials



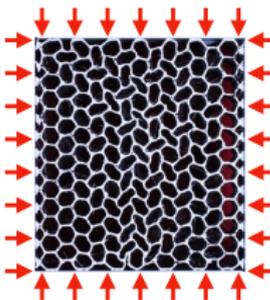
Diffuse buckling, *but* non-uniform patterns: various amplitude, various modes (Periodic AM)



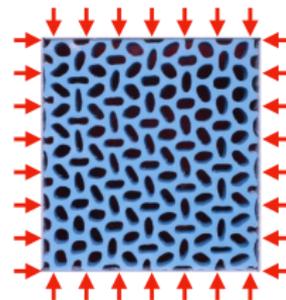
[Kyriakides et al. 1999]



[Karagiosova et al. 2008]



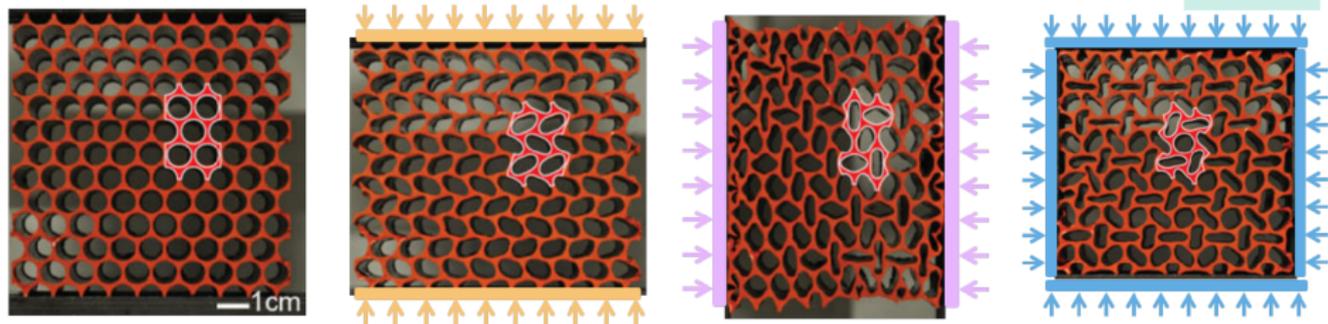
(our 1st disappointment)



(a later one 2023)



Buckling of architected materials



- ▶ (Almost) uniform buckling patterns
- ▶ Tunable wave propagation
- ▶ Controlled dissipated energy
- ▶ [Shan et al., 2014]



Another application

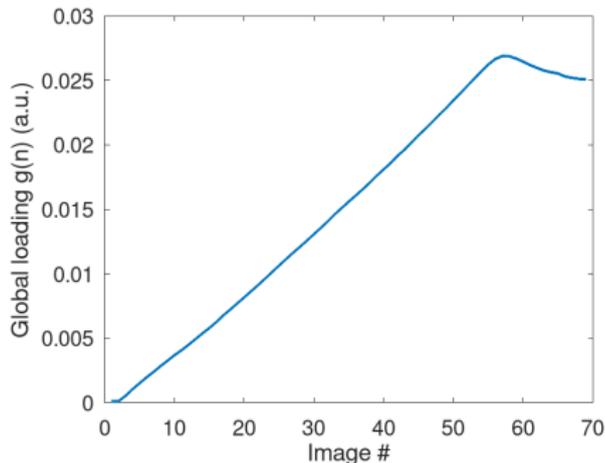


- ▶ The change in the cell geometry controls the overall stiffness



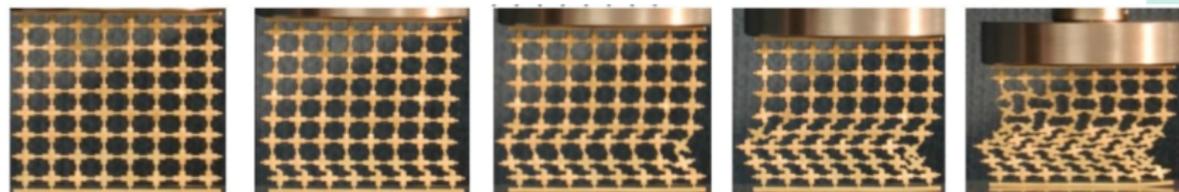
Experimental buckling

- ▶ Impose the displacement, measure the applied force and detect a force peak



- ▶ Distinguish local / global buckling ?

Using images?

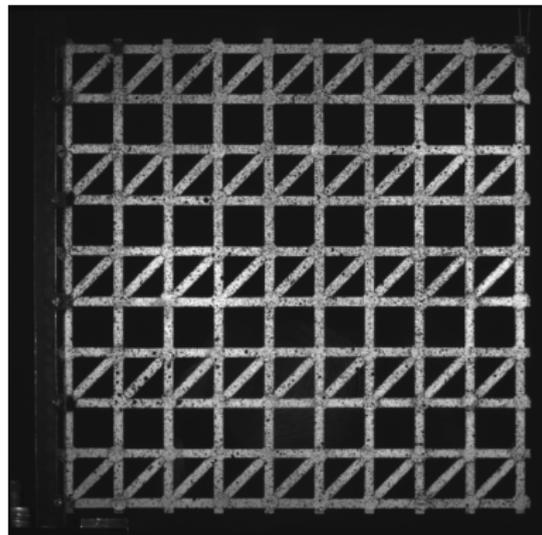


[Ghaedizadeh *et al.* 2016]

- ▶ (Possibly) easy to obtain
- ▶ ... but rarely used
- ▶ Objective : to make the most of these images !
 - ▶ Detect the buckling threshold?
 - ▶ Detect local buckling?



Using images = using DIC ?



- ▶ Patterning required
- ▶ Integrated DIC : imposed beam kinematic (no beam bending)
- ▶ The degrees of freedom are the nodes displacements

- ▶ Alternative when using periodic materials?

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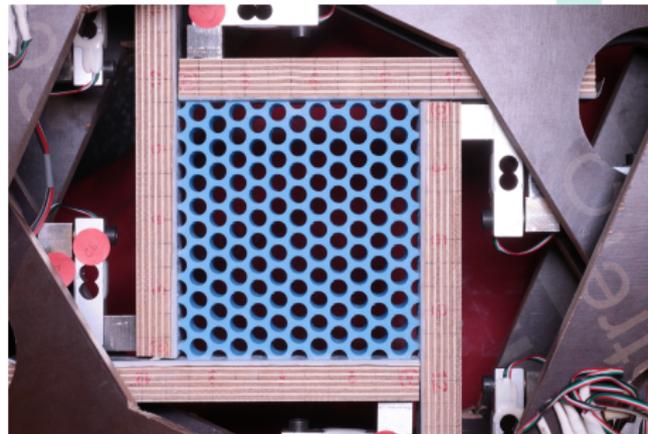
Results

Conclusions



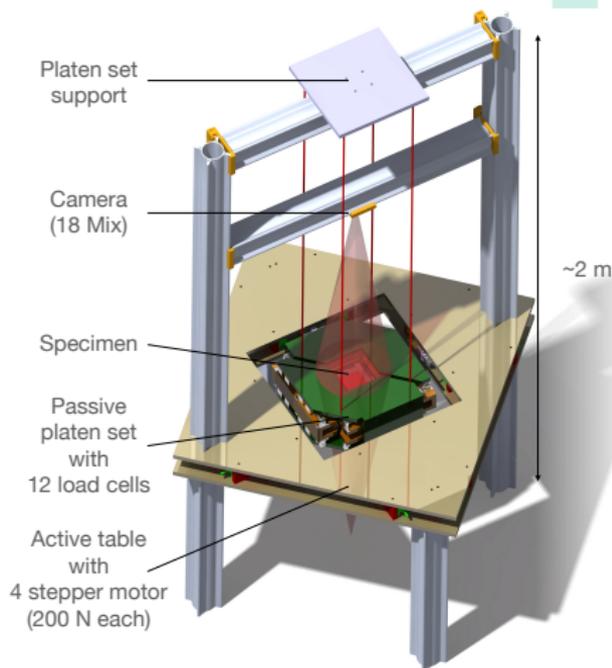
Material

- ▶ Silicon rubber $E \simeq 1.9$ MPa
- ▶ Cast in a 3d-printed mold
- ▶ $101 \times 105 \times 24$ mm

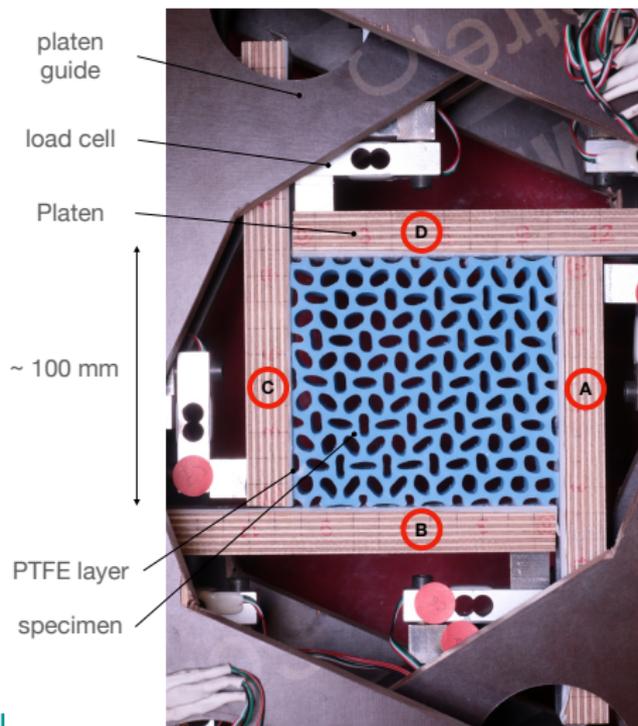


Biaxial loading

- ▶ Inspired from [Hambly, 1969]
- ▶ 8 load cells
- ▶ Loading $T(s, n) \simeq f(s) \times g(n)$
- ▶ s : cell number, n : time index

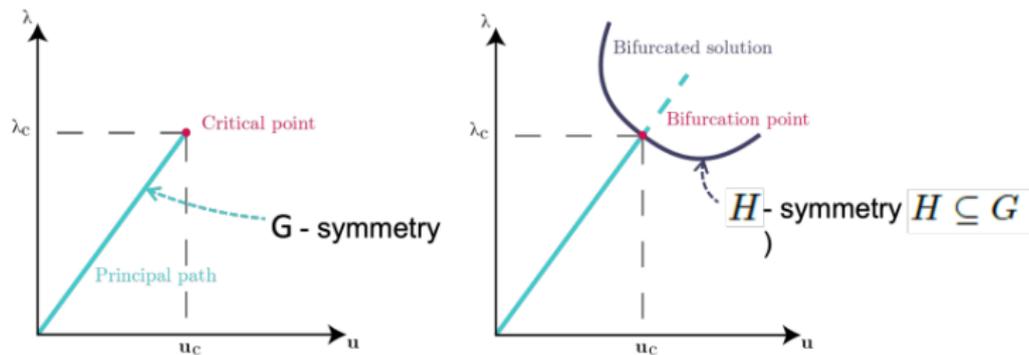


Imaging

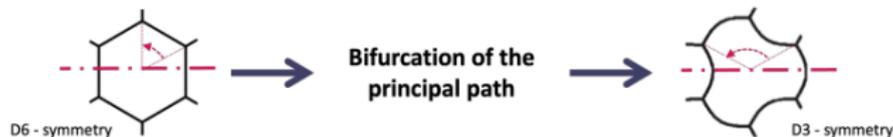


- ▶ Displacement-controlled equibiaxial compression tests
- ▶ Strain rate $< 10^{-3} \text{s}^{-1}$
- ▶ Images $P(x, y, n)$ obtained with a reflex camera (3648×5472 pix)

Idea



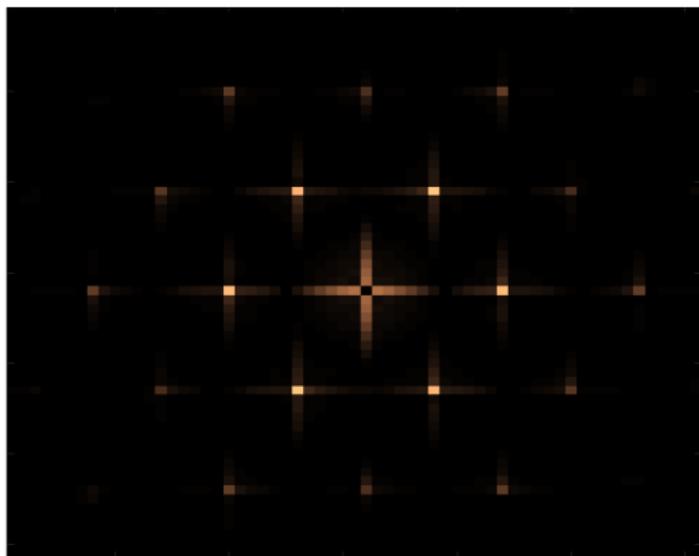
For example :



- ▶ Generic bifurcation breaks symmetry
- ▶ [Sattinger, 1979]
- ▶ Move to reciprocal space !

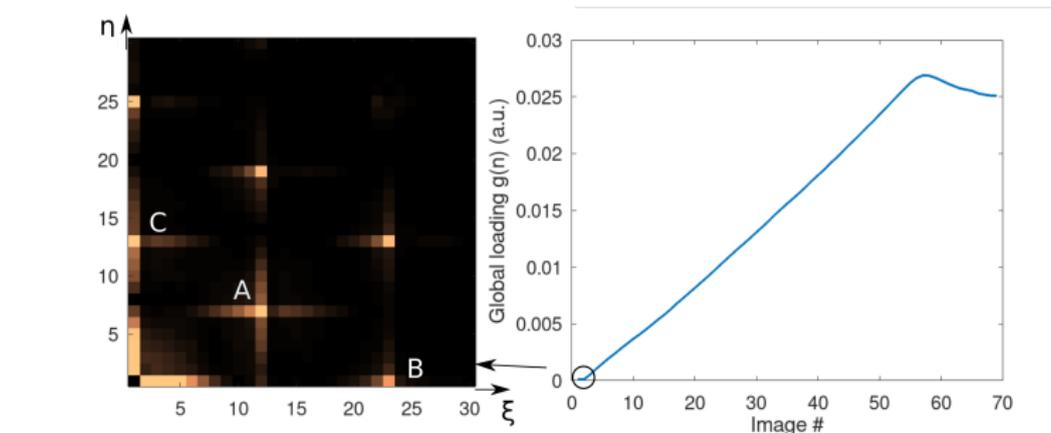
Idea

- ▶ $\hat{P}(x, y, n)$ is the windowed ROI
- ▶ $\tilde{P}(\xi, \eta, n) = \int \hat{P}(x, y, n) \exp(-2i\pi(x\xi + y\eta)) dx dy$



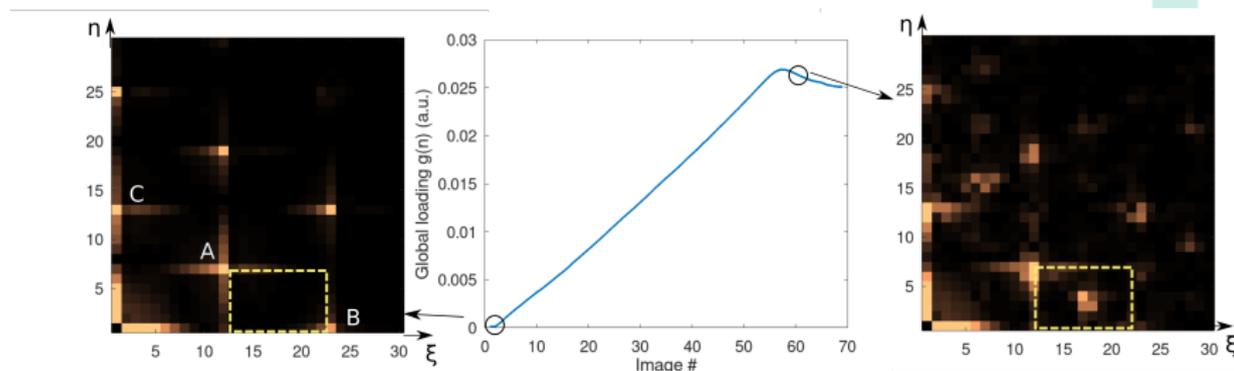
- ▶ Well-defined peaks
- ▶ Hexagonal symmetry

Idea



- ▶ Define an empty spectral region (ESR)
- ▶ For instance : the AB rectangle

Idea

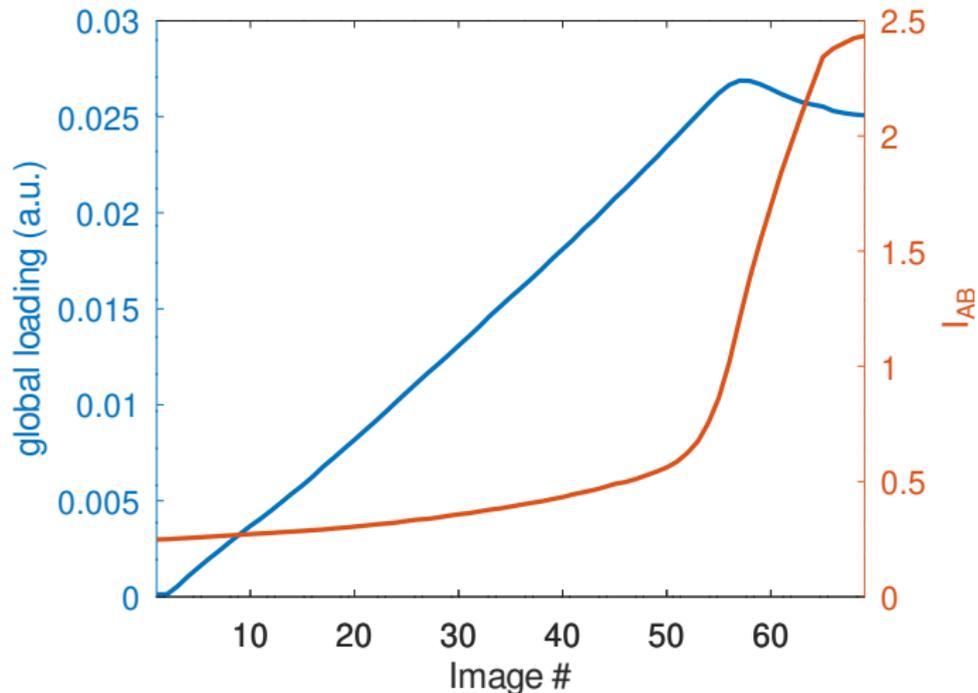


- ▶ Symmetry breaking results in additional (blurry) peaks
- ▶ Track the instabilities using

$$I_{AB}(n) = \frac{\int_{AB} \|\tilde{P}(\xi, \eta, n)\| d\xi d\eta}{\|\tilde{P}(A, n)\| + \|\tilde{P}(B, n)\|}$$

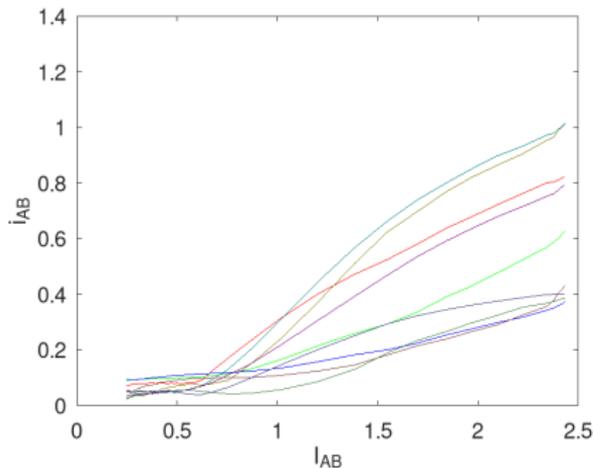
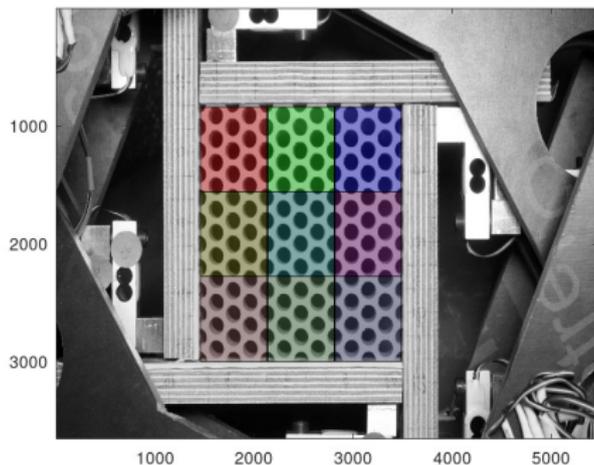
- ▶ One could choose another ESR instead of AB

Global scale



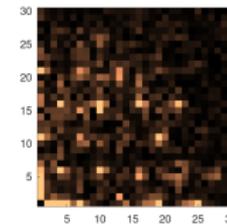
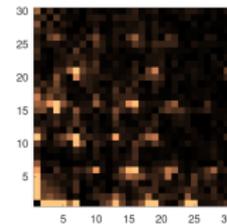
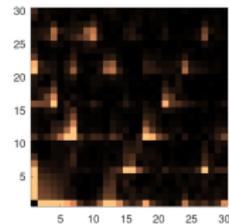
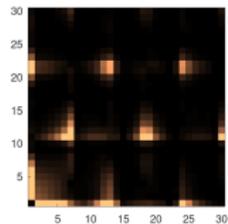
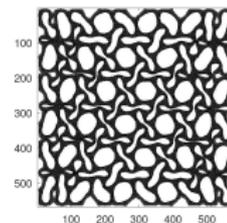
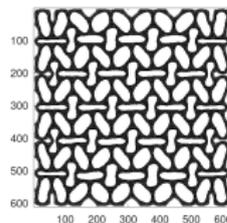
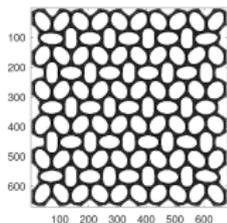
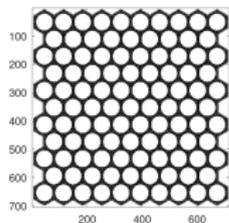
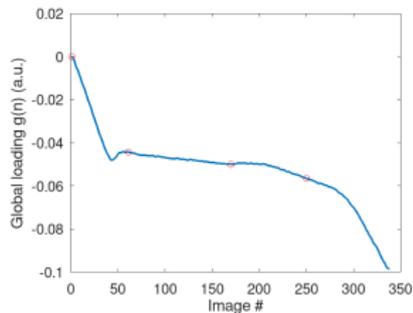
- ▶ ROI = full sample, ESR = AB
- ▶ Excellent agreement

Local scale



- ▶ ROI \subset full sample, ESR = AB
- ▶ Global bifurcation for $I_{AB} \simeq 1$
- ▶ The onset of buckling is not uniform

Computed images

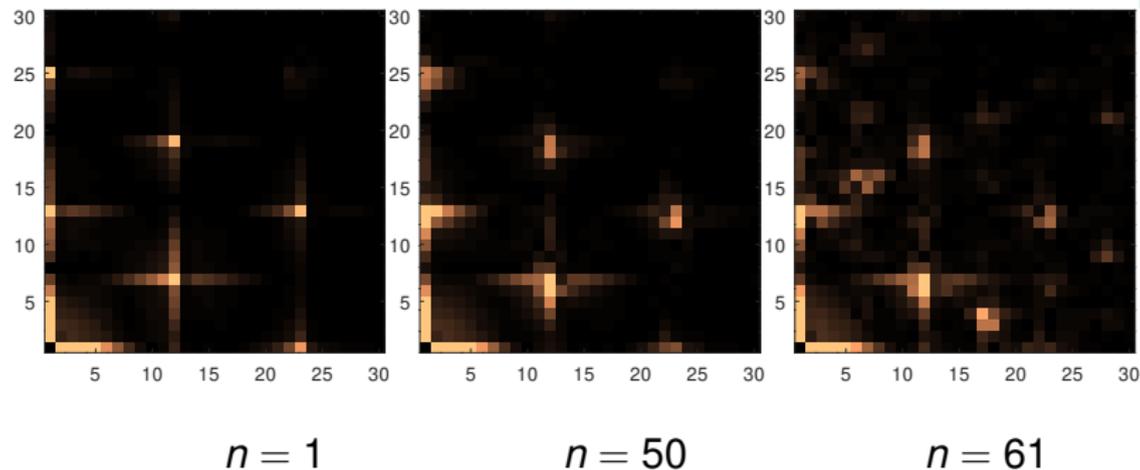


Conclusions

- ▶ Very simple image processing to detect buckling from images
- ▶ No patterning
- ▶ Validated wrt usual buckling indicator
- ▶ Buckling is resolved in both time and space : global/local buckling are discriminated
- ▶ Works with any images (experimental, computed...)
- ▶ [Poncelet et al., 2023]
- ▶ IBIS : Imaging BifurcationS
<https://zenodo.org/records/8252796>



Idea



Uniaxial tension

