Negative Stiffness Mechanical Metamaterials: A Review

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Abstract. Metamaterials have thrived in recent years, with negative stiffness mechanical metamaterials emerging as an essential branch. Extensive research in the past decade has yielded fruitful results in this field. This work encompasses an in-depth exploration of the origin of negative stiffness behavior, along with detailed demonstrations of the implementation mechanism and construction methods used in negative stiffness mechanical metamaterials. Furthermore, the paper highlights the diverse range of applications for these metamaterials, including energy absorption, advanced actuators, deployable and morphing structures, vibration control, and more. Lastly, a brief glimpse into the future development direction of this metamaterial is proposed.

Contents

1. Introduction

The concept of metamaterials originated in the field of electromagnetic materials [1–6] but has since been extended to various domains, encompassing optical [7–9], thermal [10–14], acoustic [15–19], and mechanical metamaterials [20–28]. Mechanical metamaterials are a type of structural materials that achieve unique mechanical properties through microstructure design rather than altering chemical composition [29–31]. In the present day, mechanical metamaterials mainly include [27] pentamode metamaterials [32], compression-twist coupling metamaterials [33–37], ultra-property metamaterials [38–40], and metamaterials with negative constant such as negative compressibility [41–43], negative stiffness [44–46], negative thermal expansion coefficient [47], and negative Poisson's ratio [48–58]. Additionally, drivable [59], adaptive [60–62], programmable [63] metamaterials, as well as origami and kirigami materials [64–69], also fall within the realm of mechanical metamaterials.

Negative stiffness mechanical metamaterials (NSMMs) have gained significant attention in recent years as a critical area of metamaterial research. They are known for their unconventional mechanical properties and vast potential applications. NSMMs exhibit traits such as elastic buckling, multistability, and negative stiffness, which make them suitable for various purposes, as depicted in Fig. 1. These applications include energy absorption [76], actuators [78], deployable structures [79], morphing structures [80], vibration control [77], and more [81, 83].

Research on negative stiffness behavior has a historical foundation dating back to the 1930s [84]. Negative stiffness behavior is characterized by an increase in deformation of a structure or material resulting in a decrease in load [85, 86], or by a region on the load-displacement curve with a negative tangent slope. It's important to note that the study of negative stiffness structures typically excludes behavior caused by structural fracture or failure. Some literature also refers to this behavior as negative incremental stiffness [87, 88]. The occurrence of negative stiffness behavior is often accompanied by a snap-through phenomenon, depicted by the red line in Fig. 2.

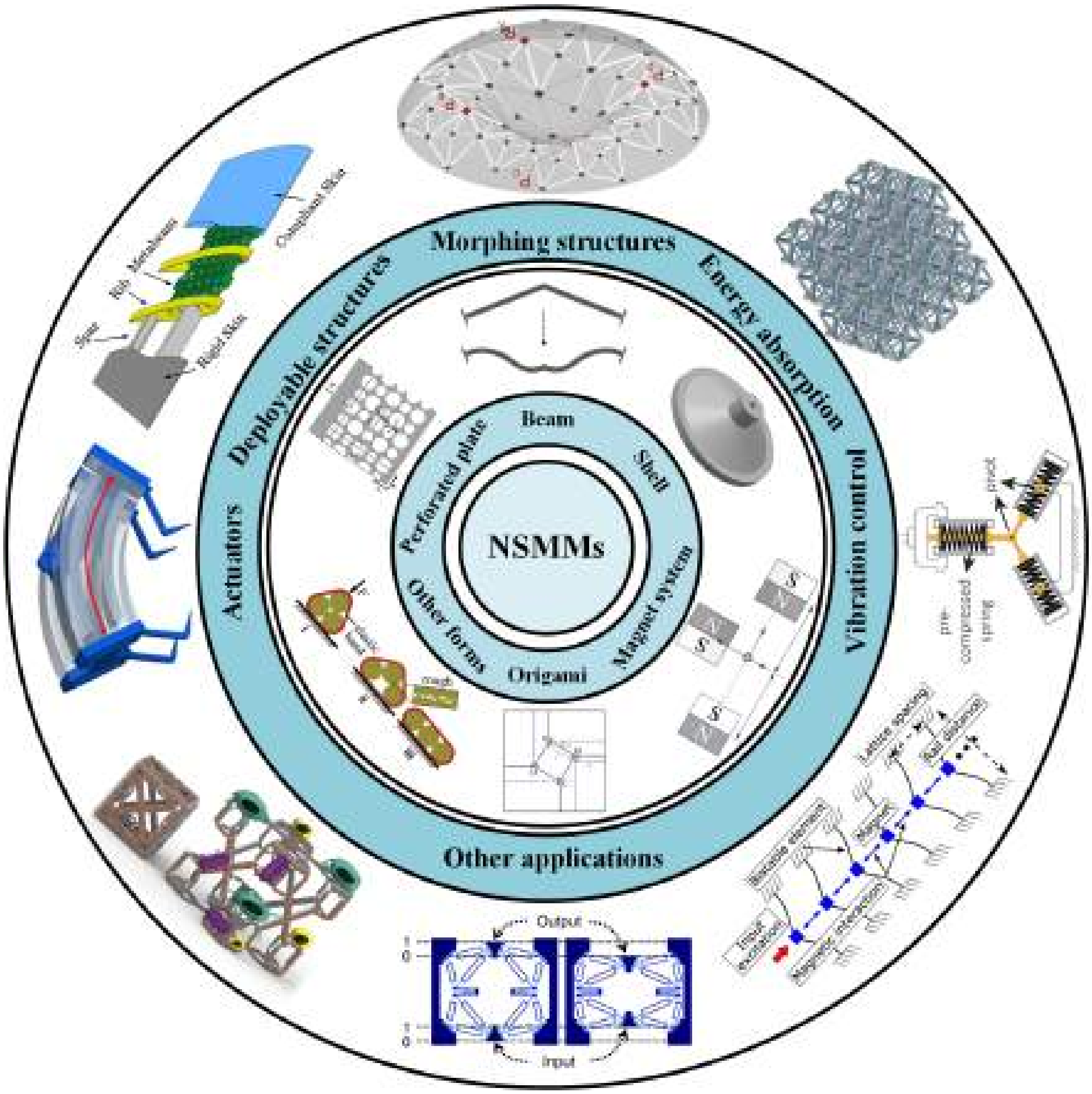

Figure 1: Overview of negative stiffness mechanical metamaterials' construction concept [70–75] (Copyright 2019, Elsevier. Copyright 2020, Elsevier. Copyright 2021, Elsevier. Copyright 2022, Elsevier. Copyright 2019, Jhon Wiley and Sons. Copyright 2016, Royal Society of Chemistry) and possible application field, including energy absorption [76] (Copyright 2019, Elsevier), vibration control [77] (Copyright 2017, Elsevier), actuator [78] (CC BY 4.0), deployable structures [79] (Copyright 2022, Elsevier), morpging structures [80] (Copyright 2021, Elsevier), and etc [80–83] (CC BY 4.0).

Snap-through refers to the dynamic transition of an elastic system's equilibrium from a critical point to a stable equilibrium point that is not adjacent [74]. Previously, structural instability was strictly avoided due to the catastrophic consequences it could entail, such as building collapses [89]. However, in the 21st century, there has been a shift in academic understanding towards skillfully utilizing unstable phenomena rather

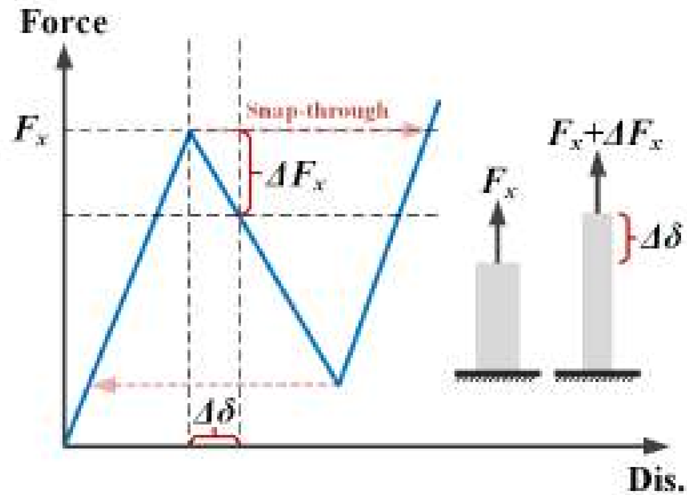

Figure 2: Diagram of negative stiffness response: increase of deformation results in a decrease in load, $\Delta F_x < 0$.

than solely avoiding them. NSMMs are a result of harnessing structural instability for practical applications [89].

Negative stiffness mechanisms have found widespread use in industries, particularly in the field of micro-electromechanical systems (MEMS) [90–92]. This is due to their advantages of simple structures, low cost, fast response, and the absence of a power supply requirement. Furthermore, emerging applications like microfluidics [93] have demonstrated significant potential. In contrast, the development of NSMMs started relatively late, and research in this area is primarily focused on structural design. Several mechanical issues pertaining to NSMMs remain unresolved, and there is limited exploration of their applications. The objective of this review is to introduce the construction methods of NSMMs and highlight their potential applications. It is intended to serve as a valuable reference for scholars working in related fields.

2. Construction/Concept of negative stiffness mechanical metamaterials

Negative stiffness mechanical metamaterials (NSMMs) are typically comprised of an array of negative stiffness elements, displaying periodic negative stiffness responses when subjected to loading [94]. The literature [95] demonstrates early examples of NSMMs exhibiting periodic characteristics. NSMMs can be further categorized as monostable or multistable. Multistable NSMMs can maintain deformed shapes, while achieving this effect with monostable NSMMs is challenging. Structures such as curved beams (cosine beams) and inclined beams (V-shaped beams) have gained popularity in NSMM design due to their simplicity and extensive research on their nonlinear responses. These structures have accumulated substantial early work [96–98] in the field. In addition to these, multi-magnet systems [72], perforated plate structures [63], rotating shell structures [84], origami [73], and other structural forms [99,100] are commonly employed in NSMMs design, as depicted in Fig. 1.

2.1. Negative stiffness mechanical metamaterials based on beam elements

Curved beams (cosine beams) and inclined beams (V-shaped beams) are commonly used in the design of NSMMs [45, 101–105] due to their simplicity. Early designs involving curved beam elements include two-dimensional NSMMs [101] (Fig. 3(a)) and micro-scale NSMMs [45], both of which exhibit negative stiffness characteristics under compressive loads. Curved beam elements can also be employed to create NSMMs with negative stiffness under tensile loads [88]. These structures achieve negative stiffness and multistable behavior through the buckling instability of the beams. Typically, these beam structures are made of soft materials, although some research has explored their implementation with metal substrates [106–109].

NSMMs [110–112] can also be constructed using inclined beam elements (Fig. 3(b)). Both curved and inclined beam elements can be designed as a double-layer configuration [113] (Fig. 3(c)), which effectively prevents local asymmetrical buckling and maintains structural performance [114, 115]. Comparative studies [116] between inclined and curved beam elements reveal that curved beam elements offer slight advantages in mechanical performance and designability, while inclined beam elements are simpler in terms of structure.

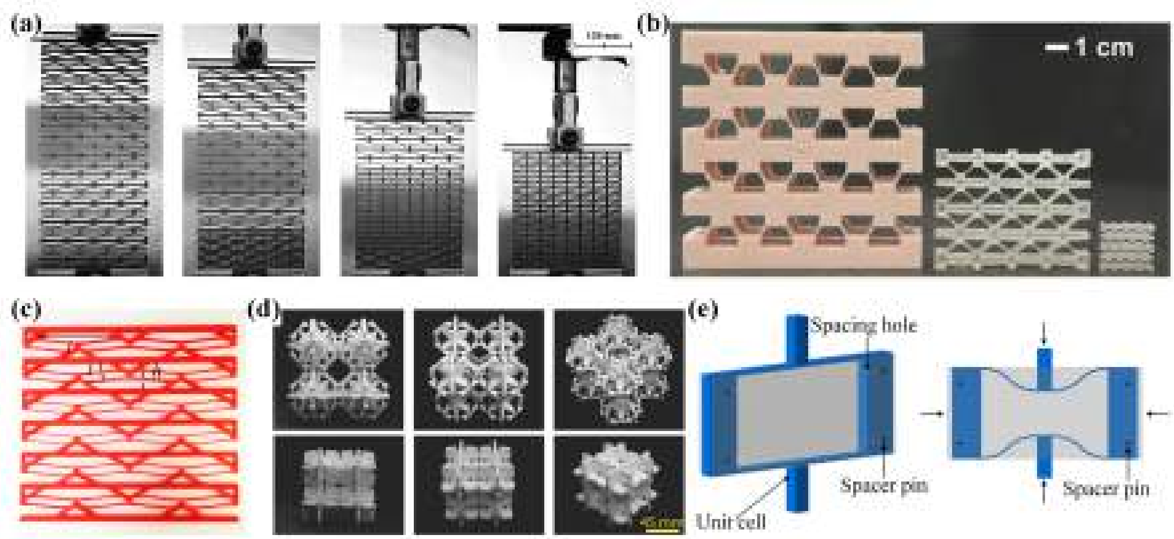

Figure 3: (a) NSMM with curved beam elements [101] (Copyright 2015, Elsevier), (b) NSMM with inclined beam elements [110] (Copyright 2015, Jhon Wiley and Sons), (c) NSMM with double-layer beam elements [113] (CC BY 4.0), (d) NSMM with variable cross-section beams [111] (Copyright 2016, Jhon Wiley and Sons), (e) pre-compressed beam element [117] (Copyright 2020, Elsevier).

In addition to curved and inclined beam elements, NSMMs can also be designed using other beam structures, such as pre-compressed beams [117] and variable cross-section beams [118–120]. Variable cross-section beams [111] are characterized by varying thicknesses along the axial direction. Careful design of the beam's thickness and width in the axial direction allows for specific performance requirements to be achieved. Fig. 3(d) [111] showcases multi-stable reconfigurable metamaterials constructed using variable cross-section beams. These materials offer significant shape and volume changes, and the incorporation of variable cross-section beams notably enhances their energy absorption capacity. Pre-compressed beam structures refer to beams that have been compressed and buckled along their axial direction [117], as depicted in Fig. 3(e). Furthermore, porous beam elements generated through topological optimization [121–123] have been utilized in NSMM construction. Research results [121] have demonstrated that porous beam structures exhibit approximately twice the energy dissipation of solid beams, leading to improved performance in terms of energy absorption.

2.2. Negative stiffness mechanical metamaterials based on perforated plate

Perforated plate structures, featuring periodically distributed holes with regular shapes (such as circles, squares, and rhombuses) on an elastic plate (Fig. 4(a)), can exhibit negative stiffness and multistable properties [124]. These structures, commonly known as phase transition metamaterials, undergo structural transformations through instability. Topological design allows these materials to achieve negative stiffness or negative Poisson's ratio effects under compression or tension loading [125]. Current research on these materials primarily focuses on phase transition paths, phase transition morphologies, and the regulation of various functions before and after phase transition, such as acoustic [126] and optical [127] regulation, as well as energy absorption [63].

From a developmental perspective, perforated plate structures are derived forms of beam-type NSMMs. In theoretical studies, these structures are often approximated as periodic structures composed of variable cross-section beams. Similarly, previous research [128–130] has also explored phase transition structures constructed using periodic beam structures. Although the emphasis may not primarily be on the negative stiffness effect in these structural materials, due to their reliance on the instability principle and significant similarities, they are also classified within the scope of NSMMs.

In recent years, significant progress has been made in the development of perforated plate structures, leading to numerous notable research studies. For instance, Jie et al. [131] and Jiang et al. [132] explored the use of membrane structures with periodically distributed pores, combined with shape memory materials, to achieve optical regulation. Bastiaan et al. [75] designed a tunable, multi-stable, negative stiffness metamaterial utilizing a plate structure with periodically distributed pores (Fig. 4(b)). This study demonstrated the ability to regulate negative stiffness behavior and damping performance by adjusting the transverse pre-strain of the structure. Extensive research on perforated plate structures has been carried out by Bertoldi et al [124, 126, 133–135]. They investigated the influence of pore shape on structural buckling modes, proposed

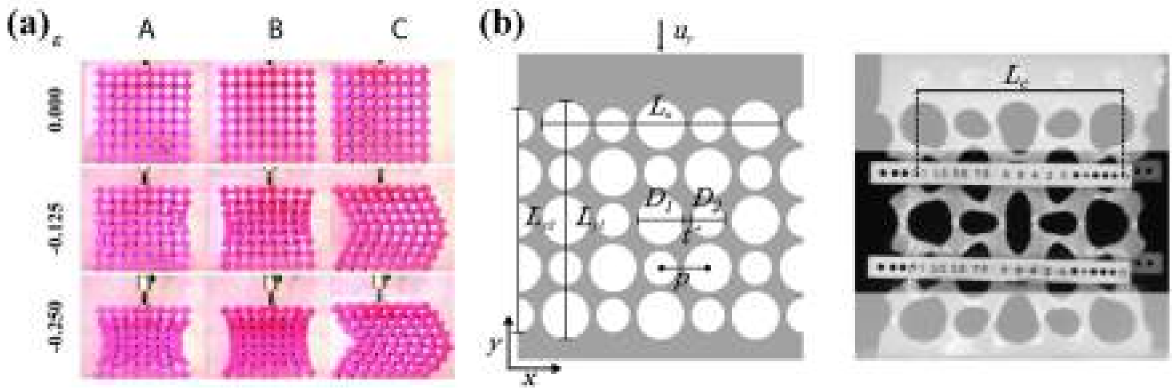

Figure 4: (a) Typical perforated plate structures [124] (Copyright 2012, Jhon Wiley and Sons), (b) tunable and multi-stable NSMMs based on perforated plate structures [75] (Copyright 2005, Royal Society of Chemistry).

a porous phase transition structure in the form of a cylindrical shell [136], utilized the phase transition behavior of perforated plate structures for structural bandgap regulation [133], and designed new types of actuators [134] using such structural materials. Furthermore, the development of microscale perforated plate structures [137] and tunable perforated plate structures [138] has also greatly expanded the potential application scenarios for these structural materials.

Indeed, many perforated plate structures have inherent monostable characteristics, meaning that their deformed states require external forces to be maintained. To achieve specific functionalities, these structures often rely on the coordination of external physical fields [139, 140]. Various methods are employed for state transformation and stability maintenance in such materials, including temperature fields [141], magnetic fields [142], shape memory materials [132], pneumatic drive [143], and more. These external influences facilitate the implementation of desired functions in the perforated plate structures.

2.3. Negative stiffness mechanical metamaterials based on shell structures

Shell structures, such as spherical and conical shells (Fig. 5(a-b)), can exhibit negative stiffness and multi-stable behavior when subjected to loading [144–148]. Compared to other negative stiffness elements, shell structures are advantageous in terms of tuning due to their cavity property [149]. Additionally, shell structures often possess pseudo-bistable characteristics, meaning that after maintaining a deformed steady state for a period of time, the structure gradually returns to its initial state, as shown in Fig. 5(c). These pseudo-bistable characteristics [149] hold great promise in the field of deployable structures, as the structure can automatically revert from the deformed state to the initial state without external force intervention. Given these unique properties, spherical and conical shells are commonly employed in the design of NSMMs.

The common telescopic straw we often use for beverages is actually composed

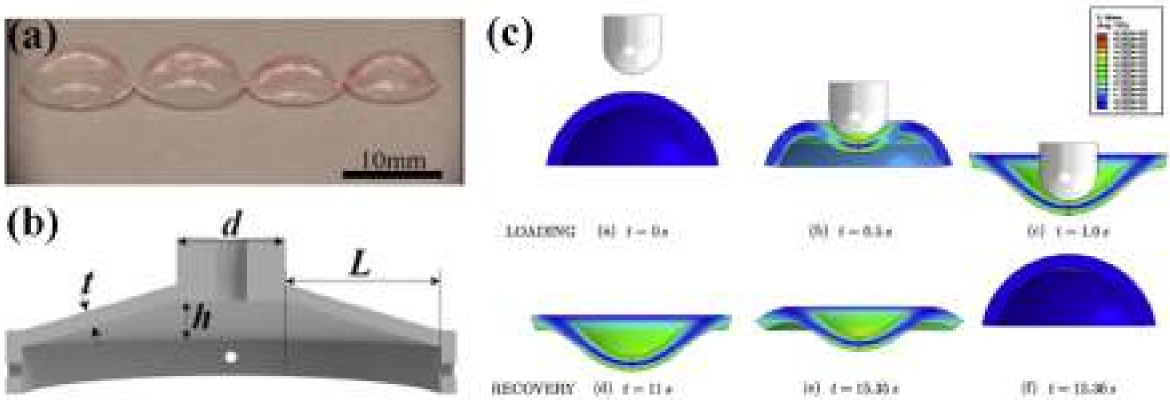

Figure 5: (a) Spherical shell structure [145] (Copyright 2014, Elsevier), (b) conical shell structure [147] (CC BY 4.0), (c) pseudo-bistable characteristics of shell structures [146] (Copyright 2012, Elsevier).

of multiple conical shells arranged in series. The ability of the straw to stretch and shrink reflects the presence of a multi-stable phenomenon. Drawing inspiration from this design, a 3D pixel mechanical metamaterial [150] with negative stiffness and multistable properties has been developed (Fig. 6(a)). These metamaterials exhibit remarkable mechanical programmability. Tan et al. have also designed NSMMs utilizing conical shells, including the tridirectional NSMM [71, 149] and the bio-inspired NSMM [147]. Research results have shown that the bio-inspired NSMM (Fig. 6(b)) outperforms certain commercial packaging materials, such as air bubble film and foams, in terms of mechanical properties.

Spherical shells are widely utilized in the construction of NSMMs [152] as well. Udani et al. [144] introduced a programmable metamaterial with highly tunable stiffness by utilizing locally bistable spherical shells, as depicted in Fig. 6(c). Meanwhile, Jia et al. [153] presented a mechanical metamaterial employing thin spherical shells that exhibits negative stiffness, negative bulk modulus, and negative Poisson's ratio simultaneously. Apart from spherical and conical shells, curved shell structures [151, 154–156] also possess negative stiffness and bistable characteristics. Fig. 6(d) displays a typical NSMM [151] constructed using curved shell elements. Currently, mainstream shell structures [157–159] are generated by rotating and sweeping beam elements, while diverse design options are available with certain special shell forms [160].

2.4. Negative stiffness mechanical metamaterials based on multiple magnets system

The interaction between magnets can be characterized by repulsion and attraction, which is determined by the relative positions of the magnets. By harnessing this magnetic interaction, it is possible to achieve negative stiffness and develop bi-stable systems [72]. The advantage of magnet-based interaction lies in its non-contact nature, which eliminates concerns related to structural fatigue and offers the potential for extended service life. Previous studies [161] have shown that the strength of

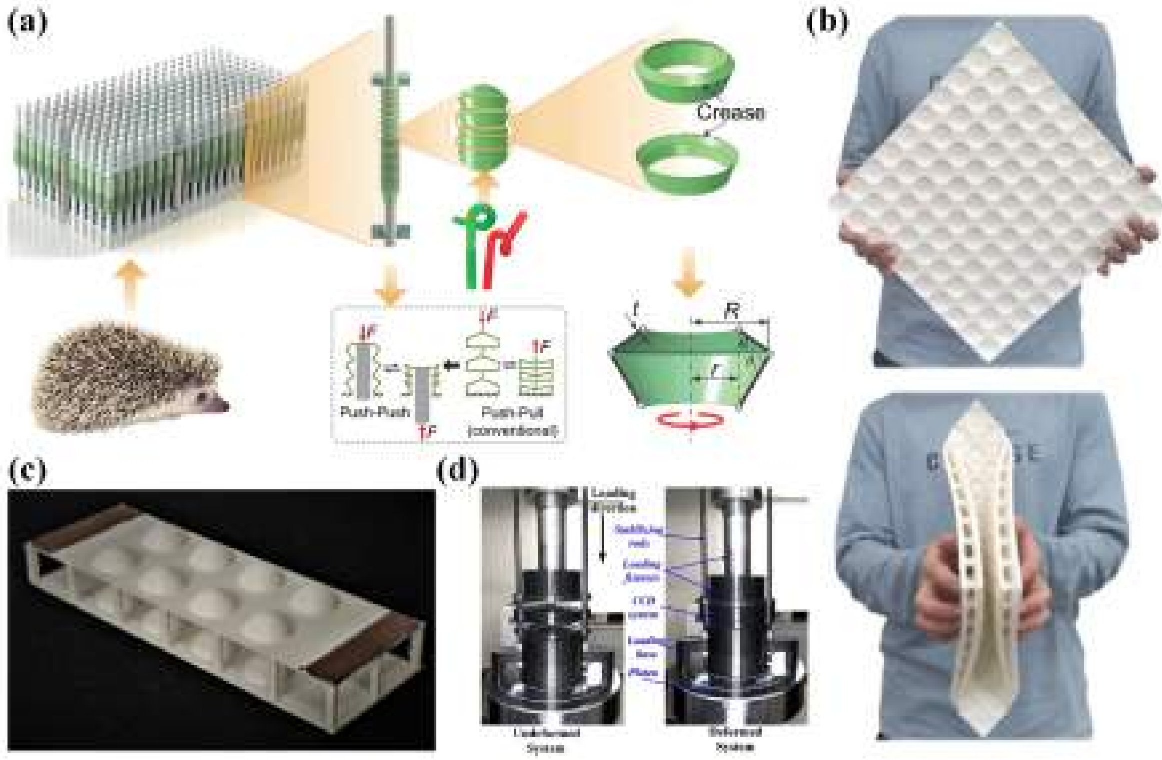

Figure 6: (a) An 3D pixel mechanical metamaterial [150] based on conical shells (Copyright 2019, Jhon Wiley and Sons), (b) bio-inspired NSMMs [147] based on conical shells (CC BY 4.0), (c) programmable metamaterials with highly tunable stiffness [144] based on spherical shells (Copyright 2021, Elsevier), (d) typical NSMMs constructed with curved shell element [151] (Copyright 2019, Elsevier).

the magnetic force is directly proportional to the square of the magnetic induction intensity. Therefore, introducing magnets with high magnetic induction intensity, such as superconducting magnets [162], has the potential to enhance the mechanical performance of metamaterials.

In recent years, numerous studies [72, 163–169] have focused on harnessing the potential of magnets to construct NSMMs. Alderson et al. [163] and Dudek et al. [164] designed double-negative mechanical metamaterials that exhibit negative stiffness and negative Poisson's ratio simultaneously (Fig. 7(a-b)). Tan et al. [72] conducted theoretical analyses to determine the key parameters that influence the mechanical performance of magnet systems and identified the optimal system layout for energy trapping. Additionally, Tan et al. [165] introduced a shear-induced NSMM capable of effectively cushioning glancing mechanical impacts (Fig. 7(c)). Seyedkanani et al. [166] developed a mechanical metamaterial with negative incremental torsional stiffness by arranging permanent magnets in a circular pattern. This design was then utilized to create a tunable fluid-free rotary metadamper that dissipates energy through repeated snap-back instabilities (Fig. 7(d)).

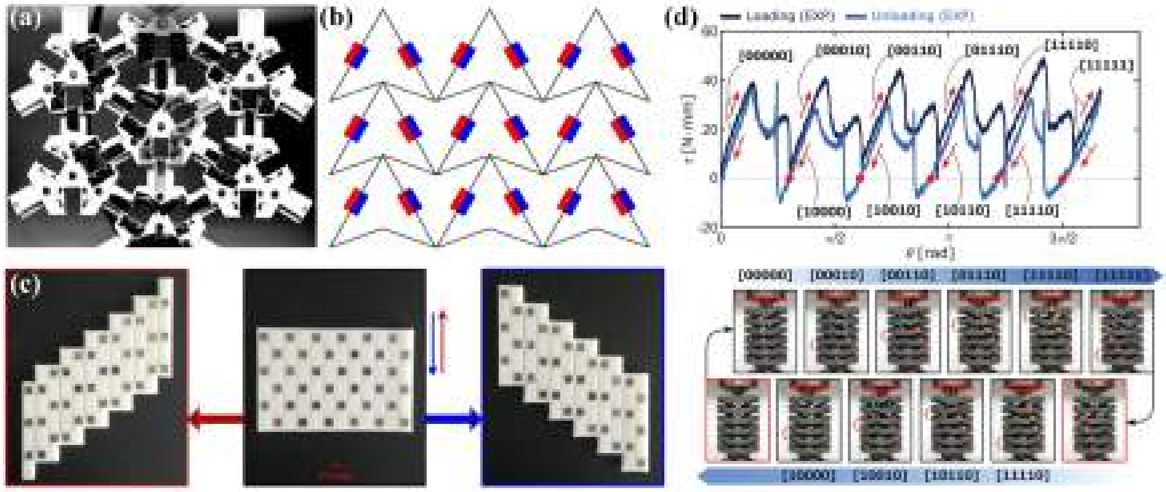

Figure 7: (a-b) Double-negative mechanical metamaterials [163, 164] based on multiple magnets system (Copyright 2016, Jhon Wiley and Sons. Copyright 2018, The Royal Society(U.K.)), (c) shear-induced NSMM [165] based on multiple magnets system (Copyright 2019, Elsevier), (d) negative incremental torsional stiffness mechanical metamaterial [166] based on multiple magnets system (Copyright 2022, John Wiley and Sons).

2.5. Negative stiffness mechanical metamaterials based on origami

Origami [170–180] and kirigami [181–183], the art of paper folding and cutting, also exhibit negative stiffness and multistable behavior. Origami has evolved into a design framework applicable to various engineering fields [175]. Multistable origami/kirigami structures have garnered significant attention [174] due to their potential to enhance controlling stability and improve energy utilization efficiency during shape-reconfiguration processes. Fang et al. [175, 184], through extensive research, have made notable contributions to this field. Their work includes proposing folding multi-stable stacked-origami [175], programmable self-locking origami mechanical metamaterials [184], and multistable origami metamaterials (Fig. 8(a)) with reprogrammable mechanical properties [177]. They have also studied the nonlinear dynamical characteristics of multi-stable series origami structures [185]. Furthermore, Chen et al. [174] presented a novel class of multistable origami honeycombs that are lightweight, scalable in three-dimensional space, and offer flexible and easy designability (Fig. 8(b)). Filipov et al. [170] explored the mechanical behavior of origami hyperbolic paraboloids and constructed a bistable thin sheet structure (Fig. 8(c)).

Indeed, kirigami, the art of paper cutting, can be applied to construct NSMMs as well. Pasini et al. [186] perforated various cut motifs into a rubber sheet, introducing a class of architected materials that exhibit both auxeticity (negative Poisson's ratio) and structural bistability. Many multistable kirigami metamaterials [183, 187–189] have been developed based on similar strategies. Furthermore, certain kirigami structures

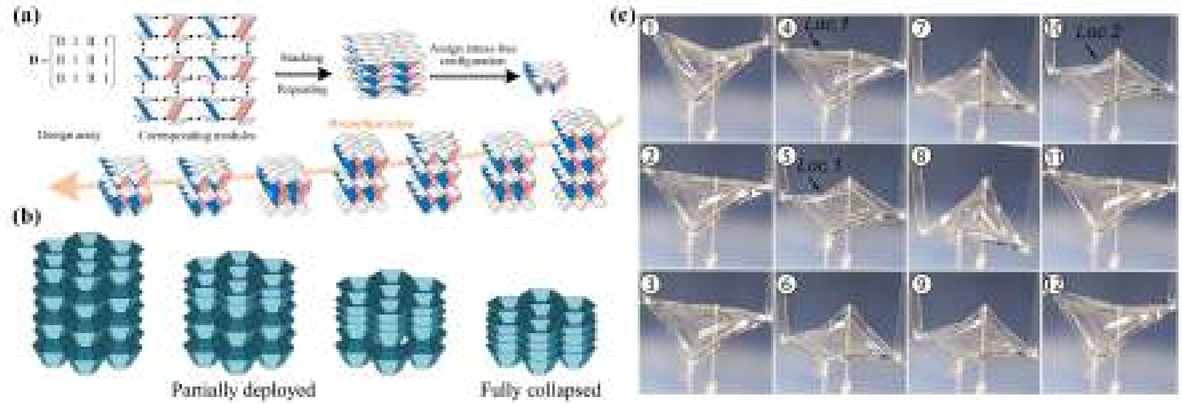

Figure 8: (a) Multistable origami metamaterials with reprogrammable mechanical property [177] (Copyright 2023, Elsevier), (b) Multistable origami honeycombs [174] (Copyright 2023, Elsevier), (c) bistable thin sheet structure [170] (Copyright 2018, Elsevier).

can exhibit negative stiffness or multistability under out-of-plane loading. For instance, Yang et al. [181] demonstrated that by adjusting the geometric configuration of a kirigami structure, bistable behavior can be achieved, and the mechanical properties can be controlled through locally reversible steady-state transformations. Virk et al. [182] proposed a kirigami structure that exhibits a negative stiffness effect under flat compression.

2.6. Other novel structural forms

Column buckling is indeed a common form of structural instability, and elastic rod structures are frequently employed in the construction of NSMMs [190–196] (Fig. 9(a-b)). Research studies [193] have indicated that energy-absorbing materials utilizing elastic rod structures exhibit significant potential for high-strain impact applications. One advantage is that their performance is minimally affected by loading rate, making them effective in absorbing and dissipating energy under dynamic loading conditions.

The phenomenon of instability caused by mutual compression of substructures is another promising mechanism for achieving negative stiffness. One example of this is the 'snap-fit' structures [197], which are typical mechanical metamaterials that utilize mutual compression to obtain negative stiffness properties (Fig. 9(c)). Bertoldi et al. [198] designed a phase transition metamaterial by alternating soft crystals of different sizes. Under loading, the compression of the crystals causes a change in the relative positions of smaller soft crystals, resulting in a phase transition effect. Jin et al. [74] employed a similar concept to design an energy-absorbing multi-stable metamaterial that combines rigid components and stretchable components (Fig. 9(d)). The stretchable component maintains the structure's integrity and controls the mode of phase transition. Friction between the rigid components dissipates energy when the structure is subjected to a load. The sleeved structure is another example that

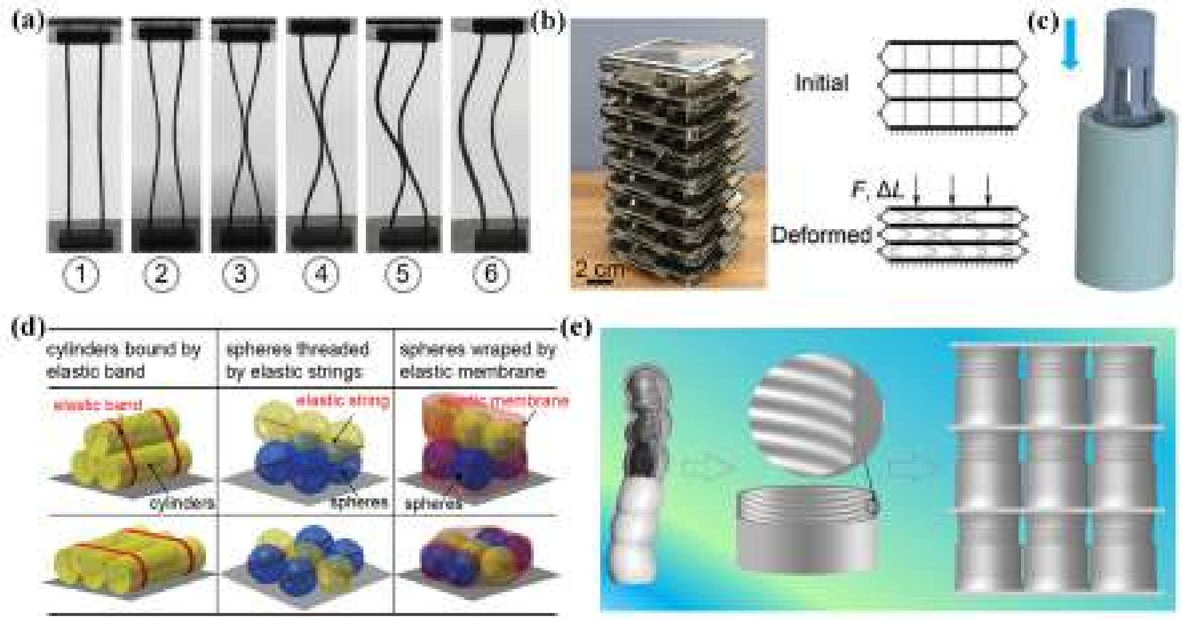

Figure 9: (a-b) NSMMs based on elastic rod structures [190, 193] (Copyright 2020, Elsevier. Copyright 2021, Jhon Wiley and Sons), (c) NSMMs based on snap-fit behavior [197] (Copyright 2023, Elsevier), (d) multi-stable metamaterial combining rigid components and stretchable components [74] (Copyright 2019, Jhon Wiley and Sons), (e) sleeve-typed NSMMs [99] (Copyright 2019, Elsevier).

utilizes the compression-instability mechanism to achieve negative stiffness and multi-stable effects. Zhu et al. [99] designed a sleeve-typed NSMM with toothed structures on cylinder walls. The friction-compression and instability between the inner and outer sleeves not only achieved negative stiffness and multi-stable effects but also exhibited high energy absorption performance (Fig. 9(e)). Other NSMMs designed based on this mechanism can also be found in the literature [199–201].

In addition, there are some less popular schemes for achieving negative stiffness behavior, such as the design of omnidirectional NSMMs based on spring (elasic wire) [202], and the block negative stiffness structures [203]. The laminates [204] can be equipped with multistable property, but are rarely used to constrcut mechanical metamaterials.

3. Application of negative stiffness mechanical metamaterials

The application exploration of NSMMs is still in the laboratory stage. Potential application fields mainly include [205] energy absorption, actuators, deployable structures, morphing sturtucres, vibration control, etc. The following are the details:

3.1. Energy absorption

NSMMs exhibit remarkable potential in the field of energy absorption, with applications found across various sectors including transportation vehicle collisions, athlete protection, and safeguarding precision instruments and valuable items. While traditional energy-absorbing materials rely on mechanisms such as plastic deformation of metals, fragmentation of brittle materials, and velocity-related viscoelastic processes, these methods typically suffer from repeatability and velocity-related effects. In contrast, NSMMs offer effective solutions to these challenges, providing advantages such as the ability to accommodate large deformations, absence of rebound after impact, and adjustable impact response amplitudes [206]. In terms of development, early NSMMs [207] relied on the damping properties of base materials for energy absorption, but recent advancements have introduced two innovative mechanisms: the energy trapping mechanism [110, 150, 160] and the twinkling mechanism [45, 46, 101, 104, 208, 209], which enable efficient energy absorption.

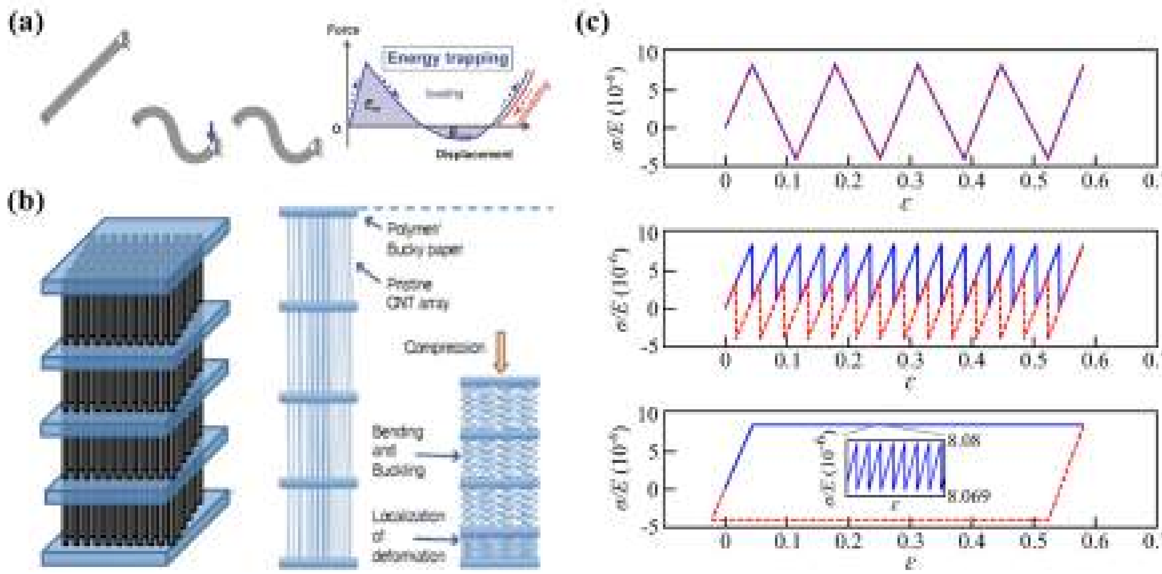

Figure 10: (a) Energy trapping mechanism of multistable metamaterials [110] (Copyright 2015, Jhon Wiley and Sons), (b) micro-scale multilayer metamaterials [210] (Copyright 2011, American Chemical Society), (c) twinkling mechanism [101] (Copyright 2015, Elsevier).

Shan et al. [110] firstly demonstrated the energy trapping mechanism with a beam typed NSMM. This metamaterial has the capability to capture mechanical energy from external sources and convert it into strain energy stored within the structure itself, as depicted in Fig. 10(a). To prevent structure recovery and the release of strain energy, it is essential for this type of metamaterial to possess multi-stable properties. The energy trapping mechanism offers several advantages, including reusability, effective suppression of shock acceleration response, and minimal velocity effects. Building upon this concept, Pan et al. [150] designed a pixel metamaterial that not only captures energy but also exhibits self-adaptive characteristics, thereby ensuring stable protection of impacted objects and preventing secondary damage.

The "twinkling" mechanism represents an intriguing and significant energy absorption strategy in NSMMs. Daraio et al. [210,211] first discovered this phenomenon in micro-scale multilayer metamaterials, as illustrated in Fig. 10(b), and conducted experimental verifications. The crux [101] of this mechanism lies in the relationship between the energy absorption efficiency and the number of cells connected in series. When only a small number of cells (e.g., 1, 2, 3, etc.) are connected, the loading and unloading response curves exhibit extensive overlap, indicating minimal dissipated energy. However, as the number of connected cells increases, the energy absorption efficiency progressively improves. At a certain point, the rate of change in energy absorption and dissipation efficiency starts to diminish, eventually reaching a plateau, as depicted in Fig. 10(c). The "twinkling" mechanism suggests that the energy absorption and dissipation efficiency of NSMMs is independent of velocity effects, and the viscoelasticity of the base material has minimal influence on the structure's energy absorption efficiency. The "twinkling" mechanism has been experimentally and theoretically verified by Frenzel et al. [45] and Liu et al [104]. Frenzel et al. [45] designed a micro-scale negative stiffness energy absorption device using curved beams and provided a physical explanation for the "twinkling" phenomenon. They posited that the "twinkling" behavior is akin to the hysteresis observed in the loading and unloading process of metal materials. When a single metal crystal acts as a nonlinear spring unit for tension and compression, hysteresis does not occur. However, the mechanical model of numerous gathered metal crystals resembles countless springs in series. Under loading and unloading conditions, this configuration manifests the hysteresis phenomenon.

The reusability of NSMMs is attributed to the fact that their structural deformation remains within the elastic range. To achieve significant recoverable local strain during loading, soft materials with high elastic strain are commonly employed as the base material in most NSMM designs. However, the utilization of soft materials poses a challenge to the energy absorption performance of NSMMs due to their low strength and stiffness. Wang et al. [108] theoretically demonstrated that even introducing a stronger base material like stainless steel narrows down the design space available, thereby further limiting the performance of NSMMs. Using traditional metal materials as the base material will actually lower the performance under the premise of ensuring reusability. As a result, enhancing the performance of NSMMs has become a crucial issue in their development process [212].

Recent studies have offered some solutions to enhance the energy absorption capabilities of NSMMs. Pasini et al. [107] utilized rigid materials to design a multistable metamaterial and demonstrated its durability through cyclic testing. The metamaterials presented, composed of sturdy base materials, exhibit both bistability and durability even after enduring 10,000 cycles. As illustrated in Fig. 9(e), Zhu et al. [99] introduced a sleeve-type multistable structure founded on the 'friction-compression'

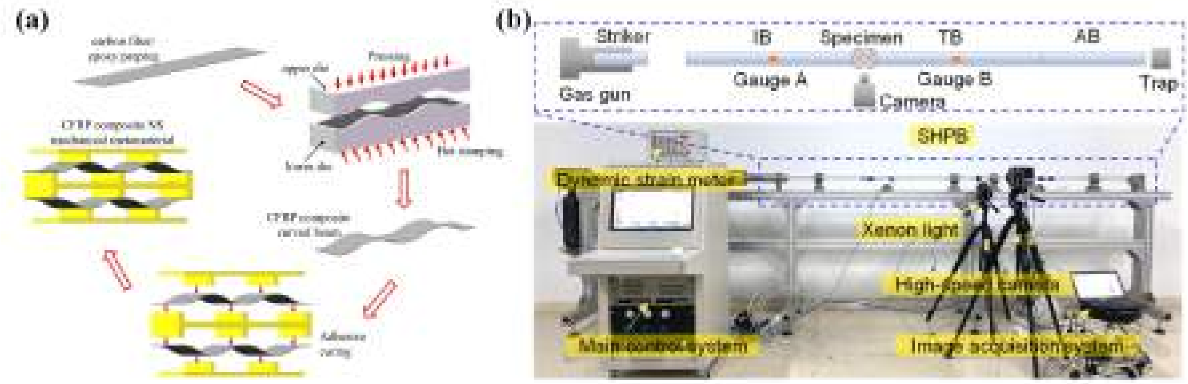

Figure 11: (a) Negative stiffness mechanical metamaterial made of fiber-reinforced resin-based composites [213] (Copyright 2022, Elsevier), (b) experiment setups of high-speed impact on NSMMs [214] (Copyright 2023, Elsevier).

(mutual compression) instability mechanism. This innovation achieves remarkably high energy absorption performance by leveraging friction between the inner and outer sleeves during the loading process.

Addressing the demanding prerequisites of the 'twinkling' mechanism, which necessitates a specific number of serial units, and the energy trapping mechanism, which relies on materials with multistable characteristics, various studies have explored the integration of 'composite materials' to enhance the energy absorption capabilities of NSMMs. For instance, Cortes et al. [106] introduced a composite NSMM configuration by interconnecting curved beam units with viscoelastic polymers in parallel, all while preserving negative stiffness behavior. Meanwhile, Tan et al. [70] proposed a strategy that involves incorporating fillers to enhance the mechanical performance of NSMMs. Furthermore, techniques such as structural topology optimization [215–218], machine learning [118], and the introduction of fiber-reinforced resin-based composites [213, 219], as illustrated in Fig. 11(a), have been employed to optimize the performance of such structures."

Beyond the aforementioned investigations, a substantial body of research [110, 150, 220–224] has delved into the response characteristics of NSMMs subjected to dynamic impact at low speeds. These studies have yielded promising results, highlighting NSMMs' ability to effectively dampen acceleration response amplitudes. Multistable metamaterials, in particular, exhibit intriguing traits, including the absence of rebound and the prevention of secondary damage [110]. Li et al. [214] conducted dynamic mechanical assessments of NSMMs subjected to high-speed impacts. The findings underscored NSMMs' capacity to maintain a layer-by-layer collapse mode even when subjected to high-speed impacts. Furthermore, to adapt NSMMs to complex impact environments, researchers have introduced various structural enhancements. These innovations include the proposal of multi-directional NSMMs [76, 102, 225–228] and cylindrical NSMMs [229–231].

Apart from NSMMs, there are also some new structural materials that have reusability. These mainly include soft structural materials [232], aerogel materials [38], and metal micro-lattice materials [233], among others. From the current state of research, NSMMs have greater advantages in buffering and energy absorption over these other materials.

3.2. Actuators

Compared to traditional soft actuators, bistable and multistable soft actuators have at least following advantages [234]: fast movement and amplified force owing to the snap-through behavior, no additional energy consumption to maintain the deformed shape, rich available deformed configurations. Moreover, these actuators can be driven by various forms of physical excitation, including magnetic fields [235], pneumatic pressurization [236], temperature [237], moisture [238], light [239], and electricity [240].

Many of the negative stiffness (bistable) elements mentioned above find practical applications in the development of advanced soft actuators. For instance, Chi et al. [241] constructed pre-curved 2D beam-like bending actuators and 3D doming actuators by bonding stress-free active layer with embedded pneumatic channels to a uniaxially or biaxially pre-stretched elastomeric strip or disk (Fig 12(a)). Faber et al. [242] employed multistable patterned dome-shaped sheets to design a pneumatic driven soft gripper, where the gripping and releasing actions are achieved through the transformation of the domes' multistable states (see Fig. 12(b)). Additionally, Kaufmann et al. [243] assembled a robotic arm using bistable origami modules, as depicted in Fig. 12(c). These origami modules offer versatility, allowing for low bending stiffness or behaving like a stiff link with the capability to switch between their two stable states. Further pertinent research on this subject can be found in the literature [243–245].

The innovative actuators mentioned above have paved the way for the creation of a wide range of novel soft robots. For instance, Yang et al. [134] have designed a series of soft robots by integrating pneumatic driving devices with a sealed perforated plate structure. These soft robots are capable of performing multiple functions, including gripping, underwater propulsion, and crawling. Researchers believe that these soft robots address the limitations of traditional 'hard' robots, such as their heavy weight and low efficiency. Additionally, soft robots are more cost-effective. Chen et al. [247] have developed a fish-shaped robot that achieves directional propulsion through the bistable mechanism of a curved beam. This robot is powered by shape memory material, which undergoes deformation upon temperature stimulation in water, causing the curved beam structure to snap and enabling directional swimming. Yin et al. [78, 246] have applied the bistable mechanism to design soft robots with enhanced mobility efficiency, such as the butterfly stroke-like soft swimmer (see Fig. 13(a)) and spine-inspired soft robots (see Fig. 13(b)). The snap-through behavior inherent in negative stiffness (bistable) structures allows for small-scale driving to trigger a large-scale response, resulting in improved response efficiency [78, 248–250].

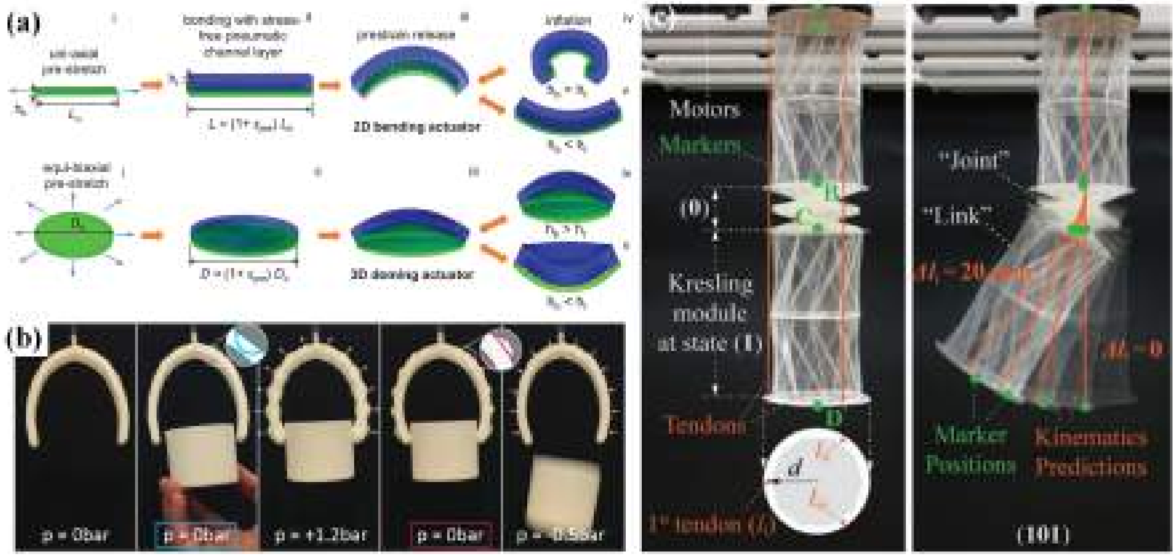

Figure 12: (a) 2D beam-like bending actuators and 3D doming actuators [241] (Copyright 2020, Jhon Wiley and Sons), (b) pneumatic driven soft gripper based on multistable patterned dome-shaped sheets [242] (CC BY 4.0), (c) robotic arm via assembling bistable origami modules [243] (Copyright 2014, Mary Ann Liebert Inc.) .

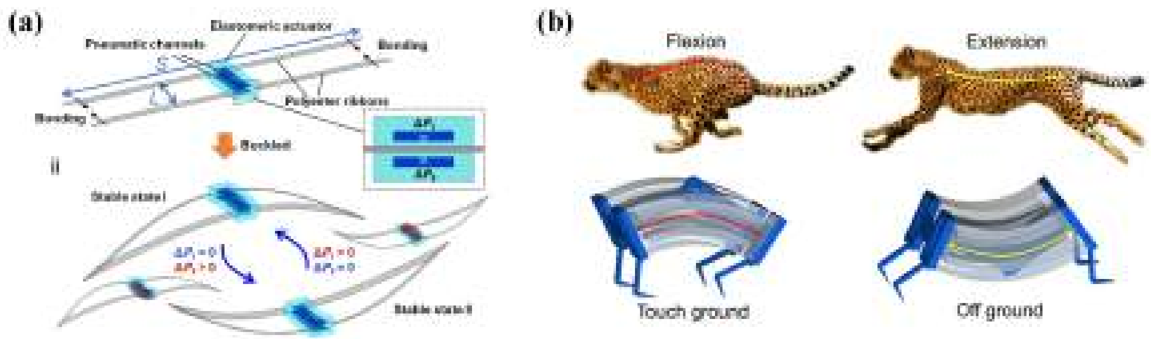

Figure 13: (a) Butterfly stroke–like soft swimmer [246] (CC BY 4.0), (b) spine-inspired soft robots [78] (CC BY 4.0).

3.3. Deployable structures

Deployable structures hold significant promise across various industrial sectors due to their ability to undergo predictable transformations and assume multiple predetermined configurations while following specified paths [251]. In recent years, multistable mechanical metamaterials have gained prominence in the design of deployable structures. These materials offer the advantage of maintaining structural stability in various states, including the initial state, under loading, and in a deformed state. They also enable high stretch ratios and volume changes before and after achieving a steady-state transition [111].

Friedman et al. [252] proposed a periodic cylindrical truss deployable structure, as shown in Fig. 14(a). The structure consists of a relatively soft elastic truss and a relatively hard transverse fixed frame, exhibiting a snap-back phenomenon during loading. Haghpanah et al. [111] and Chen et al. [253] independently designed a series of reconfigurable metamaterials using beam structures, which can achieve large volume and shape changes before and after deformation. Inspired by origami art, Melancon et al. [171] designed rigid-walled deployable structures that are multistable and inflatable. These structures can be deployed through a single fluidic pressure input and can form metre-scale arches and emergency shelters. Bobbert et al. [254] used bi-stable elements to design deployable meta-implants, which are compact in retracted state, allowing them to be brought to the surgical site with minimum invasiveness, and deployed to take their full-size load-bearing shape after in place (Fig. 14(b)).

Compared to the traditional deployable structures, many novel drive methods have been taken to transform the multistable deployable structures. For example, Zareei et al. [255] realized structure with a bistable linkage as a robust mechanism quickly deployed via transition waves. Che et al. [256] demonstrated that the pseudo-bistability of the printed viscoelastic metastructures can be tuned by adjusting the temperature, and the multistbale structure can be deployed by triggering the pseudo-bistability transformation. These easily driveable deployable structures will greatly facilitate the application of NSMMs.

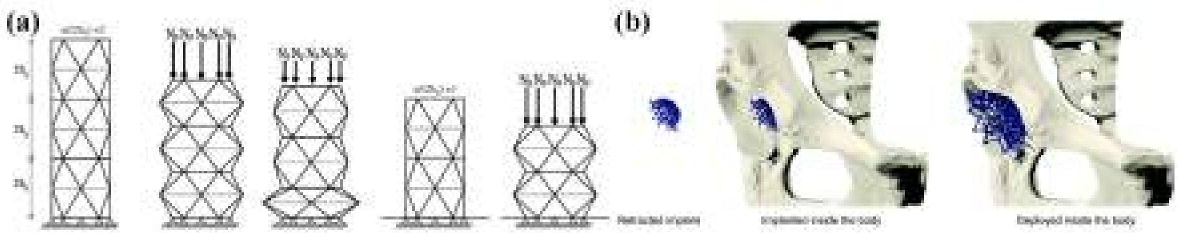

Figure 14: (a) Deployable truss structure [252] (Copyright 2013, Elsevier), (b) deployable meta-implants [254] (CC BY 3.0).

3.4. Morphing structures

The definitions of deployable and morphing structures can be somewhat nebulous, and their characteristics often exhibit significant overlap. To distinguish between the two structural forms, authors have summarized their differences as follows: Deployable structures, sometimes referred to as foldable structures, primarily emphasize the rate of volume change or stretch ratio before and after structural deformation. These structures typically exhibit periodic patterns. In contrast, morphing structures are more commonly employed to achieve self-functional changes through structural deformation [257].

Morphing structures, harnessing the advantages of multi-stable behavior, have found extensive applications in our daily lives [259], exemplified by commonplace items

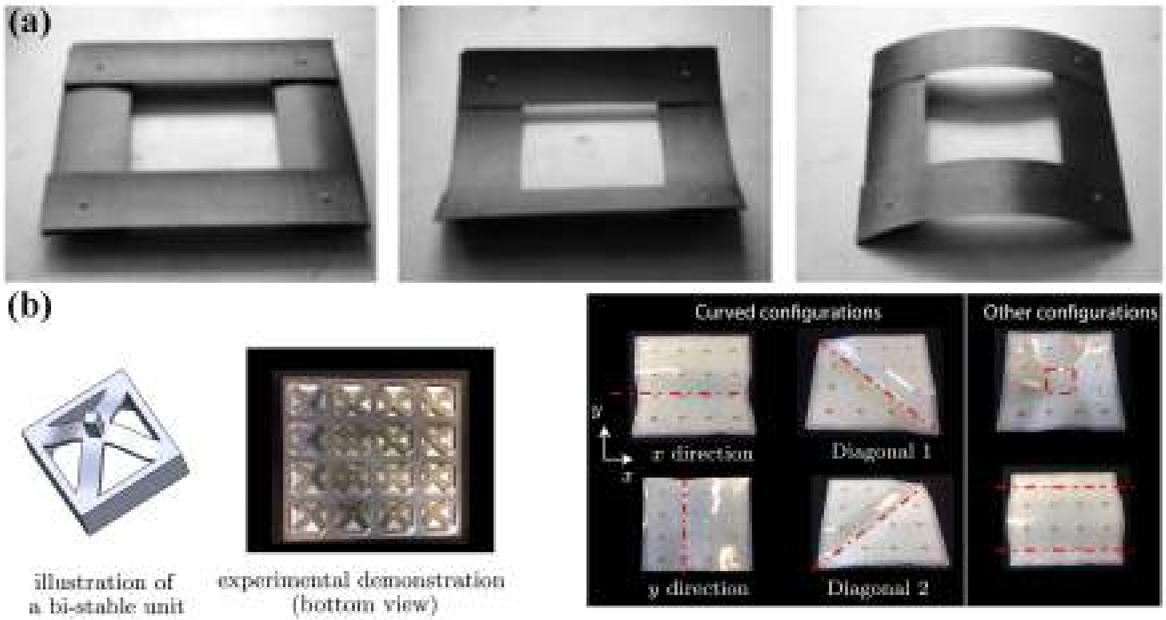

Figure 15: (a) Morphing structure with three stable states: plane, concave, and convex [204] (Copyright 2012, Elsevier), (b) morphing meta-surfaces [258] (CC BY 4.0).

like hair clips and glasses cases. These everyday objects incorporate clever designs that leverage multi-stable behavior to achieve structural deformation. Moreover, research and applications involving multi-stable structures extend to various domains [260, 261]. Daynes et al. [262] devised a flap device boasting two equilibrium states, capable of deflecting downward by 10 degrees and maintaining robust load-bearing capacity in this configuration without spontaneous rebound. Schults et al. [263] introduced a bistable convex rectangular wing structure, capable of undergoing steady-state transformation under torsional forces. Furthermore, multi-stable structures have been explored in the realm of scalable wings [79]. Dai et al. [204] designed a morphing structure with three stable states—plane, concave, and convex—by combining four identical bistable plates, as illustrated in Fig. 15(a). Another significant category of multi-stable morphing structures includes reconfigurable meta-surfaces [80, 257, 258] (see Fig. 15(b)), which hold great importance in aeronautical systems and building applications."

The morphing structures discussed earlier rely on external stimuli to initiate state transitions but can maintain their stable states without the need for ongoing external forces once deformation occurs. In contrast, there exists another category of morphing structures that lack inherent multi-stable behavior but can change and sustain their stable states through external stimuli. These structures primarily utilize shape memory materials, fluid-flexible composite materials, mechanical driving mechanisms, and other approaches to achieve deformation [264]. Compared to these alternative structures, multi-stable morphing structures offer distinct advantages, such as simpler designs and lower power consumption. This makes them particularly advantageous in various applications.

3.5. Vibration control

The application of NSMMs in the field of vibration control is mainly divided into three categories: one category utilizes the nonlinear mechanical response of NSMMs to improve the damping performance; another category utilizes the multi-stable property of NSMMs to adjust the bandgap of the structure and achieve the effect of vibration isolation [265]; the third category is to introduce positive stiffness structures into NSMMs to construct quasi-zero stiffness metamaterials [266–268].

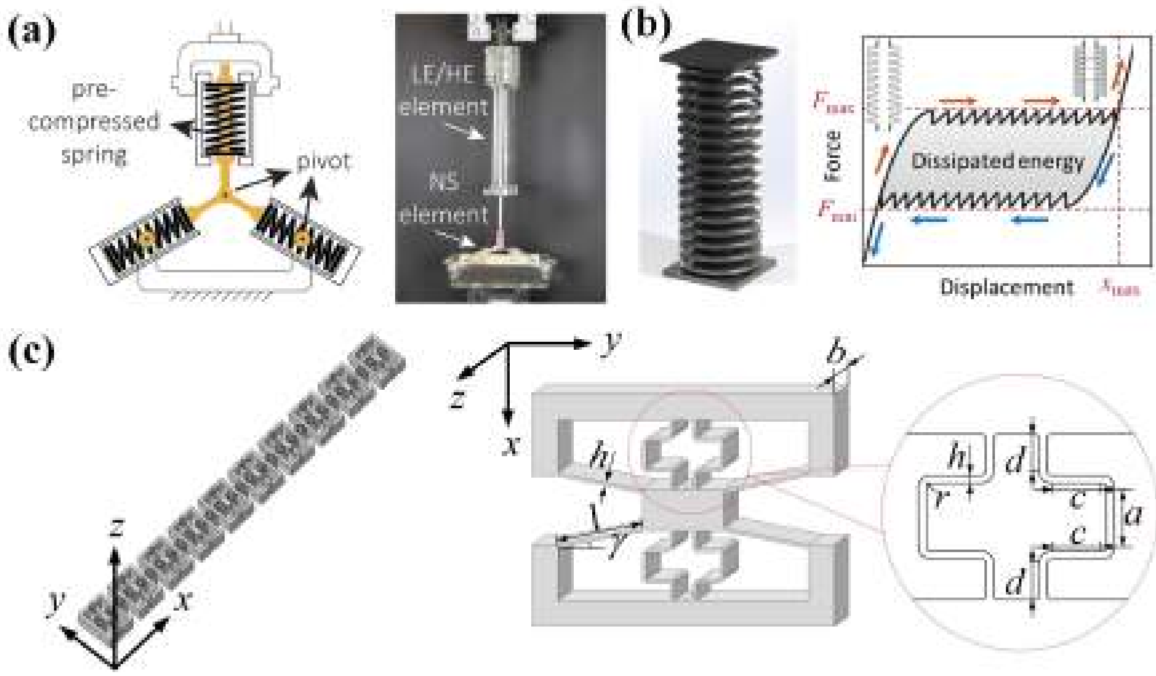

Figure 16: (a) Improving structure's damping through negative stiffness elements [77] (Copyright 2017, Elsevier); (b) vibration isolator based on negative stiffness mechanical metamaterials [148] (Copyright 2020, Elsevier); (c) quasi-zero stiffness mechanical metamaterial [268] (Copyright 2020, Elsevier).

Haghpanah et al. [77] introduced an innovative damper design that involves connecting positive stiffness springs in series with a negative stiffness structure, as depicted in Fig. 16(a). This design exploits the snap-through behavior of the negative stiffness element induced by the series spring to achieve damping performance that surpasses the limits of conventional structural materials. The fundamental principle of inducing snap-through behavior in the negative stiffness element through a series spring can be found in the referenced literature [269]. Leveraging this principle, Sefi et al. [148] devised a novel type of isolator, as illustrated in Fig. 16(b). The study demonstrates that, regardless of variations in vibration amplitude or frequency, the force transmitted from the environment through the isolator remains filtered below the designated threshold.

The multi-stable metamaterials can be used to achieve vibration isolation by controlling the bandgap. Bertoldi et al. [126] studied the influence of deformation and steady-state transition of perforated plate structures under axial compression on the bandgap. The research results show that the bandgap of the structure changes slowly before the plate structure reaches the buckling critical value. Once the structure becomes unstable, the bandgap will undergo a sudden change. The researchers believe that this completely reversible steady-state transition process can be applied to the design of acoustic switches. Shan et al. [133] improved the tunable properties of material dynamic response by controlling the loading direction to trigger different mode transition paths of elastic perforated plate structures consisting of triangular and circular hole arrays. Chronopoulos et al. [270] added negative stiffness units to periodic honeycomb materials and studied the sound insulation performance of this integrated material by theoretical methods. The research shows that this combined design has excellent sound insulation performance over a wide frequency range. The essence of this research is also to use the local resonance of negative stiffness units to achieve the change of the bandgap. Similar work with the above research ideas can be seen in references [271, 272].

In addition to the above perforated plate metamaterials, a small amount of research has utilized NSMMs with beam elements to realize wave control. For example, Meaud et al. [273, 274] designed an elastic wave propagation control system that can be used as an acoustic switch, using a multi-stable structure composed of curved beams. Goldsbeery et al. [275] designed an acoustic metamaterial using curved beams, with unique properties of certain five-mode metamaterials. There are also cases where vibration reduction has been achieved through phase transition characteristics of other structural forms. For instance, Babaee et al. [139] designed a tunable bandgap metamaterial using an elastic spiral structure.

Zhou et al. [268] have conducted extensive research in the field of quasi-zero stiffness metamaterials, such as proposing a quasi-zero stiffness metamaterial consisting of folding beams and buckling beams, as shown in the Fig. 16(c). The material can open an elastic wave band gap in the low-frequency region while ensuring load-bearing performance. Considering the great demand for low-frequency vibration reduction and isolation, the quasi-zero stiffness isolation mechanism based on NSMMs has great application potential.

3.6. Other applications

Apart from the applications mentioned above, NSMMs also have a wide range of potential applications in other fields. For example, NSMMs have good application prospects in areas such as packaging processes [276], medical implants [83, 163, 186], customized mechanical responses [150, 277–285], vibration energy harvesting [286], logic gates [81, 183, 187, 287, 288], non-reciprocal materials [289, 290], mechanical storage devices [159, 291] (Fig. 17), and etc.

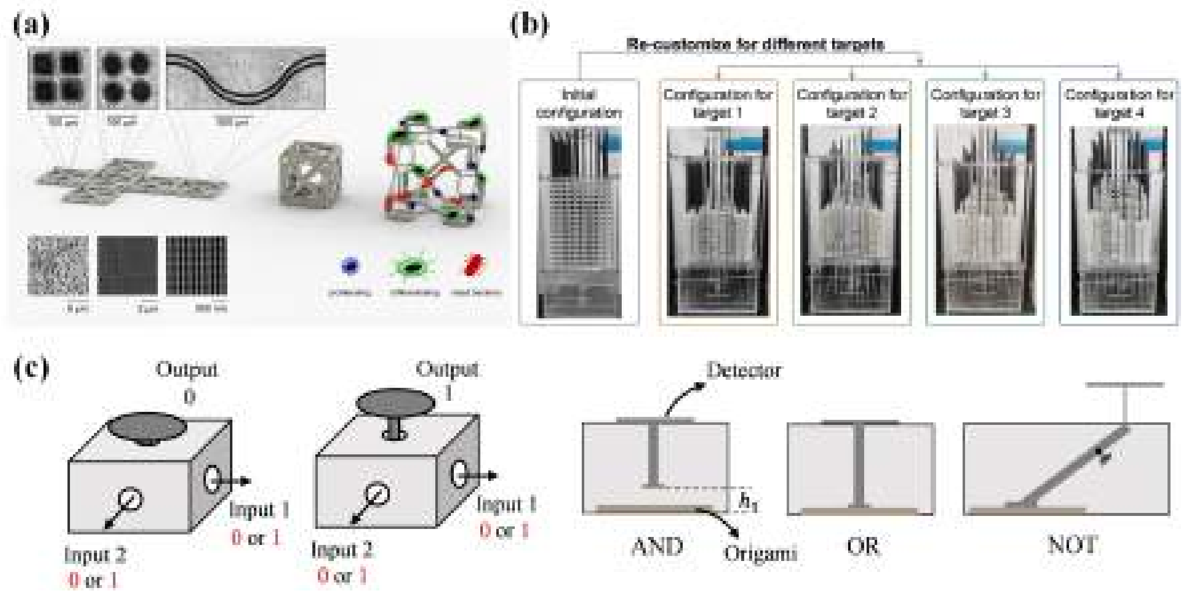

Figure 17: Application of NSMMs in (a) medical implants [83] (CC BY 4.0), (b) customized mechanical responses [277] (Copyright 2021, Jhon Wiley and Sons), (c) logic gates [183] (Copyright 2021, Elsevier).

4. Conclusion

This paper offers an in-depth review and introduction to the evolving landscape of NSMMs and their current applications. Recent strides in additive manufacturing technology have propelled NSMMs to the forefront, unveiling vast potential across various domains, including energy absorption, vibration and noise mitigation, deployable structures, and medical implants. While NSMMs have shown remarkable promise, they remain in a developmental phase, with considerable ground to cover before reaching widespread commercial maturity. In the realm of energy absorption, NSMMs face the challenge of bridging the performance gap compared to other materials. Addressing this hurdle can be achieved through topological optimization and the incorporation of high-performance base materials. In the field of vibration and noise reduction, NSMMs must harness the capabilities of smart materials to deliver superior performance across broader frequency ranges. Deployable structures and medical implants emerge as promising areas for immediate NSMM applications, but further optimization is imperative to align performance with practical use cases. The future holds the potential for NSMMs to revolutionize various industries, but it requires ongoing research, development, and strategic application to fully unlock their capabilities.

5. Acknowledgement

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