Single Machine Scheduling with Solar Panels and Batteries

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1 Introduction

It is a fact that the information and communication technology (ICT) domain has an impact on the environmental crisis, that is not negligible at all. The amount of energy consumed by data centers or HPC centers continues to grow year after year [4]. The amount of electricity required by data centers worldwide could account about 3 % by 2030 [3]. To decrease their carbon impact, a credible option is to totally disconnect data centers from the traditional power grid as shown in [1, 2]. One consequence of this design is that ICT resources are subject to energy intermittency. In this paper, we show, with this first study focusing on a single-machine platform only powered by renewable energy, how simply adding a storage device such as one battery, even small, can facilitate task scheduling by providing the supplementary energy needed to complete all tasks, for example.

1.1 Framework

We consider a set $\mathcal{T}$ of n tasks that should be scheduled on a single processor. Each task $j \in \mathcal{T}$ is characterized by a processing time p_j. The preemption of tasks is not allowed.

In order to run the single processor, a constant power P_r is required. The platform is supplied by renewable energy, and thus the power is intermittent by nature. In order to formalize this, we consider the horizon composed by two alternating periods: *daylight period* during which solar energy is produced at a constant rate P_p and *night period* during which no energy is produced. In this work, we consider that $P_r = P_p = P$. Let D_k and N_k be the duration of the daylight and the night periods, respectively. We call *phase* k the couple of the k-th night period and the k-th daylight period, in this order, i.e., the horizon starts by a night period.

A battery of capacity B (in Wh) is available in the platform. We assume that $B \geq P \cdot \max_j\{p_j\}$. During a daylight period, we can use the produced solar energy either to execute a task on the processor using a constant rate of power P or to charge the battery at a rate $\eta_c \cdot P$, where $\eta_c \leq 1$. In other words, there is a loss of a $(1 - \eta_c)$ fraction of energy while charging the battery. During a night period, we can use the battery's charge accumulated in the previous daylight period(s) to execute a task. However, using (discharging) the battery incurs again energy loss. In order to execute a task using the required power P, we need to consume energy at a rate P/η_d, where $\eta_d \leq 1$. In this work, we consider that $\eta_c = \eta_d = \eta$. Let $\bar{b}_k$ be the energy charge level (in Wh) of the battery at the end of the k-th daylight period (or equivalently at the end of the k-th phase). By convention, let $\bar{b}_0 \in [0, B]$ be the energy charge level of the battery at the beginning of the horizon. Assuming that E_k^c is the energy charged on the battery during the daylight period k and E_k^d is the energy discharged during the night period k, the following relation describes the evolution of the battery charge over the phases:

$$\bar{b}_k = \bar{b}_{k-1} + \eta_c \cdot E_k^c - \frac{E_k^d}{\eta_d}.$$

Our objective is to minimize the number of used periods. In a sense, this corresponds to the makespan of the schedule.

2 Results

We are interested in two versions of this problem: with and without loss of energy.

In the more general case where the use of the battery is subject to loss of energy, we have the following theorem whose proof is based on a reduction from the Partition problem. In the instance constructed in our proof, when a solution to the Partition problem exists, an optimal schedule for our problem executes all jobs in two daylight periods, without using the battery. Then, it is NP-complete to decide if we need to use a third day or to use the battery.

Theorem 1 *When energy losses are present, the problem is $\frac{3}{2}$-inapproximable.*

Inspired by the Bin-Packing problem and using the well-known First-Fit-Decreasing (FFD) algorithm, we obtain the following theorem. We emphasize that the proposed algorithm does not exploit the battery, nevertheless this is sufficient to settle the problem in this settings.

Theorem 2 *When energy losses are present, FFD achieves a $\frac{3}{2}$-approximation ratio.*

In the case where the battery operates without energy losses, the battery capacity plays a crucial role in the complexity of the problem. More precisely, if no battery is available in the system ($B = 0$), the proof of Theorem 1 can be directly adapted to obtain the same inapproximability result. On the other hand, if the battery capacity is large enough, we propose the greedy algorithm ASAP, which considers the tasks in an arbitrary order and schedules each task as soon as possible, regardless the origin of the used energy (solar energy produced during daylight or energy available in the battery).

Theorem 3 *If $B \geq P \cdot \max_j\{p_j\}$, ASAP is optimal when battery usage incurs no energy loss.*

Finally, for intermediate values of battery capacity, we introduce a polynomial time algorithm, called G-FFD, which combines reductions and properties from $P||C_{max}$ and Bin-Packing, and the following theorem holds.

Theorem 4 *Without loss of energy, G-FFD achieves a $\frac{13}{9}$-approximation ratio, comparing to an optimal solution which cannot use the battery. At the cost of additional computing phases, this ratio can be arbitrarily close to $\frac{11}{9}$, if the battery capacity is considered to be a constant.*

References

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