MILP Formulation for the Static Carpooling Problem with Flexible Roles and Detours

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Abstract. We study a novel carpooling problem modelled as an extended Dial-a-Ride Problem (DARP) incorporating flexible roles, time windows, detours, and heterogeneous fleet of vehicles. Unlike traditional services, where users are either drivers or passengers, this variant allows users to be registered both as driver and passenger, and lets the optimization process determine which role will be assigned to the user. We propose an initial Mixed-Integer Linear Programming (MILP) formulation for this problem. The goal is not only to match travellers and define their itineraries, but also to optimize the matching in terms of user satisfaction and ecological impact. We discuss the hierarchy between the different objectives and introduce a method to incorporate comfort into the optimization process.

Our numerical experiments demonstrate that both the number of requests and the number of stops significantly impact the problem's complexity. The proposed algorithm remains efficient when the combined number of requests and stops is approximately sixteen or fewer. Finally, we validate the use of the bonus value method to prioritize the allocation of preferred roles.

Keywords: Extended Dial-A-Ride Problem, Mixed Integer Linear Program, Flexible Roles with Preferences, Carpooling Exact Solving.

1 Introduction

The use of individual cars for daily journeys in France shows significant disparity between urban centers like Paris and rural areas, which reflects differences in infrastructure, lifestyle and public transport availability. Indeed, while in Paris, only 4.3% of trips are made by car [1], the car usage reaches 93% of trips in rural areas [2]. At the same time, the average occupancy rate of a car is 1.3 people per car. This rate is slightly the same in urban and rural areas. In this context, carpooling services play a key role in achieving decarbonization of the transport sector which is the largest contributor to greenhouse gas emissions [3]. Academic research on carpooling optimization mainly focuses on ride matching methods, carpooling efficiency optimization, or demand prediction approaches. However, only few optimization studies allow detours for drivers [4][5]. In addition many of these works focus on the cost sharing policies or leave the process of selecting the detours to the drivers [6]. Furthermore, no work from the literature introduced the flexible roles with preference. Which means that a user can accept to be both driver or rider, with some preference for one mode or another. In this paper, we address the optimization of a new static flexible carpooling model focused on the satisfaction of the users, where the drivers and passengers agree to make detours as long as it does not violates the time constraints or the vehicle capacity constraints. In this model, the user declares in their request whether they are a driver, a passenger, or if they accept both roles, with a preference. The proposed problem is modeled by a Mixed-Integer Linear Programming (MILP) system which allows the exact solving of the instances. The remaining paper is organized as follows. In the next section, we review the main methods used in the literature to address the Dial-A-Ride Problem (DARP). In Section 3, we focus on our studied extended DARP called Flexible DARP (F-DARP) incorporating flexible roles and detours. Then we give a formal modeling of the problem as Mixed-Integer Linear Programming system. In Section 4, we present the main results of the experiments on different scenarios with different sizes. Conclusions and perspectives are given in the last section.

2 Related Works

Carpooling is a relatively old practice. During World War II, it emerged as a wartime necessity due to fuel rationing in Europe and U.S.A, organized among workers or neighbors. The carpooling interest was renewed during the oil crisis in 1973 but it was mostly informal or employer-organized. First online carpooling matching services appeared in conjunction with the early digital platforms (1990s-2000s) and take the form of rudimentary web-based bulletin boards or email systems. The rise of smartphones accelerated the use of app-based carpooling with the emergence of major players like BlaBlaCar, Karos and SPLT. All these Off-the-shelf solutions work as a matching platform that centralizes the carpooling demands and offers and manages the agreements between drivers and riders. A carpooling formulation can be derived from the DARP, as both involve shared rides with multiple passengers traveling between different pickup and drop-off locations. The DARP was first formulated in 1978 [7]. It was originally used to describe professional door-to-door transport systems, such as elderly transportation services , or bus services [8]. The main objective is typically to minimize transportation costs while satisfying as many requests as possible. In this paper, we focus on the static version of the problem [9]. Various types of carpooling are discussed in the literature. The differences typically lie in the objective of the system, the roles of the users, or the definition of departure and arrival points. In [10], the objective is to minimize the number of cars on the road in order to avoid congestion, reduce traffic jams, and lower CO2 emissions. In their formulation, the roles (driver or passenger) are fixed beforehand, and all the users have different pickup and drop-off locations. In contrast, [11] focuses on minimizing costs, including delays, vehicle usage, and travel distances. In their approach, all users can act as both drivers and passengers. They are colleagues who share one common point: the workplace. Drivers are chosen to take turns alternately. [12] aims to maximize both the number of satisfied requests and the overall social satisfaction of the matches. In this case, travelers also share a common destination (a university) and the roles (driver or passenger) are once again set beforehand. Finally, [13] presents a multi-criteria formulation that considers, among other factors, travel cost, delays, safety, social compatibility, comfort, and waiting times. The model includes free departure or arrival locations, and driver and passenger roles are established in advance. In this paper, we aim to address an F-DARP model which allows detours, considers time windows, and supports flexible user roles. Specifically, users can request to be a driver, a passenger, or accept both roles with a preference for one. Additionally, users do not necessarily share common departure and arrival points. Our strategy is to consider the user's point of view, and to optimize their experience, in order to build loyalty and ultimately reduce the ecological cost of transportation. Our objectives take into account both user satisfaction and ecological criteria. Up to now, no MILP formulation has been proposed to model the problem addressed in this paper.

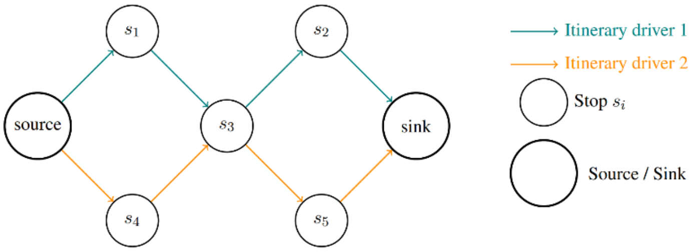

Fig. 1. Single-commodity flow network for the carpooling problem.

3 Problem Formulation

The F-DARP problem is modeled as a graph (see Fig. 1) by a set of stops s ∈ S spread across the studied area. A stop refers to a designated location within the geographical area where passengers can board or exit a vehicle. The travel time between stops s_1 and s_2 ∈ S is denoted $\delta_{s_1 s_2}$. A user u ∈ U, submits their car journey request from an origin to a destination, sometimes passing through other stops along the way, also called "vias". The set of stops requested by user u ∈ U is S_u. We denote S_O^u the origin point, and S_A^u the arrival point of the user u. A single request can concern several people . For each stop s ∈ S_u, the user u ∈ U indicates the number of people to be picked up p_s^u, and dropped off d_s^u. A user specifies one role among four options: driver only (od), passenger only (op), mainly passenger (mp), or mainly driver (md). We denote by od_u, op, md_u, $mp_u \in \{0,1\}$ the role of user u. Each potential driver u indicates the capacity of their vehicle, C_u. They also indicate the maximum number of detours lv_u they accept. Users also provide a time window $[h_{su}^{-}\ ,\ h_{su}^{+}\ ]$ during which they wish to arrive at each stop. We assume that returning to the same stop s ∈ S within a user's trip is not allowed. Two fictitious stops are introduced: the source $\overline{s}$ and the sink $\underline{s}$. All carpools will fictitiously start from the source and end at the sink. We call *satisfied* a request for which a complete response is given, respecting the user's constraints (time windows, capacity, role ...). The user's role is *satisfied* if his preferred role is assigned in the solution. The F-DARP problem can be seen as a problem of matching users for carpooling. For each carpool group, we must define a detailed itinerary indicating who is picked up, where, and when. Additionally, each participant must be assigned a role either as a driver or a passenger. Finally, a departure time must be scheduled for each carpool. These decisions must comply with the specific constraints and preferences of each user

3.1 MILP Formulation

F-DARP is modeled as a MILP. The proposed MILP is based on the flow-based formulation of the DARP [14, 15], where each stop corresponds to a node, and the path between two stops corresponds to an edge. To eliminate subtours, we use the [16] method.

3.2 Variables

The matching between users is described by the binary variable $x_{uu'}$ which is equal to 1 if user u' travels in the car driven by user u, and 0 otherwise. If user u is a driver, then $x_{uu} = 1$. Γ_{su}^{+} and Γ_{su}^{-} are continuous variables representing the departure and arrival time to stop s of driver u. If user u is not assigned as driver, then $\Gamma_{su}^{+} = 0$ and $\Gamma_{su}^{-} = 0$ for all s ∈ S. $F_{ss'u}$ is a positive integer which denotes the flow between s ∈ S ∪ $\{\overline{s}\}$ and s' ∈ S\\$\{s\}$∪$\{\underline{s}\}$, associated to carpool u. The binary variable $y_{ss'u}$ defines the itinerary of the driver u. More clearly, $y_{ss'u}$ = 1 if the driver u travels directly from stop s ∈ S ∪ $\{\overline{s}\}$ to stop s' ∈ S\\{s}∪$\{\underline{s}\}$. If user u does not drive, then $y_{\overline{s}\underline{s}u} = 1$. The integer variable Q_{su} indicates the number of occupied seats in the car driven by u when departing from stop s ∈ S ∪ $\{\overline{s}\}$. By definition, $Q_{su} = 0$ for all u ∈ U. Finally, B_u represents the bonus of the role assigned to driver u ∈ U. If the driver's request is not satisfied, then $B_u = 0$. If the preferred role is assigned, then $B_u = 0.002$; if the request is satisfied but the preferred role is not granted, $B_u = 0.001$.

3.3 Objective Function

This section outlines the objectives considered in the optimization model and how they are aggregated into one single objective function. The primary objective relates to the user's satisfaction and it is measured using two components. The first component computes the number of satisfied requests. The user role preferences are incorporated within the second component of the satisfaction objective using the bonus values B_u. The second objective is related to the ecological benefits obtained by minimizing the number of used vehicles. Consequently, the objective function is formulated as follows (1):

$$\max 9 \sum_{u \in U} \sum_{u' \in U} x_{uu'} + \sum_{u \in U} B_u - \sum_{u \in U} x_{uu} \quad (1)$$

Satisfying more user requests is considered more important than achieving a better ecological outcome. That is why request satisfaction is weighted by 9 (maximum personal vehicle capacity). Second, minimizing the number of vehicles takes precedence over assigning users preferred roles ($B_u < 1$).

3.4 Constraints

The constraints can be sorted into six groups from A to F. These constraints use two sets of values M_1, and M_{2u} with : M_1= maximum time, and $M_{2u} = |S_u| + l_{vu}$, ∀u ∈ U. Constraints in Group A establish the matching between users. They constrain who can travel together and the user's final role. One request cannot be assigned to more than one driver, and is not necessarily assigned. A driver is their own passenger. The requested roles must be respected. At each stop, the vehicle's occupancy must not exceed its capacity. Constraints of Group B define the itineraries. An itinerary is created for each user, fictitiously departing from the source and arriving to the sink. If a user is not designated as a driver, their route corresponds to a fictitious empty itinerary. The stops must be visited in the correct order. The first and the last stop of each itinerary must match those of the driver. A car must arrive at a stop as often as it departs. Each stop leads to exactly one next stop. These constraints also enhance user satisfaction by preventing repeated visits to the same stop. Constraints in group C use [16]'s method to eliminate subtours by sending flow from source to sink, each node consuming one unit. This limits detours and mitigates the "taxi effect". The initial amount of flow for each carpool is set using l_{vu}. The amount of flow arriving into a node equals flow out +1. A route that is not included in an itinerary cannot carry a positive flow. Constraints group D set the departure and arrival times at each stop for each carpool. Arrival must precede departure at each stop. Passenger's time windows must be met for each stop. Travel times between stops are considered. The constraints in group E assign a bonus value to each user. This bonus is set to 0, 0.001, or 0.002, depending on the role assigned to the user and whether the request is satisfied or not (2-5). Group F constraints establish the domains of definition of the variables.

$$B_u = 0.002 \sum_{u' \in U} x_{u'u} \qquad \forall u \in U \; s.t. \; op_u = 1 \; (2)$$

$$B_u = 0.002 \, x_{u'u} \qquad \forall u \in U \; s.t. \; od_u = 1 \; (3)$$

$$B_u = 0.001(1 - x_{uu}) + 0.002 x_{uu} \qquad \forall u \in U \; s.t. \; md_u = 1 \; (4)$$

$$B_u = 0.001 x_{uu} + 0.002(1 - x_{uu}) \qquad \forall \, u \in U \; s.t. \; mp_u = 1 \; (5)$$

4 Numerical Experiments

We present now the numerical experiments to assess the effectiveness of the proposed MILP model. We first describe the used data-set, and then we expose the conducted experimentation in order to analyze both the strengths and limitations of the MILP approach.

Table 1. Mean of the runtime (s) for FrBvCM according to NbS and NbR in the scenarios

		NbR								
		2	3	4	5	6	7	8	9	10
NbS	6	0.17	0.24	0.38	0.67	1.37	2.89	5.95	13.92	49.72
	7	0.34	0.52	1.15	4.12	11.65	38.99	116.94	161.84	-
	8	0.40	1.07	5.48	27.66	196.60	298.86	483.54	601.70	-
	9	0.45	3.19	61.48	207.33	470.63	606.90	-	-	-
	10	0.93	49.26	250.76	475.72	502.36	-	-	-	-
	11	4.68	168.84	391.03	-	-	-	-	-	-

4.1 Instances Description and Experimental Setting

We generated a dataset of 2840 scenarios, each containing two to ten requests. User roles follow these rules: at most 30% “op”; at least 50% “od” or “md”; and the remaining 20% assigned “mp”, “md”, or “od”. Each scenario includes six to eleven possible stops within Montbéliard (France). Travel-time matrices are obtained from Google Maps, and departure time windows span 15 minutes. For each parameter combination (number of requests, role distribution, number of stops), four instances are generated. MILP scenarios are solved with CPLEX on an Intel Core i5-1035G1 (64-bit, 1.19 GHz), with a 600-s time limit.

4.2 Results

This section evaluates the MILP formulation from several perspectives: scalability and the impact of role preferences on problem complexity and solution quality. Both DARP and F-DARP are NP-hard; consequently, model complexity grows exponentially with the number of stops (NbS) and requests (NbR). Scenarios with NbS + NbR $\leq$ 16 are generally solved within 600 s (Table 1), and complexity increases faster with NbS than with NbR. We examine role-preference effects on solution quality and computation time by comparing the MILP model with a simplified version using only three roles (“od”, “op”, “both”). The model in Section 3.1 is denoted Flexible Roles and Bonus-Value Carpooling Model (FrBvCM), and the simplified version FrCM. The same scenarios are used for both models, where the roles “mp” and “mp” are replaced by the “both” role for FrCM. The ratio of satisfied roles in a solution is the number of times the preferred roles is attributed over the number of requests. Over 2400 scenarios, FrBvCM and FrCM give the same solutions in 88% of time, and FrBvCM gives a solution with a better ratio of satisfied roles in 12% of time. Figure 2 breaks down the results and shows the evolution of execution time as a function of the NbR, with a separate line for each NbS. From the plot, we can observe that the trend for FrCM is consistently lower than FrBvCM, but the gap is narrow, suggesting that both methods perform similarly in terms of speed. In addition, FrCM and FRBvCM reduce total CO2 emissions by 4.12% and 5.46%, respectively, compared to a hypothetical scenario in which each user (driver or passenger) travels alone by car. Carbon emission reductions are expected to increase as the number of carpooling offers grows, due to greater opportunities for effective matches.

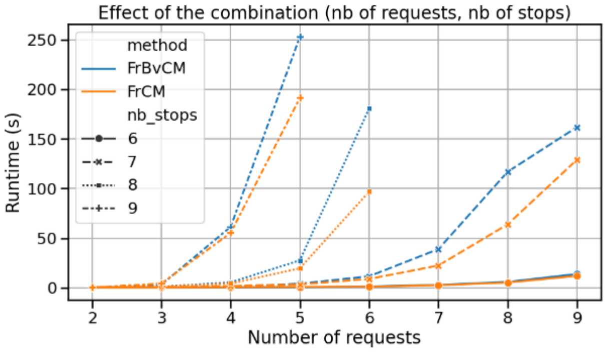

Fig. 2. Comparison between FrBvCM and FrCM regarding the effect of the NbS and NbR on the execution time.

Given the small execution-time difference, and since FrBvCM yields better user satisfaction and lower carbon emissions, it can be considered the better formulation.

5 Conclusion and Future Work

In this study, we investigate an extended carpooling problem with time windows, flexible detours, and flexible user roles. We propose a first MILP formulation for this problem using [16]'s technique to prevent subtours. The objective of the optimization is to maximize the users satisfaction (mobility request + role preference), while reducing the ecological cost. The performed tests show that the problem's complexity is directly related to the number of requests and the number of stops in the scenario. The exact resolution remains efficient when the combined number of requests and stops does not exceed sixteen. We also compare our model to a baseline model, observing that while our model improves user satisfaction and reduces carbon emissions, it has a slight higher execution time. Looking ahead, we aim to reduce complexity by removing implicit variables and introducing additional constraints to improve algorithmic convergence. In addition, we plan to introduce vehicles' power-train type and avoided carbon estimation.

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