

Design of a metallic half-wave plate based on birefringent metamaterial for applications in the visible range

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Abstract. The main objective of this study is to design an artificial birefringent metamaterial to operate within the visible spectrum. This plate consists of a C-Shaped Aperture Array (CSAA) engraved into a silver metallic film deposited on a glass substrate. The artificial anisotropy of this structure is generated by the simultaneous excitation of two guided modes with different effective indices. These excitations occur within the same C-Shaped aperture, at the same wavelength, for two distinct states of orthogonal polarization of the electric field. They propagate at different velocities, resulting in a phase difference $\Delta\phi$ between the transmitted electric fields. We show that by adjusting the thickness of the silver layer, it is possible to achieve a phase difference of π corresponding to a Half-Wave Plate (HWP). This design is highly innovative, being more compact and efficient, exhibiting enhanced transmission of $T = 75\%$ at a wavelength of $\lambda = 727\text{nm}$, with a thickness of $H = 146\text{nm}$. Such a configuration results in an extraordinary birefringence of $\Delta n = 2.5$.

Keywords: Birefringence, Metamaterials, Half-Wave Plate, FDTD, Visible range.

1. Introduction

The dependency of enhanced transmission [1–4] on the geometric arrangements of certain apertures has introduced the potential to create birefringent metallic plates [5–9], generating a phase difference between orthogonal polarized waves [10–13]. The interest in this type of structure lies in its potential applications in nano-optics and photonics. For example, these plates, which allow polarization rotation, were used in polarization-division multiplexing (PDM) to double the channel capacity of optical communication systems [14]. More recent applications involved their integration into silicon photonic circuits for on-chip polarization control and manipulation [15]. Recent studies, such as the one performed by Parveen *et al* in 2024 [16], have explored tunable metasurfaces for mid-infrared absorption using vanadium dioxide (VO_2)-based designs, which hold

potential for applications in sensing and thermal management. In the literature many birefringent plates have been designed, among which the most common are the Half-Wave Plate (HWP) and Quarter-Wave Plate (QWP). Plates based on naturally birefringent materials are generally characterized by low birefringence [17,18], requiring significant thicknesses to achieve the desired phase shift, which is inconsistent with nano-optical applications. To overcome this obstacle, research is moving towards the use of metamaterials offering the possibility of designing birefringent blades that require much less space than conventional blades. Artificial birefringence could be achieved, for example, by creating arrays of micro-apertures in metallic films, under specific conditions [10–13,19–21]. The designed plate exhibits low transmission ($< 20\%$) [22–27], based on the excitation of surface plasmons [24]. To address this issue, an alternative approach was suggested, focusing on the excitation of guided modes [13]. A HWP, equipped with two rectangular apertures was especially designed for a wavelength $\lambda = 300\mu\text{m}$ for applications in the THz domain with a transmission of 90% [13]. Similarly, a QWP, operating at $\lambda = 554\text{nm}$, was developed using an array of coaxial apertures with an elliptical core, exhibiting a transmission coefficient of 44% for a metal layer thickness of 182.5nm [20]. Half and Quarter-Wave Plates, featuring sub-wavelength L-Shaped hole patterns etched into a metal film, were also proposed. These THz devices were notable for their excellent transmission as well as their remarkable birefringence, which was evaluated to be approximately $\Delta n = 0.254$ for the QWP and $\Delta n = 0.768$ for the HWP for a metal layer thickness of $H = 400\mu\text{m}$ [28]. As outlined in [29], a significant proportion of studies addressing the conversion efficiencies of wave plates in both the THz and visible ranges, with a particular focus on the impact of nanostructure shape, material type and structure layers, have been synthesized. This comprehensive review reveals enhanced conversion efficiencies, attained in reflective mode through the use of multilayer metallic materials and in transmissive mode through the deployment of single semi-conducting layers characterized by increased thickness and reduced birefringence. In a previous work, our team employed a variety of rectangular apertures in the design of a HWP, for which one of the apertures was glass-filled. This enabled the development of an operational HWP at a wavelength of $\lambda = 737\text{nm}$. The distinguishing features of this plate are a transmission of 60% and a birefringence of $\Delta n = 2.1$ which are achieved with a metal layer thickness of 175nm [30]. In regard to the subject of this article, the present study proposes the use of a C-Shaped Aperture Array (CSAA). This device operates by exciting two modes with different effective indices through the orthogonal components of the electric field, E_X and E_Y . The propagation of these modes at distinct velocities will result in a phase difference between the components of the electric field at the output of the structure. By adjusting the geometric parameters of the structure appropriately, we demonstrate that this phase shift can be easily controlled to the desired value, which is a distinct configuration from our previous work, where rectangular apertures were employed in the design of the HWP. The use of the C-shaped aperture array provides a more versatile platform for achieving tunable birefringence and precise control over phase shifts, offering new insights into the design and functionality of metallic plates.

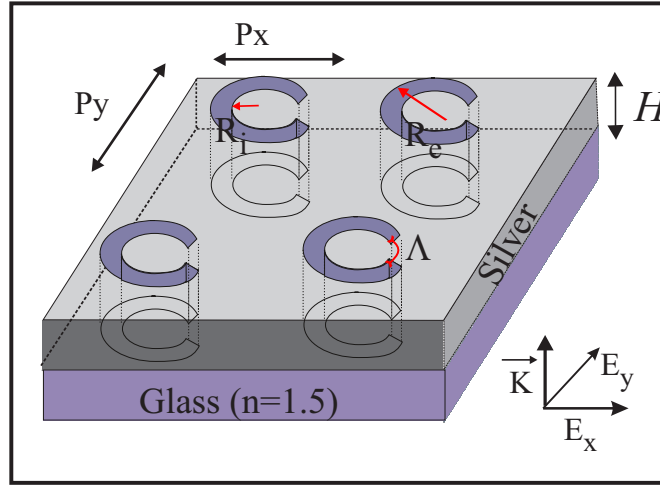


Figure 1. Design of a C-Shaped Aperture Array (CSAA) engraved into a silver film.

2. Metal plate design

The proposed and studied metal plate in this work consists of a two-dimensional **C-Shaped Aperture Array** (CSAA) [9], filled with glass ($n = 1.5$) and engraved into a metal film of thickness H , deposited on glass. These apertures have an inner radius R_i and an outer radius R_e as shown in figure 1. P_X and P_Y represent the periods along the x and y axes, respectively. The parameter Λ corresponds to the angle **sector** formed by the two ends of the C-shaped aperture.

In this study, simulations are carried out using a home-made numerical code based on the **3D** Finite-Difference Time-Domain (FDTD) **algorithm**. This code, employing cartesian coordinates, incorporates the dispersion of silver in the optical domain according to the Drude critical points model (DC2P) [31–33], with parameters that have been optimized based on experimental data from [34]. **The structure under consideration is illuminated by a plane wave at normal incidence on the substrate side (glass), and transmission is determined from the components of the electromagnetic field that are detected in the far field.**

3. Parametric study for HWP design

The objective of this study is to design a more compact HWP than those previously described in the literature. We have adopted the approach of filling the apertures with glass [30]. This approach enables us to shift the structure's **guided modes cut-off wavelengths** to higher **values** while reducing the geometrical parameters.

By adjusting the thickness H of the metal film, it is possible to achieve the desired artificial birefringence. Initially, we set the geometric parameters of the CSAA structure as follows: $R_i = 50\text{nm}$, $R_e = 100\text{nm}$, $\Lambda = 120^\circ$ and $P_X = P_Y = 250\text{nm}$ (see figure 1).

The structure has been designed to excite the $TE_{10}(l = 1)$ and $TE_{20}(l = 0)$ modes

Design of a metallic HWP based on birefringent metamaterial

4

within the visible spectrum. The $TE_{10}(l = 1)$, which has a near-zero effective index, is excited by an electric field oriented along the x -direction, while the $TE_{20}(l = 0)$ mode, which has a zero effective index, is excited by an electric field oriented along the y -direction. The transmission is calculated by varying the thickness H of the metal film for both cases of polarization E_X and E_Y .

In the overlapping region, where λ is within the range of 650 to 800 nm for both modes, the phase difference $\Delta\phi = \phi_{xx} - \phi_{yy}$ between the two transmitted components t_{xx} and t_{yy} is calculated. The results are shown in figure 2-a. The special values of the phase difference $\Delta\phi = \pi$ are indicated in black. The diagram has been overlaid with a white dashed contour indicating points where $|t_{xx}| - |t_{yy}| = 0.02$. Figure 2-a, shows the location of a point that fulfills the conditions of the HWP (point A: $H = 146\text{nm}$ and $\lambda = 727\text{nm}$).

The transmission coefficients in amplitude ($|t_{xx}|$, $|t_{yy}|$, $|t_{yx}|$ and $|t_{xy}|$) as well as the phases (ϕ_{xx} , ϕ_{yx} , ϕ_{yy} and ϕ_{xy}) spectra are depicted in figures 2-b, 2-c, 2-d and 2-e for $H = 146\text{nm}$, corresponding to the point A on the diagram of figure 2-a. The Jones matrix constructed from the previous spectra at $H = 146\text{nm}$ for $\lambda = 727\text{nm}$ is as follows :

$$\begin{aligned} t(H = 146 \text{ nm}, \lambda = 727 \text{ nm}) &= \begin{pmatrix} |t_{xx}|e^{i\phi_{xx}} & |t_{xy}|e^{i\phi_{xy}} \\ |t_{yx}|e^{i\phi_{yx}} & |t_{yy}|e^{i\phi_{yy}} \end{pmatrix} \\ &= \begin{pmatrix} 0.8791e^{i0.74\pi} & 0.031e^{i1.57\pi} \\ 0.026e^{i1.81\pi} & 0.8539e^{-i0.26\pi} \end{pmatrix} \end{aligned} \quad (1)$$

From this Jones transmission matrix, we can verify that the phase difference between the components ϕ_{xx} and ϕ_{yy} is equal to $\Delta\phi = 1.00\pi$. Furthermore, it has been determined that the transmission for the x - and y -polarization directions are almost identical, while the depolarization terms t_{xy} and t_{yx} are negligible. The useful transmission through the plate is then estimated using the expression $T = \frac{1}{2}(|t_{xx}|^2 + |t_{yy}|^2)$ [35] and is found to be 75%.

4. Optical properties of the HWP design

The properties of this designed HWP are studied when illuminated by an electromagnetic wave polarized at an angle $\theta = 45^\circ$ from the x -axis. We also performed calculations and plotted the phase difference $\Delta\phi$, as shown in figure 3-a. The obtained results corroborate the functionality of our HWP, as evidenced by the observation that $\Delta\phi = \pi$, even when illuminated by a wave polarized at an angle 45° with respect to the x -axis. The intensity transmitted through this plate is $T = 75\%$ (see figure 3-a). The inset in figure 3-a corresponds to the distributions of electric field intensities at wavelength of $\lambda = 727\text{nm}$ in the xz and yz planes. As can be observed, the energy distribution within the aperture exhibits a node along the xz plane, while it is uniform in

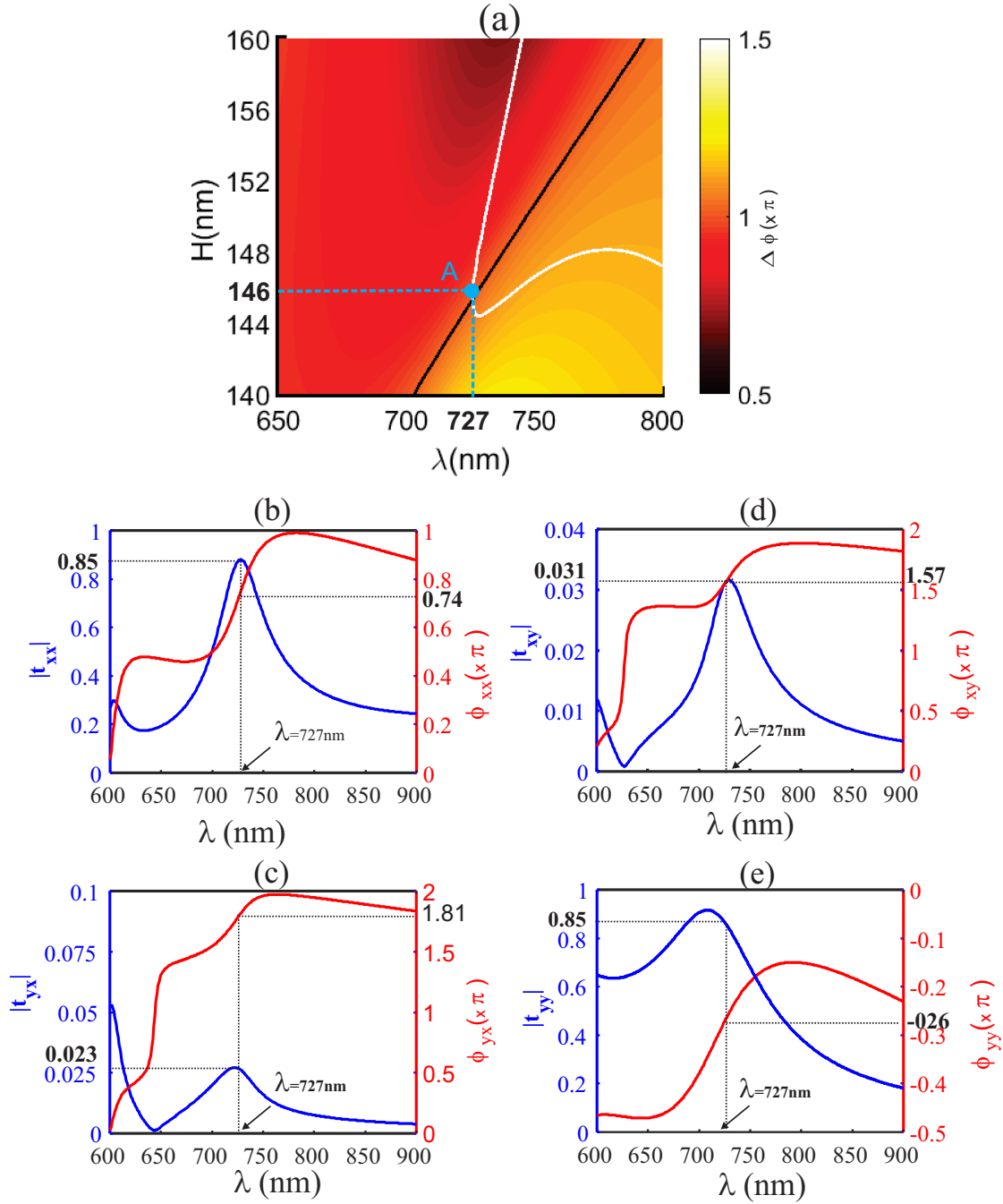


Figure 2. a) Diagram of t_{xx} , t_{yy} and phase difference $\Delta\phi$ as a function of H and λ for $P_X = P_Y = 250$ nm, $R_i = 50$ nm, $R_e = 100$ nm and $\Lambda = 120^\circ$. The white outline represents the pairs (λ, H) where $|t_{xx}| - |t_{yy}| \leq 0.02$ while the black outline corresponds to $\Delta\phi = \phi_{xx} - \phi_{yy}$. Point A corresponds to the HWP case. b) and c) Spectra of $|t_{xx}|$, ϕ_{xx} , $|t_{yx}|$ and ϕ_{yx} for polarization E_X with $H = 146$ nm. d) and e) Spectra of $|t_{yy}|$, ϕ_{yy} , $|t_{xy}|$ and ϕ_{xy} for polarization E_Y with $H = 146$ nm.

Design of a metallic HWP based on birefringent metamaterial

6

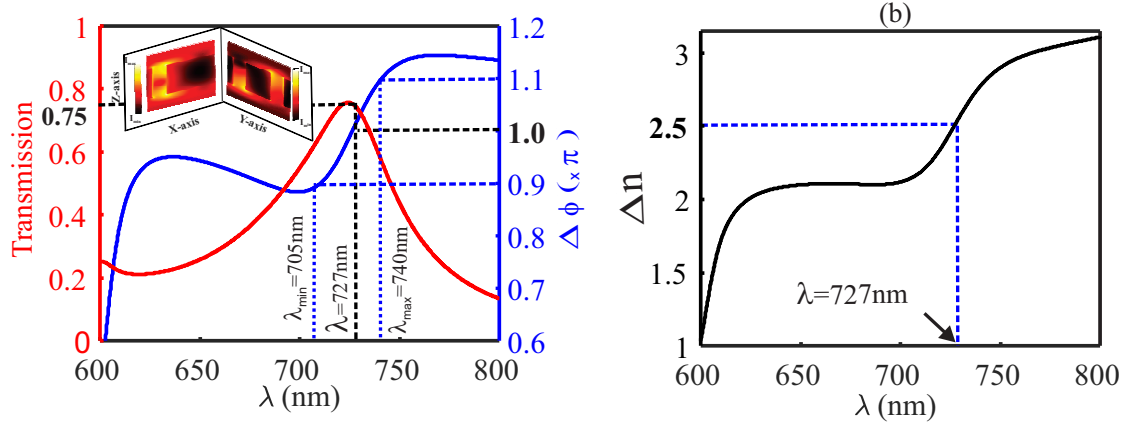


Figure 3. a) Transmission spectrum (in red) and phase difference $\Delta\phi$ (in red) plotted as a function of wavelength λ for the HWP illuminated by an incident wave that is polarized at an angle of 45° with respect to the x -axis. The inset presents a graphical representation of the electric field distributions in the xz and yz planes at wavelength of $\lambda = 727$ nm. b) Effective index curve for $TE_{10}(l=1)$ and $TE_{20}(l=0)$ modes. The geometric characteristics of the HWP are $P_X = P_Y = 250$ nm, $R_i = 50$ nm, $R_e = 100$ nm, $\Lambda = 120^\circ$ and $H = 146$ nm.

the yz plane. This confirms the simultaneous excitation of the two modes, $TE_{10}(l=1)$ and $TE_{20}(l=0)$, through the CSAA structure at wavelength of $\lambda = 727$ nm.

Assuming an acceptable margin of error on the ratio of $|t_{xx}|$ and $|t_{yy}|$, ranging from 0.8 to 1.2, and on the phase difference, ranging from 0.9π to 1.1π , around the operating wavelength, the operating bandwidth of our HWP is determined to be 35 nm wide, ranging from 705 nm to 740 nm (see figure 3-a). Furthermore, the birefringence of the designed plate was calculated using the following equation: $\Delta n_{eff} = \lambda \Delta\phi / (2\pi H)$ [13, 30], with the values of $\Delta\phi$ taken into account. At the operational wavelength of $\lambda = 727$ nm, a $\Delta n = 2.5$ is obtained (see figure 3-b), indicating a significantly compact plate in comparison to those previously discussed in the literature [29, 30].

In the case of the realistic HWP under consideration, the transmitted wave will exhibit elliptical polarization. Consequently, the deviation from a perfect HWP can be characterized by the ellipticity of the polarization of the transmitted wave. This is one of the quantities used to quantify the properties of HWP [13]. This ellipticity is related to the phase difference $\Delta\phi$ and determined by the formula: $\eta = \frac{1}{2} \arcsin(\sin(2\chi) \times \sin(\Delta\phi))$, with $\chi = \arctan(E_Y/E_X)$ [13, 30]. It is set to $\eta = -0.49^\circ$ at the operating wavelength (see figure 4-a). Note that the negative ellipticity refers to an optical property that induces a counterclockwise displacement of the light polarization, to the left, during its propagation through a HWP. The low value of ellipticity indicates that the linear polarization is preserved at the output of the plate. In addition to the previously fixed error margin for the phase difference, assuming an acceptable error margin of -0.1 rad to 0.1 rad for the ellipticity, around this value, the operating bandwidth of our HWP is reduced to 20 nm ranging from 715 nm to 735 nm, as shown in figure 4-a.

The inclination of the electric field at the exit of the plate with respect

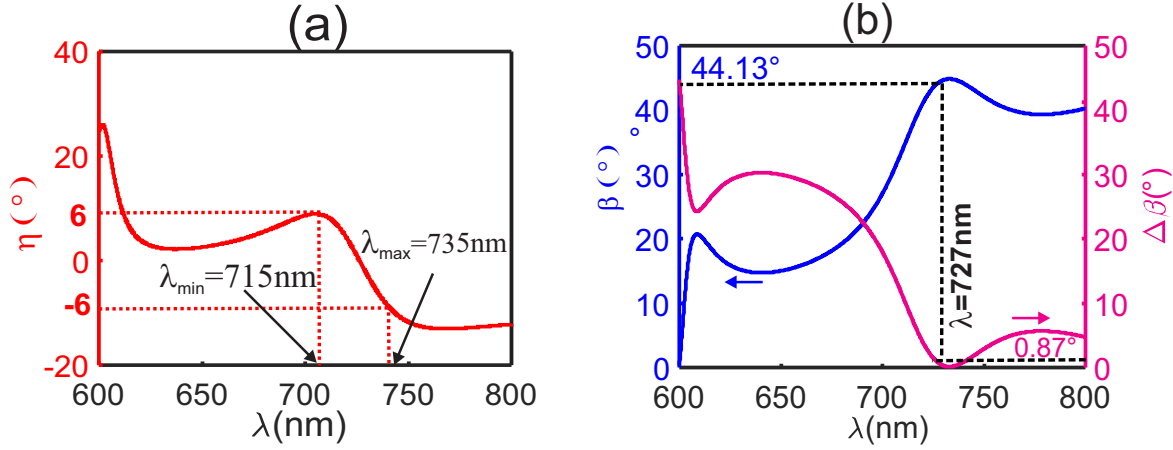


Figure 4. a) The ellipticity η (red line) and $\Delta\eta$ (green line) and b) the angle of rotation β (blue line) and $\Delta\beta$ (pink line) of the transmitted wave with respect to the x -axis. The geometric parameters are $P_X = P_Y = 250\text{nm}$, $R_i = 50\text{nm}$, $R_e = 100\text{nm}$, $\Lambda = 120^\circ$ and $H = 146\text{nm}$.

to the x -axis is determined by the angle β , which is expressed as follows: $\beta = \frac{1}{2} \text{atan}(\tan(2\chi) \times \cos(\Delta\phi))$. Figure 4-b depicts the values of the angle (β) of the transmitted electric field by our HWP relative to the x -axis. We evaluated the performance of our plate by comparing the results with a perfect HWP. The absolute error between the angle of the perfect plate ($\beta_P = 45^\circ$) and the one of our designed plate (β_D) was calculated as $\Delta\beta = |\beta_P - \beta_D|$. At a wavelength of $\lambda = 727\text{nm}$, the resulting β value was $44.13 \pm 0.87^\circ$ with a relative error of $\varepsilon = \Delta\beta/\beta_P = 1.93\%$. These results demonstrate that the optical properties of our plate are very close to those of a perfect HWP.

The effect of the designed HWP and its efficiency on a linearly polarized wave can be measured using the schematic experimental setup depicted in figure 5-a. A polarizer is placed at an angle of 45° relative to the x -axis (prior to the HWP). The incident light on the HWP is thus polarized at this angle, as illustrated in figures 5-b and 5-c).

In order to understand how a HWP modifies the nature and polarization of the incident light, an analyser is positioned at the output of the plate (see figure 5-a). The transmitted intensity is plotted as a function of the wavelength λ and the orientation angle γ of the transmitted electric field with respect to the x -axis, as well as the transmitted intensity curve as a function of the angle γ (see figures 6-a and 6-b).

Following passage through the designed HWP, the transmission intensity attains a maximum value of $T = 75\%$ at an angle of $\gamma = 135^\circ$ for the wavelength $\lambda = 727\text{nm}$, thereby that the incident field has undergone a rotation of an angle $\psi = 2\theta$ (see figures 6-a and 6-b). Figure 6-b reveals that the transmission through our plate (discontinuous blue curve) exhibits a trend analogous to that of a perfect plate (continuous blue curve). By plotting the transmission and the rotation angle γ as a function λ , it can be observed that the transmission reaches its maximum at angle $\gamma = \pi - \beta = 135.8^\circ$ (see figure 7), confirming the rotation of the polarization plane of the transmitted wave with respect

Design of a metallic HWP based on birefringent metamaterial

8

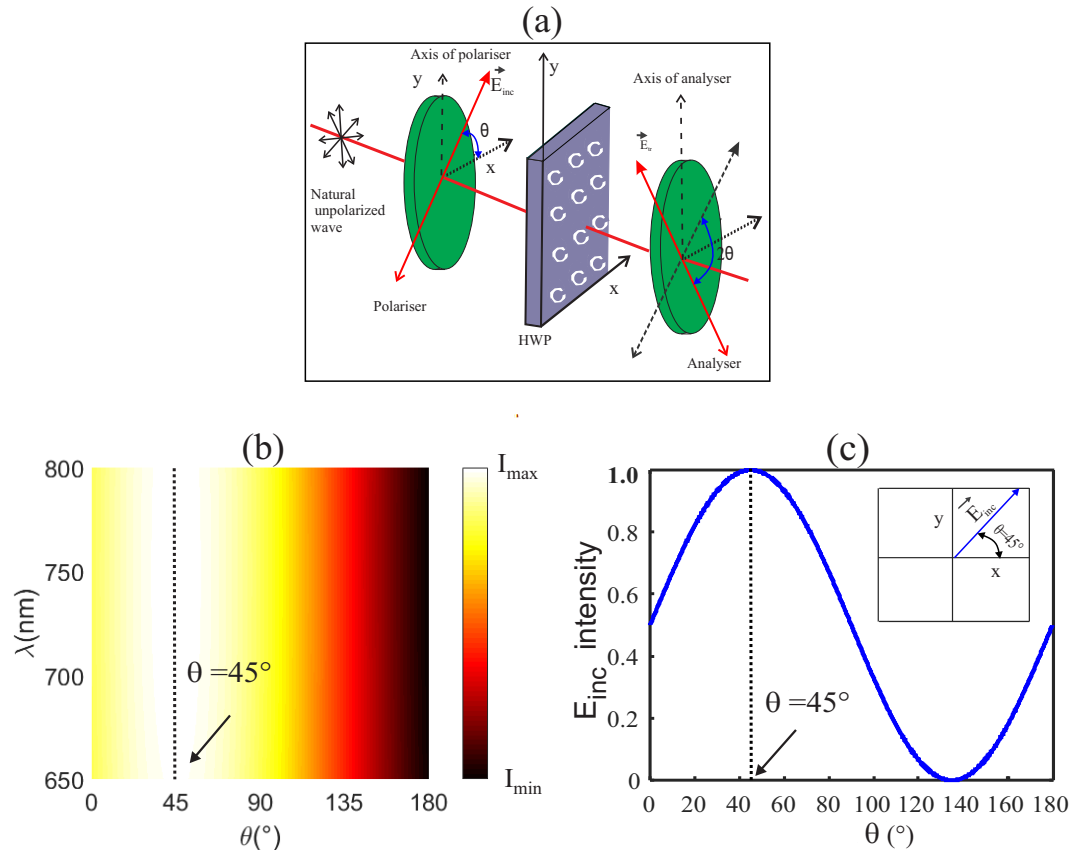


Figure 5. a) Effect of the HWP on a linearly polarized incident wave at an angle θ : The plane of incidence of the emerging light will be rotated by 2θ . b) Diagram of the incident electric field intensity as a function of the wavelength λ and angle θ of the incident electric field relative to the x -axis. c) Variation of the incident electric field intensity as a function of the angle θ .

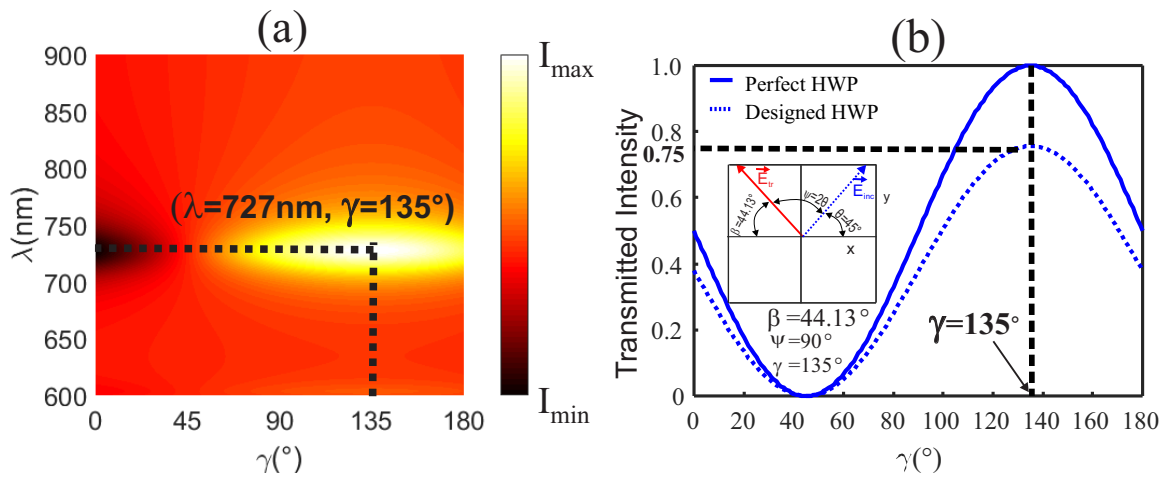


Figure 6. a) Diagram of the transmitted electric field intensity as a function of the wavelength λ and the angle γ . b) Variation of the transmitted electric field intensity as a function of the angle γ : continuous blue curve for the perfect HWP and discontinuous blue curve for the designed HWP. The insert depicts the vectors of the incident and transmitted electric fields, along with the various angles that characterize their orientations (θ , β , ψ and γ).

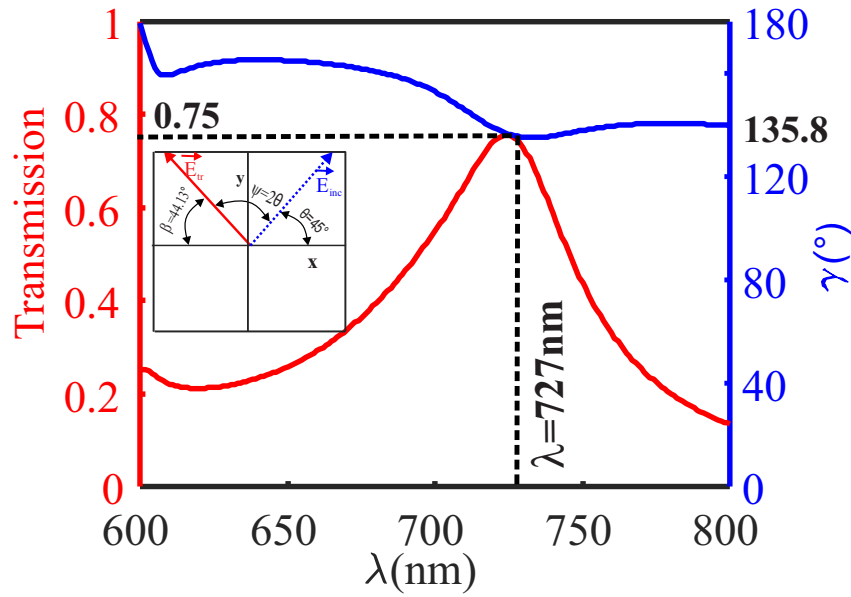


Figure 7. Transmission spectrum (in red) of the designed HWP and variation of the angle of rotation $\gamma = \pi - \beta$ (in blue) of the transmission plane with respect to the x -axis as a function of the wavelength λ . $P_X = P_Y = 250\text{nm}$, $R_i = 50\text{nm}$, $R_e = 100\text{nm}$, $\Lambda = 120^\circ$ and $H = 146\text{nm}$.

to the plane of the incident electric field. With regard to the transmitted electric field, it undergoes a rotation by an angle of $\psi = 2 \times \theta$ (see inset of figure 6-b), resulting in a rotation of the polarization plane by an angle $\gamma = |\theta + \psi| = 135^\circ$.

The aforementioned calculations confirm that our metallic HWP exhibits exceptional performance and is an excellent candidate for nano-optics.

5. Conclusion

The results of this work highlight the successful design and characterization of a functional halfwave plate (HWP) operating within the visible range. The proposed metamaterial, composed of a periodic array of C-shaped apertures etched into a silver metal layer deposited on a glass substrate and filled with glass, achieves a transmission of 75% at a working wavelength of $\lambda = 727\text{nm}$. The plate has a thickness of $H = 146\text{nm}$, resulting in a birefringence of $\Delta n = 2.5$. These features underline the innovative nature of our HWP compared to previously studied designs in the literature. Its optical properties make it a highly efficient HWP, opening promising opportunities for applications in optics and photonics.

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Design of a metallic HWP based on birefringent metamaterial 10

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