Analytical model of transverse compression of single fiber with elliptical cross-section

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Abstract

Background: The existing analytical models used to identify the transverse elastic modulus from single fiber transverse compression (SFTC) test almost all consider a circular cross-section. Only one work has proposed an adaptation for elliptical cross-section which still fails to accurately reproduce the force-displacement curve for high ellipticity.
Objectives: The aim of this study is to introduce a novel analytical model capable of faithfully replicating the transverse behavior of elliptical fibers during SFTC test.
Methods: To take into account the ellipticity of the fiber, the analytical model of Jawad and Ward is adapted using an equivalent contact radius derived from Hertzian contact theory. The proposed model is validated through a finite element analysis (FEA).
Results: From the force/displacement curve obtained with the finite element analysis, the transverse elastic modulus of elliptical fibers (with ellipticity varying from 0 to 0.8) is correctly identified, by inverse method, with the proposed model. The maximum relative difference between the identified modulus and that defined in the FEA remains less than 1.2%.
Conclusion: The analytical model proposed in this work precisely reproduces the behavior of an elliptical fiber during SFTC test, regardless of its ellipticity.

Keywords: Transverse elastic modulus, Jawad and Ward model, Hertzian contact, Finite element analysis, Inverse identification

1 Introduction

The fiber-reinforced polymer composites are currently widely used in various industries due to their excellent mechanical performance and lightweight. To accurately simulate the behavior of the composite during its life, it is necessary to know the material properties of each constituent. As fibers are generally considered transverse isotropic materials, the transverse elastic modulus E_T is required.

To experimentally characterize the transverse properties at the fiber scale (typically a few tens of µm in diameter), the established method is the Single Fiber Transverse Compression (SFTC) test which consists in compressing a fiber between a pair of plane parallel plates, as outlined in [1]. In fact, this approach was applied on various kinds of fibers such as aramid [2] or carbon [3]. Since the contact between the fiber and the platens induces a non-linear behavior, the transverse property identification is carried out by inverse method using analytical models.

Among all the equations proposed in the literature, the Jawad and Ward model [4] is the most popular. The expression of the loading plate displacement U in terms of the applied force per unit length F and the elastic properties of the fiber is based on a plane strain formulation, considering a circular cross-section. However, some fibers, such as plant fibers, cannot be considered as a circular cross-section cylinder but rather as an elliptical one [5]. Hence, the Jawad and Ward model is no longer effective to identify the transverse elastic modulus as shown by Govilas et al. [6, 7]. These authors proposed an adaptation of this model to the case of an elliptical cross-section (Fig. 1) by differentiating the influences of minor and major radii. The major radius R_m is used in the calculation of the contact half-width b while the rest of the model considers the minor radius R since its perpendicular direction to the platens influences the fiber's contraction. This analytical model is used to identify the transverse elastic modulus from FEA data. The relative difference between the transverse elastic modulus defined in FEM simulations and that identified with this adapted analytical model reaches almost 90% for a flattening ratio of 0.8, which is still relatively high (even if it is less than the value obtained with Jawad and Ward's model considering a single radius, either R or R_m).

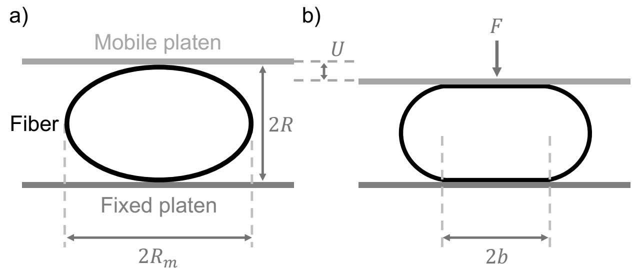

Fig. 1 Schematic diagram of transverse compression of an elliptical cross-section fiber: (a) undeformed state - (b) deformed state.

The aim of this study is to introduce a new analytical model capable of accurately describing the behavior of an elliptical cross-section fiber during SFTC test, regardless of its flattening ratio. For this purpose, an analytical model based on that of Jawad and Ward, with an equivalent contact radius derived from Hertzian contact theory, is first described. The validity of the proposed model is then assessed by identifying the transverse elastic modulus of elliptical fibers by inverse method, using data generated from finite element simulations.

2 Mathematical model

To improve the analytical models used for the identification of the transverse elastic modulus in the case of an elliptical cross-section fiber, this paper proposes a new model, based on that of Jawad and Ward, but considering an equivalent radius R_{eq} for the calculation of contact half-width b, as described in the Hertzian contact theory [8, 9]. In the contact between two cylinders (of radii R_1 and R_2) with parallel axes, the R_{eq} radius is defined as:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{1}$$

In the case of contact between a cylinder and a platen, the R_2 radius tends to infinity, leading to:

$$R_{eq} = R_1 \tag{2}$$

For an elliptical cross-section of the cylinder with minor radius R and major radius R_m, the radius of curvature R_1 in the contact zone (considering the platen parallel to the major axis of the ellipse) is defined as:

$$R_1 = \frac{{R_m}^2}{R} \tag{3}$$

For a transversely isotropic cylinder (the transverse isotropy axis being the cylinder axis), the proposed model then relies on equation (4) to calculate the contact half-width b and equation (5) to express the displacement of the cylinder U as a function of the force per unit length F and other physical quantities:

$$b = \sqrt{\frac{4F{R_m}^2}{\pi R}\left(\frac{1}{E_T} - \frac{\nu_{LT}^2}{E_L}\right)} \tag{4}$$

$$\begin{aligned} U = \frac{4F}{\pi}\Bigg[&\left(\frac{1}{E_T} - \frac{\nu_{LT}^2}{E_L}\right)\left(\sinh^{-1}\left(\frac{R}{b}\right) + \ln(2)\right) - \frac{1}{2}\left(\frac{1}{E_T}(1-\nu_{TT}) - 2\frac{\nu_{LT}^2}{E_L}\right) \\ &+ \left(-\frac{\nu_{TT}}{E_T} - \frac{\nu_{LT}^2}{E_L}\right)\frac{R}{b}\left(\sqrt{1+\left(\frac{R}{b}\right)^2} - \frac{R}{b}\right)\Bigg] \end{aligned} \tag{5}$$

with the elastic properties of the cylinder: E_L the longitudinal elastic modulus, E_T the transverse elastic modulus and ν_{LT}, ν_{TT} the Poisson ratios in the longitudinal and transverse plane respectively.

3 Finite element modelling method

To evaluate the accuracy of this new analytical model in describing the behavior of an elliptical fiber during SFTC test regardless of its ellipticity, FEM simulations of transversely isotropic elliptical fibers under transverse compression are performed with the COMSOL Multiphysics® software (v. 6.2, COMSOL AB, Stockholm, Sweden).

Nine fibers are simulated, with a minor radius $R = 15$ µm and a major radius R_m, for an ellipticity (or flattening) factor $f = 1 - \frac{R}{R_m}$ varying from 0 (circular case) to 0.8 (the flattest).

Except for the fiber geometry, the building of the FEM model is similar to that of [6, 7]. In brief, a 2D plane strain formulation is used. Only a quarter of the fiber is modeled given the symmetries along the xz and yz planes. The material properties of the tested fiber are: $E_L = 50$ GPa, $E_T = 1$ GPa, $\nu_{LT} = 0.4$ and $\nu_{TT} = 0.07$. A maximum vertical displacement of 0.5 µm is applied to the upper platen, which corresponds to a maximum displacement $U = 1$ µm, considering the symmetry. A Segregated Augmented Lagrangian contact method is used between the platen and the fiber. Furthermore, frictionless contact properties were considered in this work, as the influence of the friction coefficient having proven to be negligible [2, 10]. Then the force per unit length F is obtained by integrating the normal stress on the contact zone. Finally, triangular elements are used for the fiber mesh while the rectangular platen is meshed with free quadrilaterals. The mesh size is refined on contact. The boundary conditions and mesh of the FEM model are shown in Fig. 2.

The transverse elastic modulus E_T is then identified by fitting the results (F/U curve) obtained by the FEM simulations with the proposed analytical model using a least mean square method. The accuracy of the model is assessed by giving the relative error ΔE_T between the identified modulus E_T^{id} and the modulus defined in the simulation $E_T^0 = 1$ GPa:

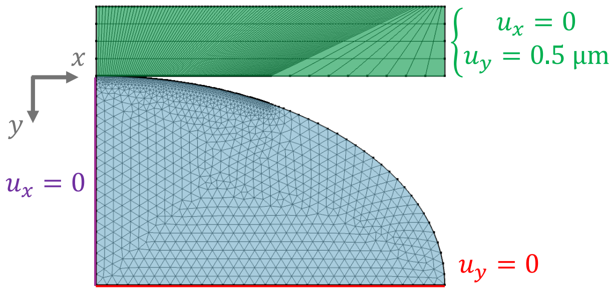

Fig. 2 Boundary conditions and mesh of the finite element model of an elliptical fiber under transverse compression.

$$\Delta E_T = \frac{E_T^{id} - E_T^0}{E_T^0} \tag{6}$$

To evaluate the model's ability to reproduce the elliptical fiber behavior, the value of the least mean squares residual is calculated:

$$\rho = \frac{1}{N}\sqrt{\sum_{i=1}^{N}(U_i^{FEA} - U_i^{model})^2} \tag{7}$$

where N is the number of points of the F/U curve, U_i^{FEA} is the platen displacement in the FEA and U_i^{model} is the fiber displacement given by the proposed analytical model.

4 Results and Discussion

The proposed analytical model is used to represent the behaviour of the nine fibers with the same material properties as in the FEA, and the results are given in Fig. 3, as well as the FEA results. In both approaches, as the ellipticity increases, the force increases for a given displacement level. Moreover, the F/U curves show that the analytical model fits well the simulation results. To quantify the accuracy of the analytical model, it is used to identify the transverse elastic modulus from FEA data. Results in terms of relative error ΔE_T and least mean squares residual ρ are given in Fig. 4. The maximum error on the identified modulus obtained is 1.2% for a flatness factor of 0.6 and the maximum residual is 2.8×10^{-16} m. That confirms the very good accuracy of the model to identify the transverse elastic modulus of the fiber.

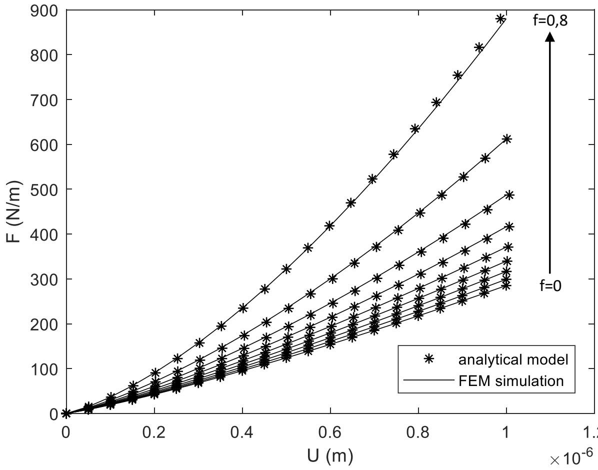

Fig. 3 Force/displacement curves for different elliptical fibers, for both analytical and FEM models.

The elliptical model proposed here is compared to the elliptical model of [6, 7] and the Jawad and Ward model, with a circular fiber of radius R or R_m. The results in terms of force/displacement curves for an elliptical fiber with $f = 0.5$ are given in Fig. 5. Fig. 5a, which gives the curves obtained for all models with $E_T = 1$ GPa, shows that the proposed model is the most accurate to fit the FEM curve. On Fig. 5b, the curves are obtained for each model with the transverse modulus identified by this same model. All the identified transverse moduli and the corresponding residuals are given in Tab 1. As expected, the identified E_T is the nearest of 1 GPa for the proposed model and moves away from the 1 GPa value for other models. These curves also show that non-linearity is better fitted by the proposed model than by the others.

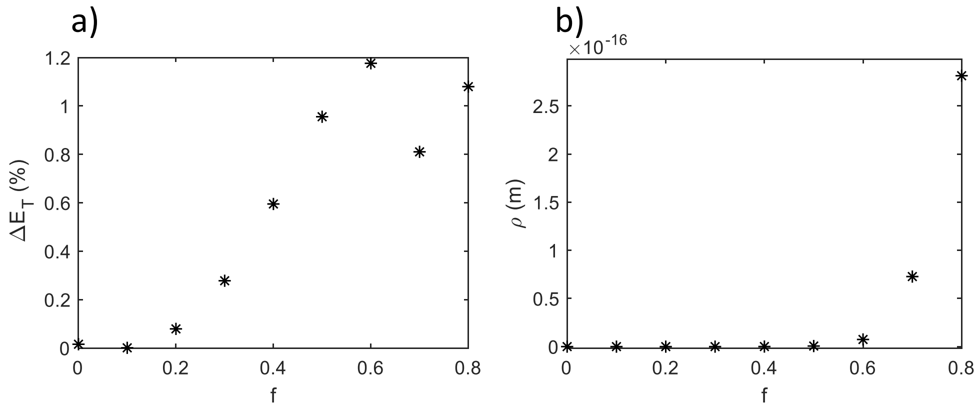

Fig. 4 Transverse elastic modulus identification results: (a) relative error ΔE_T as function of the flatness factor f - (b) least mean squares residual ρ as function of the flatness factor f.

Analytical model	E_T (GPa)	ρ (m)
Elliptical model (this study)	1.01	6.16×10^{-19}
Elliptical model (ref [6])	1.22	1.15×10^{-15}
Jawad and Ward with R	1.42	3.58×10^{-15}
Jawad and Ward with R_m	1.62	6.36×10^{-15}

Table 1 Transverse elastic modulus E_T identified from FEM results, with different analytical models and corresponding least mean squares residual ρ

5 Conclusion

This paper presents a new analytical model capable of accurately representing the transverse behavior of an elliptical fiber during SFTC test, regardless of its ellipticity. In fact, the analytical model of Jawad and Ward is adapted for elliptical cross-section using an equivalent contact radius derived from Hertzian contact theory. The very good accuracy (less than 1.2% of relative error) in identifying the transverse elastic modulus of the fiber from FEA data confirms the potential of this analytical model to be used by experimenters to analyze results from single fiber transverse compression test, and, more generally, from transverse compression of specimens with elliptical cylindrical cross-section. The analytical model has been improved with geometrical

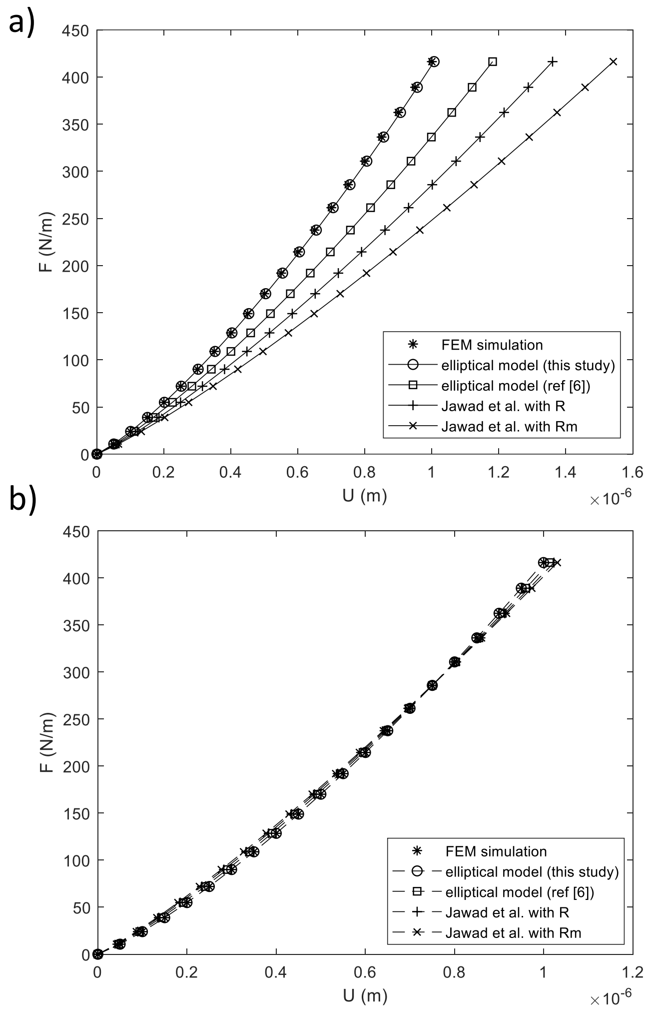

Fig. 5 Force/displacement curves for an elliptical fiber with $f = 0.5$: (a) results of FEM simulations and different analytical models with $E_T = 1$ GPa - (b) results of FEM simulations and different analytical models with the identified E_T (See Tab 1).

consideration; nevertheless, it corresponds to specific assumptions, and not takes into account other considerations as expressed in real tests, such as material behaviour (viscoelasticity, plasticity), boundary conditions (platen parallelism), loading (large deformation) or full geometry (3D variability, lumen porosity) [7].

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Declarations

- Competing interests
  The authors have no competing interests to declare that are relevant to the content of this article.
- Data availability
  The data that support the findings of this study are available from the corresponding author upon reasonable request.
- Authors contributions
  Violaine Guicheret-Retel: Conceptualization, Methodology, Formal analysis and investigation, Writing - original draft preparation, Writing - review and editing, Resources, Supervision
  Bilal Sabillah : Formal analysis and investigation, Writing - review and editing
  Vincent Placet : Conceptualization, Writing - review and editing, Resources, Supervision
  Florian Boutenel: Conceptualization, Methodology, Formal analysis and investigation, Writing - original draft preparation, Writing - review and editing, Funding acquisition, Resources, Supervision

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