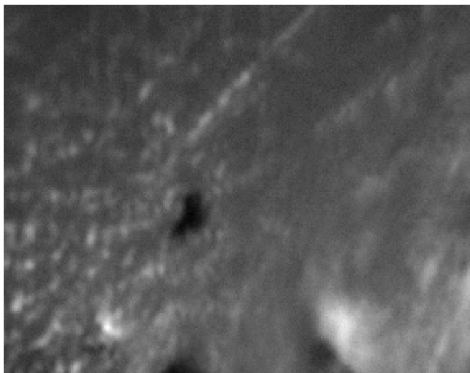


Digital Image Correlation : Background

F. Amiot

Université Marie et Louis Pasteur, FEMTO-ST Institute CNRS-UMR 6174

Introduction



THE question



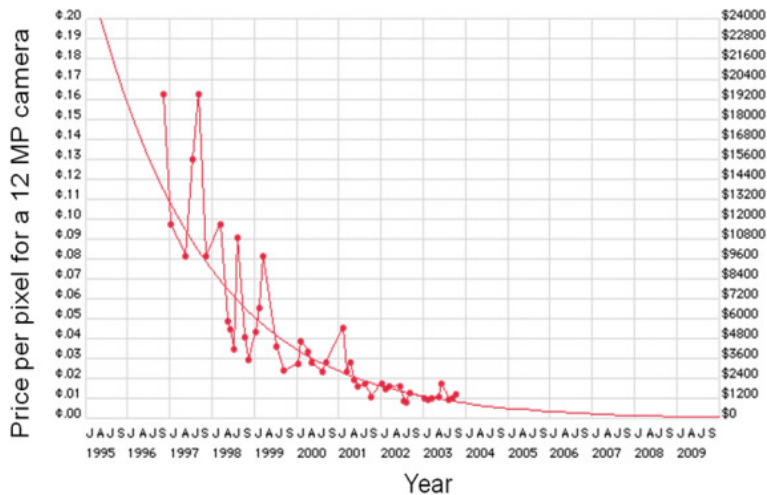
THE question



An experimental set-up to measure displacement fields?



Sensors cost



History

- ▶ Horn and Schunk, 1981, Determining optical flow, Artif. Intell. 17 (1-3).
- ▶ Chu, Ranson, Sutton and Peters, 1985, Applications of digital-image-correlation techniques to experimental mechanics, Exp. Mech. 25.
- ▶ Photomécanique conference : 1995
- ▶ 2003-2014 : CNRS national network on full-field measurements and identification
 - ▶ 15-20 research groups + 3 companies
 - ▶ 3 working groups : metrological performances, identification from full-field measurements, Infrared imaging
 - ▶ Photomechanics international conference
 - ▶ 5 co-signed papers with the outcome of the meetings
- ▶ Digital image correlation (DIC) is now a mature technique

Digital images



Digital image = a table of integers



251	251	247	243	215	170	227	239	235	235	235	231	231	227	239
121	130	142	158	154	134	198	239	235	235	235	231	231	231	231
77	73	73	77	73	81	215	239	235	239	235	231	227	227	239
81	81	81	81	81	89	182	239	239	239	235	235	231	227	231
73	73	77	81	81	85	219	239	239	235	231	231	231	227	239
73	73	73	77	77	89	186	243	239	239	235	235	231	227	231
69	69	73	77	77	89	223	243	239	235	231	231	231	227	239
69	69	73	69	77	89	194	243	239	239	235	231	231	227	235
69	69	73	73	77	89	223	243	239	235	231	231	231	231	239
69	69	73	73	77	89	194	243	239	239	235	231	231	231	239
69	69	69	73	77	81	150	178	194	210	223	227	227	231	243
69	65	69	73	73	77	81	81	85	85	89	93	101	105	117
57	57	61	65	65	69	69	69	69	69	73	73	77	77	77
49	49	49	49	53	53	53	53	57	57	57	57	61	61	61
49	53	53	57	61	57	57	53	49	49	49	49	49	49	49



Digital images



Digital image = a table of integers



251	251	247	243	215	170	227	239	235	235	235	231	231	227	239
121	130	142	158	154	134	198	239	235	235	235	231	231	231	231
77	73	73	77	73	81	215	239	235	239	235	231	227	227	239
81	81	81	81	81	89	182	239	239	239	235	235	231	227	231
73	73	77	81	81	85	219	239	239	235	231	231	231	227	239
73	73	73	77	77	89	186	243	239	239	235	235	231	227	231
69	69	73	77	77	89	223	243	239	235	231	231	231	227	239
69	69	73	69	77	89	194	243	239	239	235	231	231	227	235
69	69	73	73	77	89	223	243	239	235	231	231	231	231	239
69	69	73	73	77	89	194	243	239	239	235	231	231	231	239
69	69	69	73	77	81	150	178	194	210	223	227	227	231	243
69	65	69	73	73	77	81	81	85	85	89	93	101	105	117
57	57	61	65	65	69	69	69	69	69	73	73	77	77	77
49	49	49	49	53	53	53	53	57	57	57	57	61	61	61
49	53	53	57	61	57	57	53	49	49	49	49	49	49	49

A table of integers = a function



Image of the surface's marker

- ▶ Texture (cf Video)



Image of the surface's marker

- ▶ Texture (cf Video)
- ▶ Paint spray



Image of the surface's marker

- ▶ Texture (cf Video)
- ▶ Paint spray
- ▶ random



Image of the surface's marker

- ▶ Texture (cf Video)
- ▶ Paint spray
- ▶ random
- ▶ lightning



Image of the surface's marker

- ▶ Texture (cf Video)
- ▶ Paint spray
- ▶ random
- ▶ lightning
- ▶ grey-level images



Image of the surface's marker



- ▶ Texture (cf Video)
- ▶ Paint spray
- ▶ random
- ▶ lightning
- ▶ grey-level images

WARNING : surface displacements only!

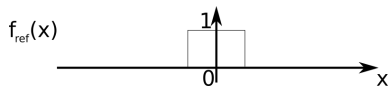


Processing images : 1D



An image in the reference state

$$f_{ref}(x)$$

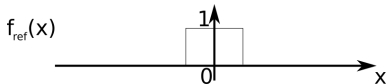


Processing images : 1D



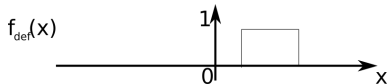
An image in the reference state

$$f_{ref}(x)$$



An image in the deformed state

$$f_{def}(x)$$



Processing images : 1D

One may use two functions :

- ▶ the cross-correlation product

$$I(u) = \int f_{ref}(x) f_{def}(x - u) dx$$

Processing images : 1D

One may use two functions :

- ▶ the cross-correlation product

$$I(u) = \int f_{ref}(x) f_{def}(x - u) dx$$

- ▶ The quadratic gap

$$J(u) = \int (f_{ref}(x) - f_{def}(x - u))^2 dx$$

Processing images : 1D



One may use two functions :

- ▶ the cross-correlation product

$$I(u) = \int f_{ref}(x) f_{def}(x - u) dx$$

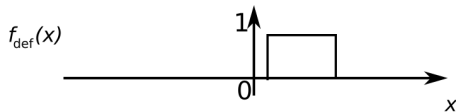
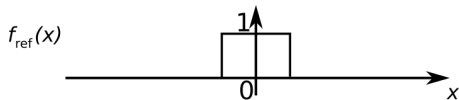
- ▶ The quadratic gap

$$J(u) = \int (f_{ref}(x) - f_{def}(x - u))^2 dx$$

Graphically build $I(u)$ and $J(u)$?

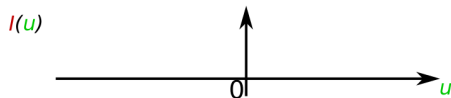
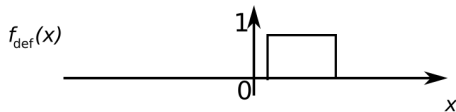
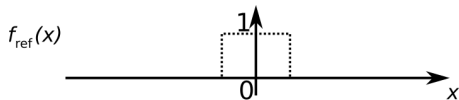


Processing images : 1D



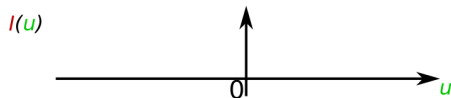
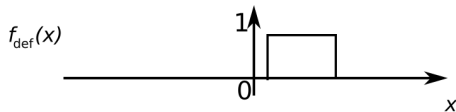
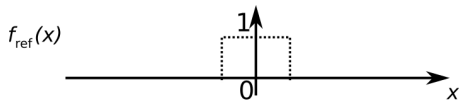
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



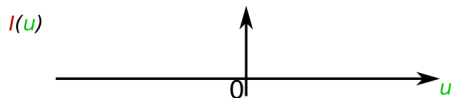
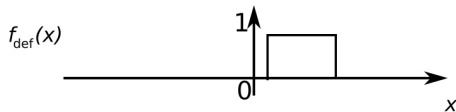
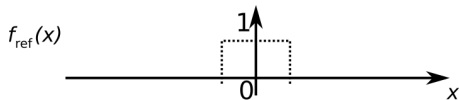
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



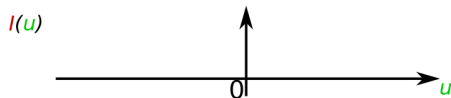
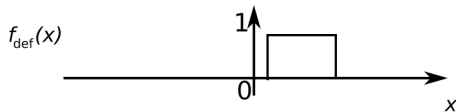
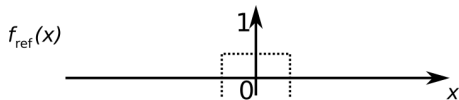
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



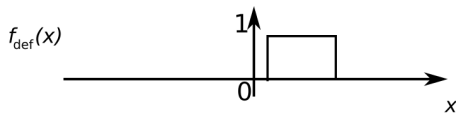
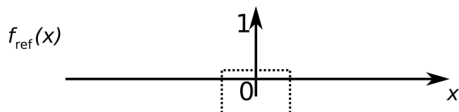
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



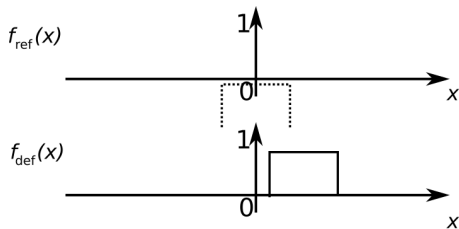
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



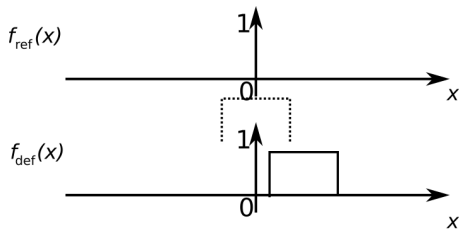
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

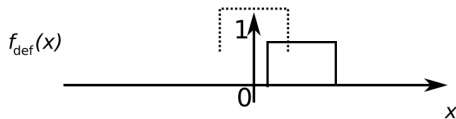
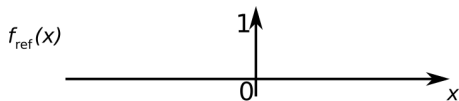
Processing images : 1D



$I(u)$

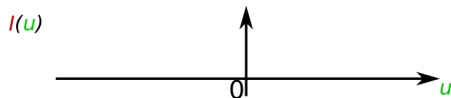
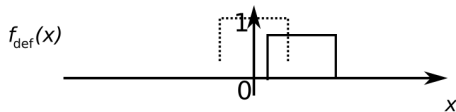
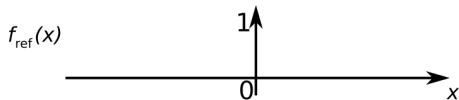
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



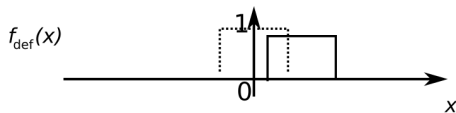
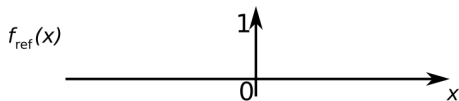
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



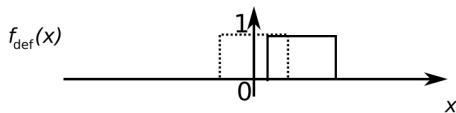
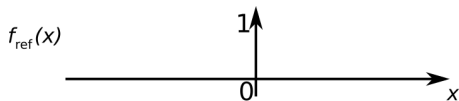
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Processing images : 1D



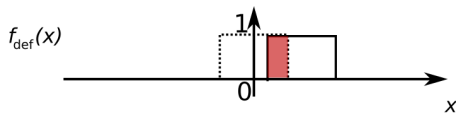
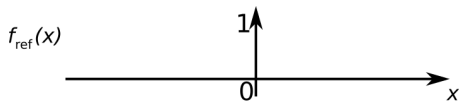
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Processing images : 1D



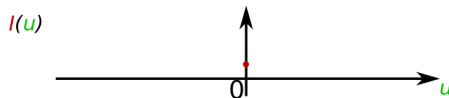
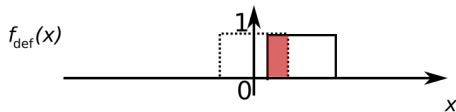
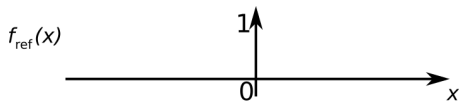
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Processing images : 1D



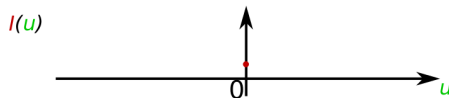
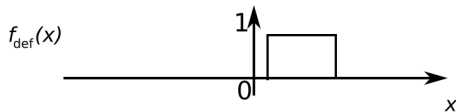
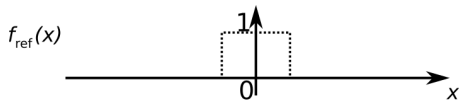
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Processing images : 1D



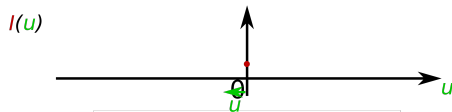
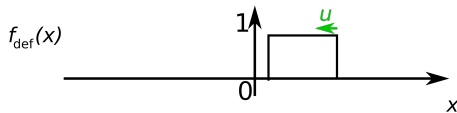
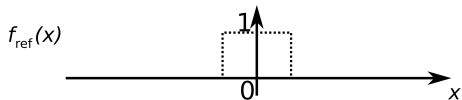
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



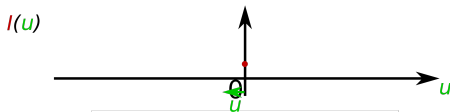
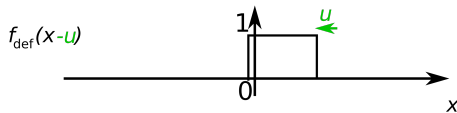
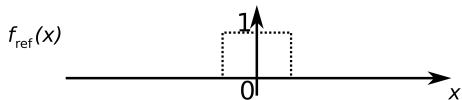
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



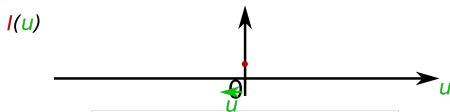
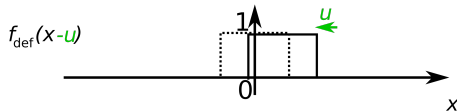
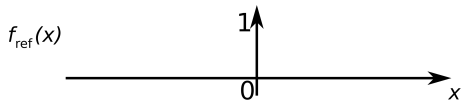
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



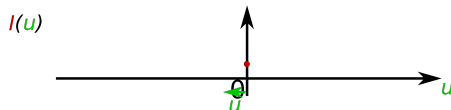
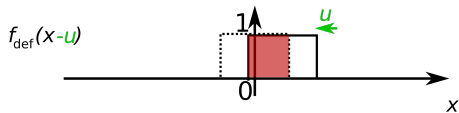
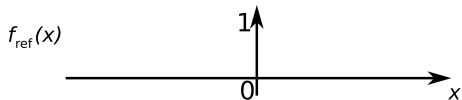
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x-u) dx$$

Processing images : 1D



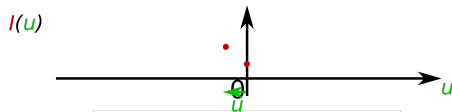
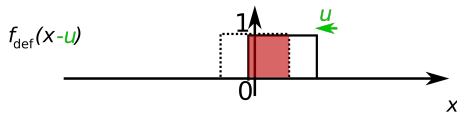
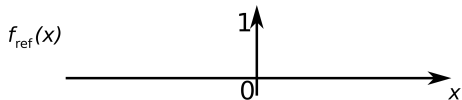
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



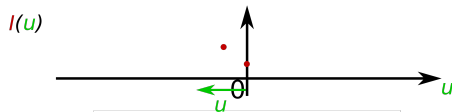
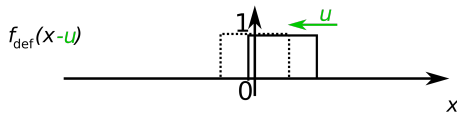
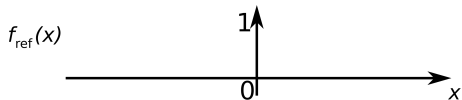
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



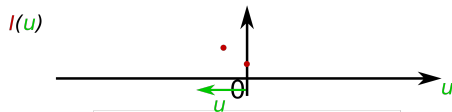
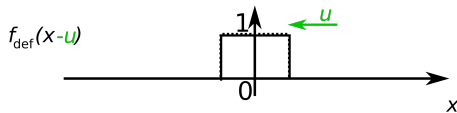
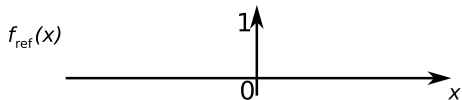
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



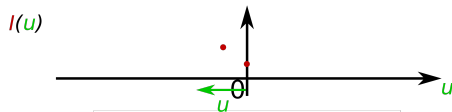
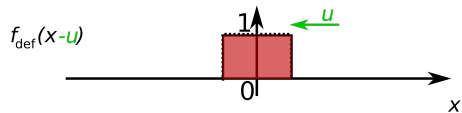
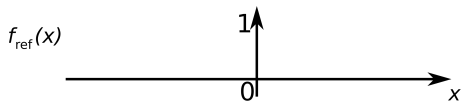
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



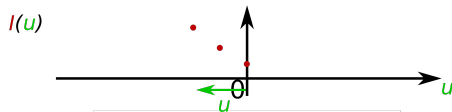
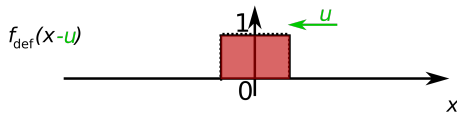
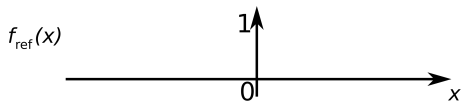
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



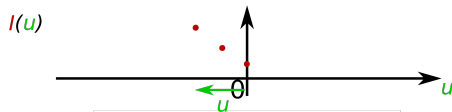
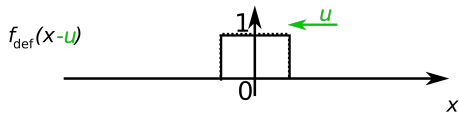
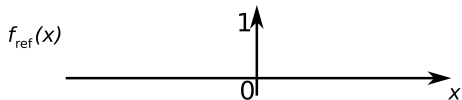
$$I(u) = \int f_{\text{ref}}(x) f_{\text{def}}(x - u) dx$$

Processing images : 1D



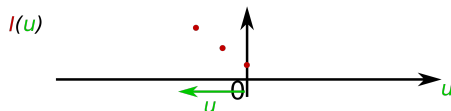
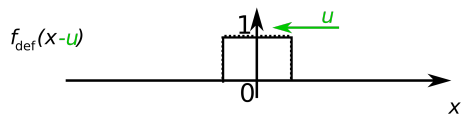
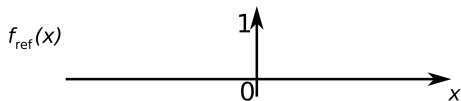
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Processing images : 1D



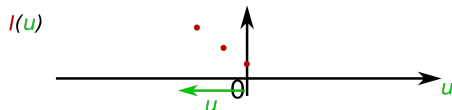
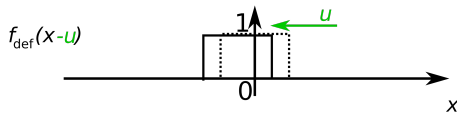
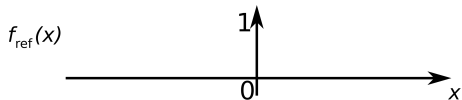
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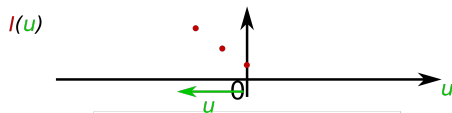
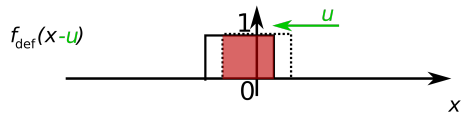
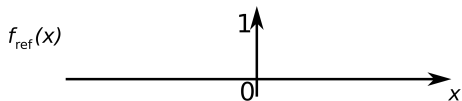
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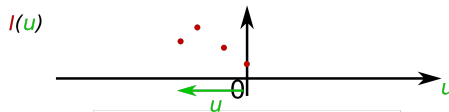
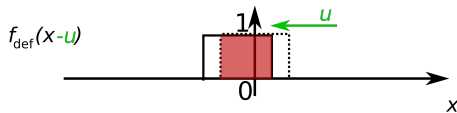
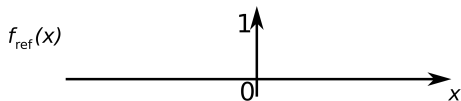
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Processing images : 1D



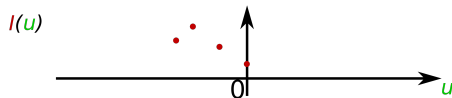
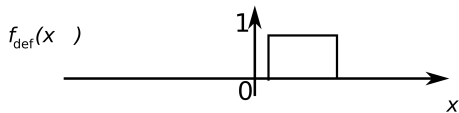
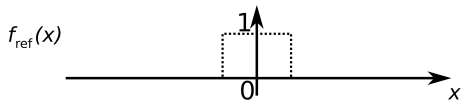
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Processing images : 1D



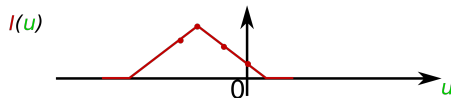
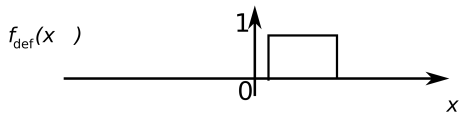
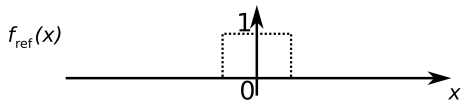
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Processing images : 1D



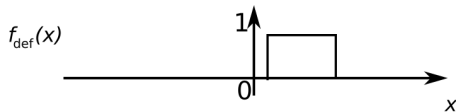
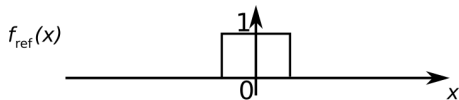
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Processing images : 1D



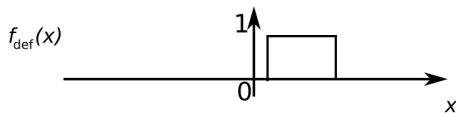
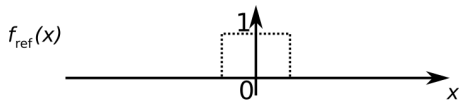
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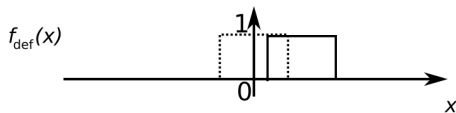
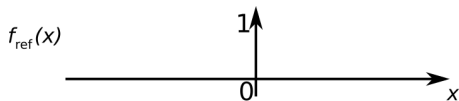
$$J(u) = \int (f_{\text{ref}}(x) - f_{\text{def}}(x - u))^2 dx$$

Processing images : 1D



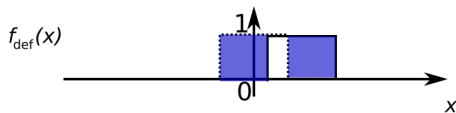
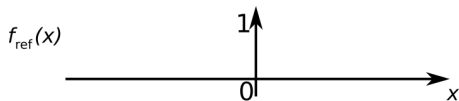
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Processing images : 1D



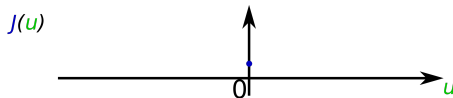
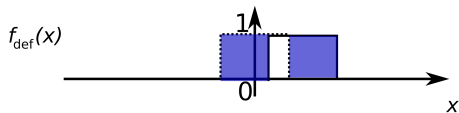
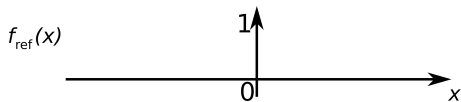
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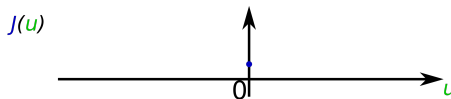
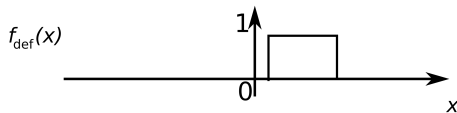
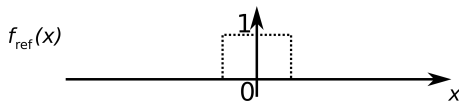
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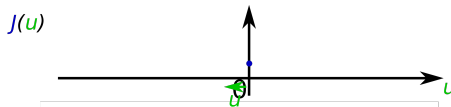
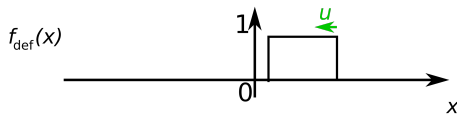
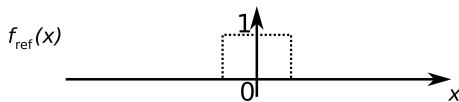
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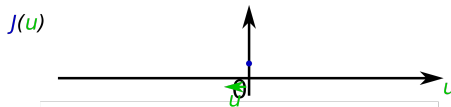
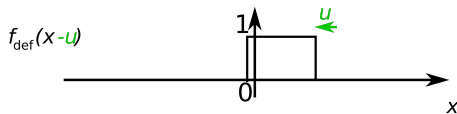
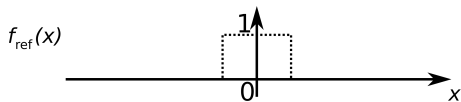
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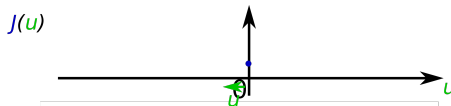
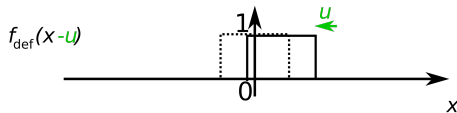
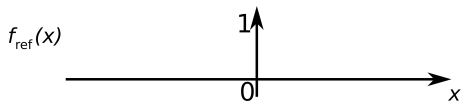
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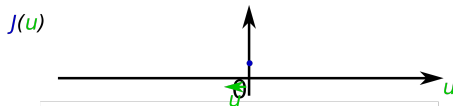
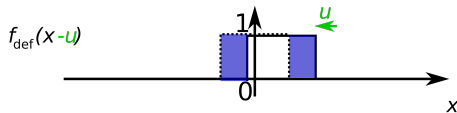
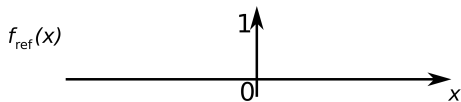
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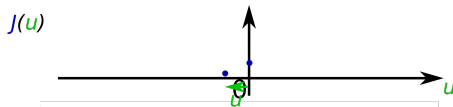
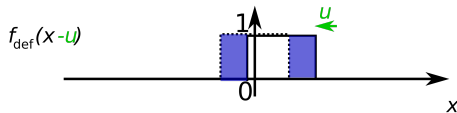
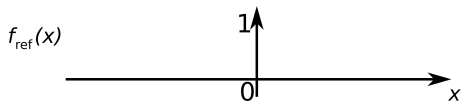
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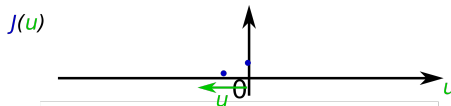
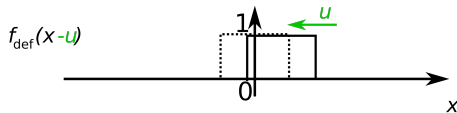
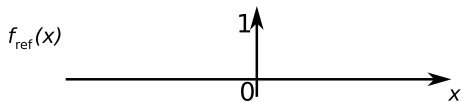
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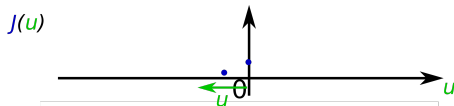
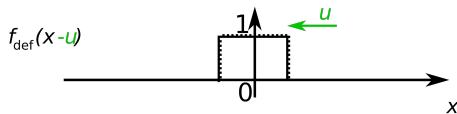
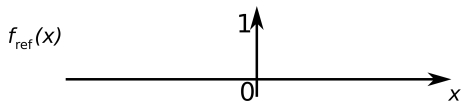
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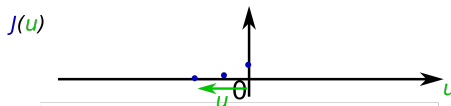
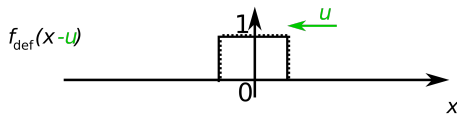
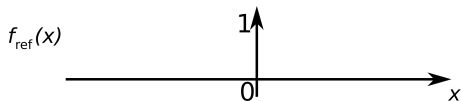
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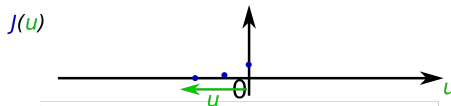
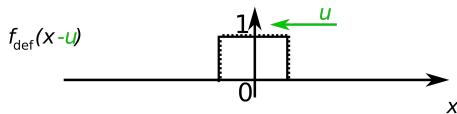
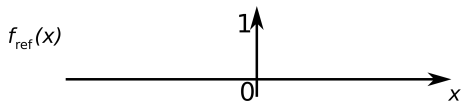
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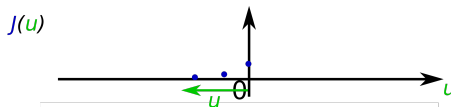
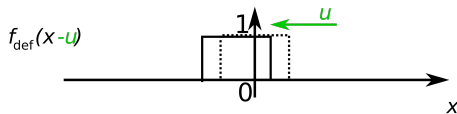
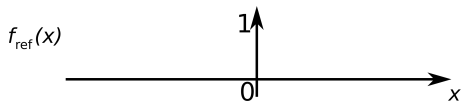
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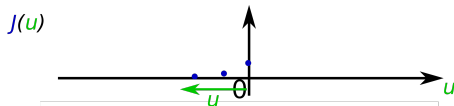
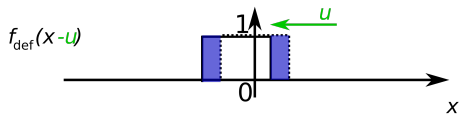
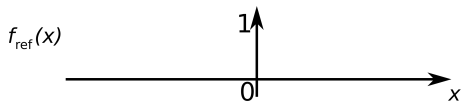
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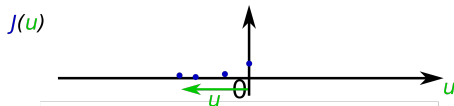
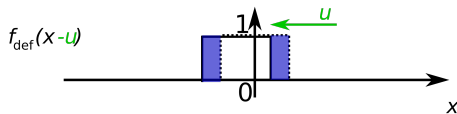
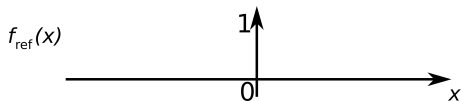
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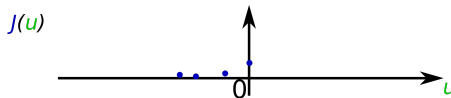
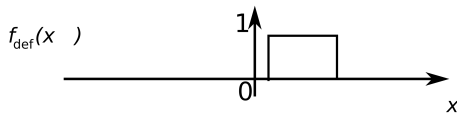
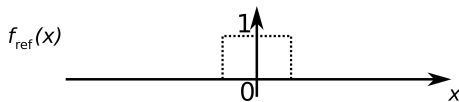
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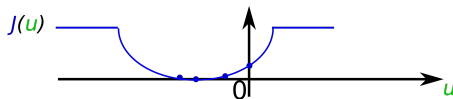
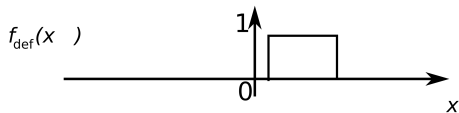
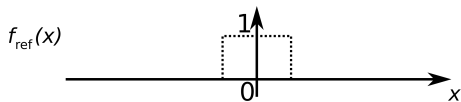
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Processing images : 1D

The displacement which transforms $f_{ref}(x)$ into $f_{def}(x)$ is obtained as :



Processing images : 1D



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- ▶ the maximizer of the cross-correlation $I(u)$



Processing images : 1D



The displacement which transforms $f_{ref}(x)$ into $f_{def}(x)$ is obtained as :

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Processing images : 1D



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Processing images : 1D



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- ▶ the minimizer of the quadratic gap $J(u)$

- ▶ Rem: the method is named after $I(u)$ (images correlation)
- ▶ Rem: the minimum of $J(u)$ is called the residual



Processing images : 1D

Remarks :

- ▶ The cross-correlation $I(u)$ may be computed efficiently (FFT)

Processing images : 1D

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- ▶ The marker's **GRADIENT** carries the information on the displacement

Processing images : 1D

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- ▶ The marker's **GRADIENT** carries the information on the displacement
- ▶ There is no possible measurement if $\frac{df_{ref}(x)}{dx} = 0$

Processing images : 2D



The quantities $\mathbf{x}$, $\mathbf{u}$ and $\mathbf{v}$ are now vectors

$$f_{def}(\mathbf{x}) = f_{ref}(\mathbf{x} + \mathbf{v}) \simeq f_{ref}(\mathbf{x}) + \mathbf{v} \cdot \nabla f_{ref}(\mathbf{x})$$



Processing images : 2D



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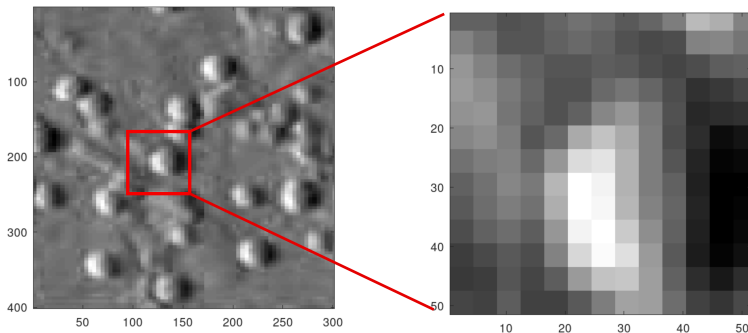
$$f_{def}(\mathbf{x}) = f_{ref}(\mathbf{x} + \mathbf{v}) \simeq f_{ref}(\mathbf{x}) + \mathbf{v} \cdot \nabla f_{ref}(\mathbf{x})$$

It is possible to get :

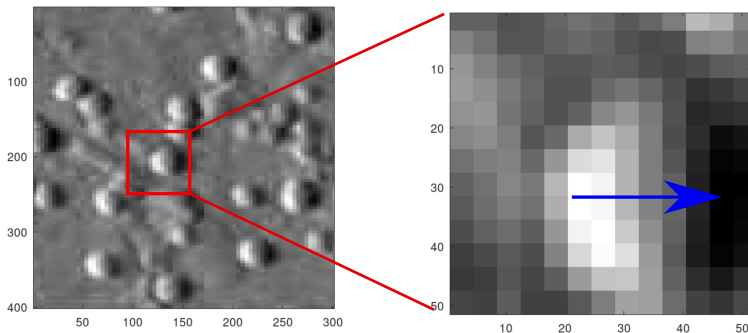
- ▶ $|\mathbf{v}| \neq 0$, $|\nabla f_{ref}(\mathbf{x})| \neq 0$ but $\mathbf{v} \cdot \nabla f_{ref}(\mathbf{x}) = 0$ if the vectors are orthogonal
- ▶ Identify $\mathbf{v}$ from $f_{ref}(\mathbf{x})$ and $f_{def}(\mathbf{x})$ is ill-posed



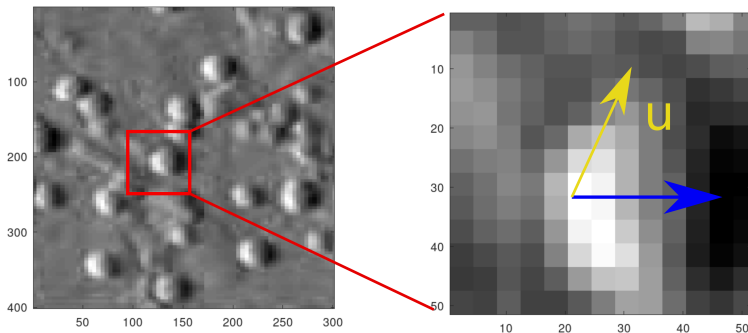
Processing images : 2D



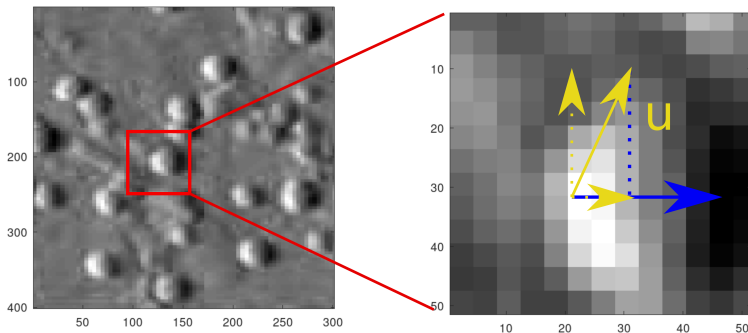
Processing images : 2D



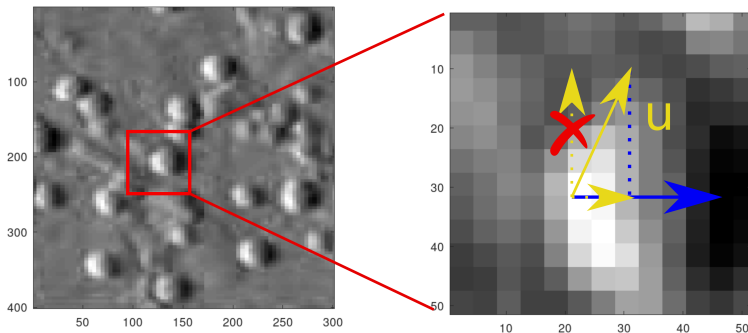
Processing images : 2D



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Regularization



The projection of the displacement field on a chosen functions basis is sought :

$$\mathbf{u}(\mathbf{x}) = \sum_i \alpha_i \phi_i(\mathbf{x})$$



Regularization



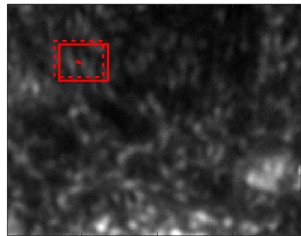
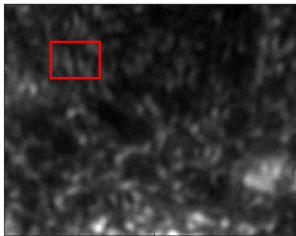
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Different groups of approaches, depending on the choice of the functions basis $\phi_i(\mathbf{x})$



Regularization with a piecewise rigid-body displacement field



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The displacement field is locally described as a rigid-body displacement

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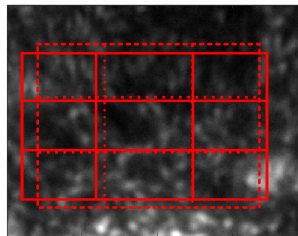
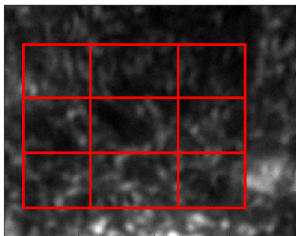
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 - ▶ Easy comparison with the results of FE computations
- ▶ Minimizing $J(u)$

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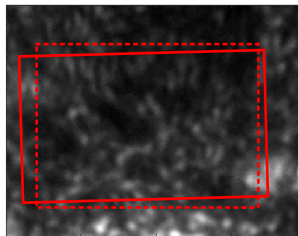
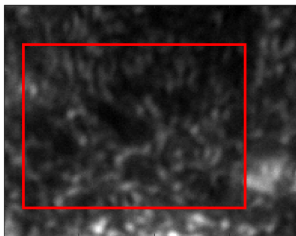
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Warning : local minimizers, large strains (or rotations)

Regularization with the displacement solution of the associated problem



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The field is described as the solution of the associated problem

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► Integrated approaches

- Hypotheses on the mechanical behavior. Ex: tensile test (along $\mathbf{x}$), uniform linear elastic material :

$$u_x = \frac{F}{ES}x + u_x^0 + \omega y$$

$$u_y = -\nu \frac{F}{ES}y + u_y^0 - \omega x$$

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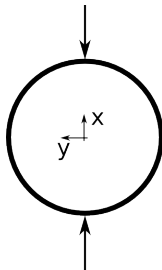
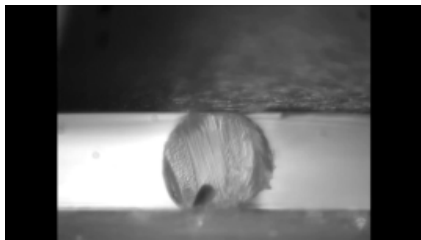
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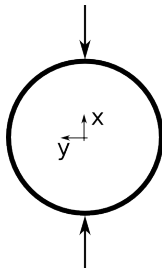
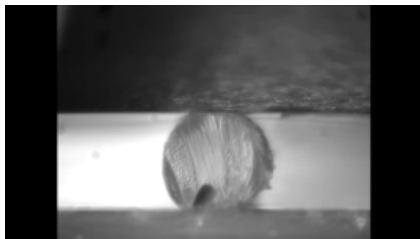
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- Resolution similar to global approaches
- The dofs have a clear mechanical meaning

Integrated approach : another example



Integrated approach : another example



Assumptions : uniform (transversely) isotropic material, diametral loading, known geometry (position and radius R). PA11 fiber, diameter $37.3 \pm 1.26 \mu\text{m}$.

Kolosov-Muskhelishvili potentials, $\kappa = \frac{3-\nu}{1+\nu}$ for plane stress :

$$\begin{aligned} v_B = \frac{F}{4\pi\mu} \left[\kappa \left(\frac{z}{R} - \log \left(\frac{R+z}{R-z} \right) \right) \right. \\ \left. - \frac{z}{R} + \frac{R+z}{R+\bar{z}} - \frac{R-z}{R-\bar{z}} - \log \left(\frac{R+\bar{z}}{R-\bar{z}} \right) \right] \end{aligned}$$

Integrated approach

Choice for the functions basis :

- ▶ 3 rigid body motions :

$$\mathbf{v}_t = v_1 + iv_2 \quad , \quad \mathbf{v}_r = v_3 i z$$



$$\frac{z}{R} - \log \left(\frac{R+z}{R-z} \right)$$

Associated dof is $\frac{\kappa F}{4\pi\mu}$

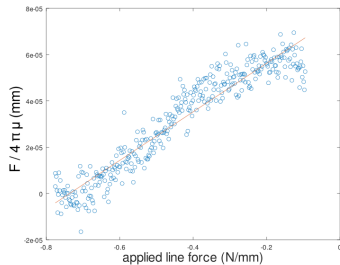


$$-\frac{z}{R} + \frac{R+z}{R+\bar{z}} - \frac{R-z}{R-\bar{z}} - \log \left(\frac{R+\bar{z}}{R-\bar{z}} \right)$$

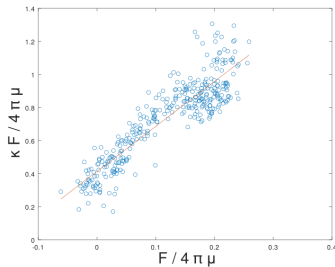
Associated dof is $\frac{F}{4\pi\mu}$

[Hild & Roux *et al.*, 2006]

Integrated approach



$$E_T = 1620 \text{ MPa}$$



$$\nu_{TT} = 0.07$$

[Placet *et al.*, 2020]

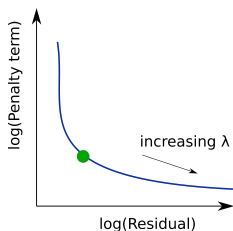
FE-like displacement field with mechanical regularization

Minimize

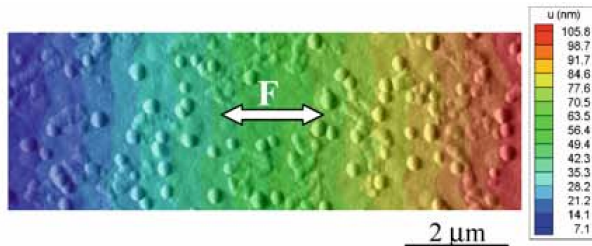
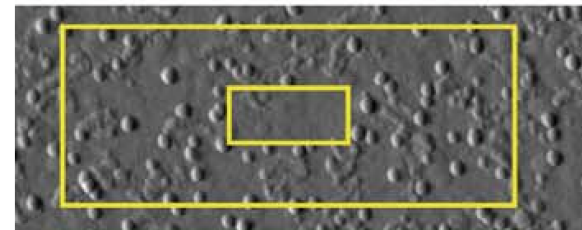
$$J_R(\mathbf{u}) = \int (f_{ref}(\mathbf{x} + \mathbf{u}) - f_{def}(\mathbf{x}))^2 dx + \frac{\lambda}{2} \|\mathbf{u}\|_R^2$$

- ▶ Many possible choices for $\|\cdot\|_R^2$:
 - ▶ Identity : the displacement is forced to be as small as possible
 - ▶ Stiffness operator : the displacement is forced to satisfy equilibrium conditions
 - ▶ ...

- ▶ Choosing λ :
 - ▶ Brute force : L-curve analysis
 - ▶ Spectral analysis : λ defines a characteristic length



AFM topographies

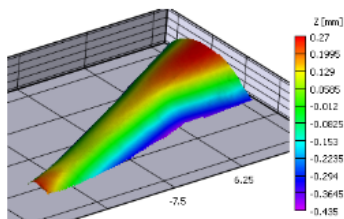
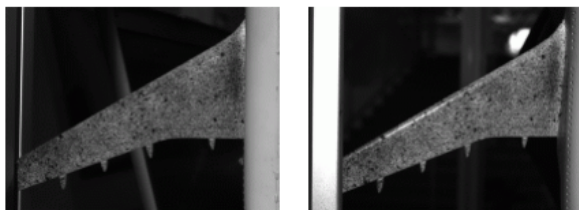


Stereo-correlation

2 cameras, for the 3D displacement of a surface :

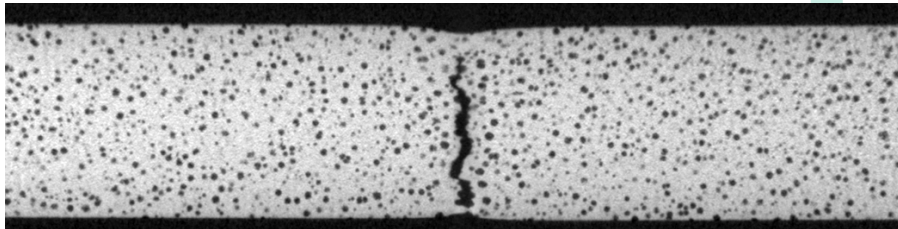


Stereo-correlation



Kinematic should be expressed in a **physical frame** (instead of the sensor's frame)

X-ray tomography

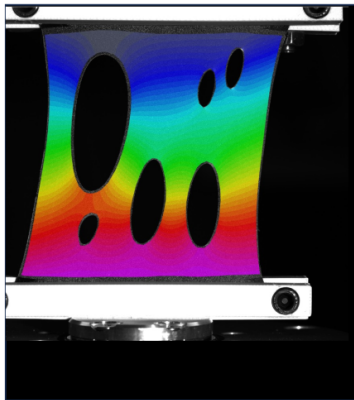


Digital volume correlation



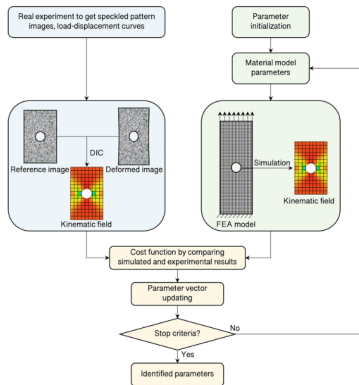
Purpose

- ▶ Measure boundary conditions
 - ▶ Soft machine
 - ▶ Pillars : substrate sink-in
 - ▶ Pillars : contact/friction
- ▶ Heterogeneous tests
 - ▶ tapered pillars
 - ▶ non uniform materials
- ▶ Strain localization
- ▶ Stress gauge
- ▶ Identify/ build constitutive laws
- ▶ ...



Identification

- ▶ Full-field measurements are very rich... which should be understood through mechanical equations.
- ▶ Many identification methods... Among which finite element model updating is probably the easiest to understand :



Interesting beyond elasticity (integrated approaches)

Beyond DIC/DVC



The global approach is a very general framework which can be used to describe any interesting image modification :

- ▶ Distorsions
- ▶ Time-integrated approaches : typically for harmonic excitation
- ▶ EBSD calibration
- ▶ crystal motion
- ▶ 3D topography reconstruction in a SEM
- ▶ Combining imaging modalities



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- ▶ Combining imaging modalities
- ▶ Stereo-correlation for pillars?
- ▶ A bridge between DDD simulations and TEM images?

Data management



The **R3XA** standard :

- ▶ Not another file format : more like a metadata format, relying on existing formats
- ▶ Sharable : so there is some structure to comply with
- ▶ Open and scalable : things may evolve...
- ▶ Sustainable : based on a wide developers community
- ▶ Easy to use

