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Inverse identification of a localized complex modulus in a cantilever beam using VPINNs and localized test functions

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Abstract. We address the inverse localization of a hidden dissipative treatment from sparse frequency-domain measurements available on a restricted observation window. A variational physics-informed neural network (VPINN) is trained on complex displacement data generated by Finite Element simulations for a 1 m clamped-free Euler-Bernoulli beam with a short viscoelastic patch bonded to one face of the beam, while the network is not informed of the material discontinuity. The governing equation is enforced on the observation window through a Petrov-Galerkin weak residual, reducing the derivative order compared with strong-form PINNs and improving stability under partial observations. To promote identifiability on the window, the unknown complex modulus is represented by a compact trainable piecewise-plateau (“bubble”) parameterization with learned transition locations and amplitudes. The contribution is a focused comparison of VPINN test-function families for inverse identification under hidden heterogeneity. On a representative case at 3490 Hz, localized RBF-type tests yield stronger identification proxies for the loss modulus than global Fourier/Legendre tests, despite similarly accurate displacement fits on the observation points.

Keywords: VPINN; inverse identification; Euler-Bernoulli beam; viscoelastic patch; complex modulus; test functions

1. INTRODUCTION

Viscoelastic surface-bonded patches are widely used to increase damping in slender structures, yet localizing and characterizing their dissipative effect from limited vibration measurements remains challenging in practice. Physics-informed neural networks (PINNs) provide a hybrid framework combining observational data with governing equations (Raissi *et al.*, 2019). In structural dynamics, Teloli *et al.* proposed a PINN-based framework for parameter identification in beam-like structures, embedding the Euler-Bernoulli equation in the loss and identifying Young’s modulus parameters from vibration responses (Teloli *et al.*, 2025). While this constitutes an important step toward equation-constrained learning, typical limitations remain when targeting localized dissipative treatments: identification primarily involves real-valued parameters, spatial variability is commonly handled through a prescribed segmentation, and sufficiently rich instrumentation is generally required to constrain the inverse problem. These aspects become critical when the heterogeneity is hidden and when measurements are available only on a restricted window. Variational physics-informed neural networks (VPINNs) mitigate several computational and stability issues associated with strong-form enforcement of high-order operators by imposing the governing equation through a weak (Petrov-Galerkin) residual (Kharazmi *et al.*, 2019, 2021). Integration by parts reduces the derivative order acting on the trial network. In addition, the choice of the test space and the quadrature rule has a direct impact on VPINN accuracy and stability (Berrone *et al.*, 2022). This observation is particularly relevant for inverse identification under partial observations, where the physics constraint is enforced only on the observation window and the excitation may lie outside the enforced domain. The present work targets inverse identification of a localized complex Young’s modulus in a clamped-free beam from complex displacement data available only on an observation window. The excitation is located upstream of this window, leading to a homogeneous residual on the enforced domain; consequently, the restriction of the response depends on unknown boundary traces at the window entrance. In this setting, identifiability relies on combining data anchoring, weak-form regularization, and an explicit low-dimensional structural prior on the modulus field. A trainable piecewise-plateau parameterization, referred to hereafter as the *bubble* representation (see Eq. (6)), is therefore adopted to model the complex Young’s modulus on the observation window. The transition locations and plateau amplitudes are learned from data, enabling the recovery of a short dissipative region without prescribing its boundaries, location, or width. A key outcome is the strong sensitivity of inverse modulus



identification to the VPINN test space. Several test-function families are compared under an identical training protocol, ranging from global bases (Fourier and Legendre) to localized radial basis function (RBF) tests, including compactly supported Wendland kernels. On a representative case at 3490 Hz, localized test functions yield stronger identification proxies for the loss modulus than global test functions, even when all configurations produce similarly accurate fits of the complex displacement on the observation points.

2. PROBLEM SETUP

We consider a 1 m clamped–free Euler–Bernoulli beam excited by a harmonic point load of magnitude $F = 1$ N applied at $x_F = 0.25$ m, as shown in Fig. 1. A short viscoelastic patch is present between $x = 0.7$ m and $x = 0.8$ m, leading to a piecewise heterogeneous constitutive response (steel/viscoelastic/steel). In the Finite Element (FE) reference model, the viscoelastic treatment is represented through an equivalent complex Young’s modulus following a Ross–Kerwin–Ungar (RKU) type modelling (Ross *et al.*, 1959); the true segmentation is *not* provided to the network. We denote the complex modulus by $\tilde{E}(x) = E_r(x) + iE_i(x)$, where E_r is the storage modulus and E_i is the loss modulus. Complex displacement data are available only on the observation window $\Omega_x = [x_{\min}, x_{\max}] = [0.5 \text{ m}, 0.995 \text{ m}]$, at $N_{\text{obs}} = 100$ points.

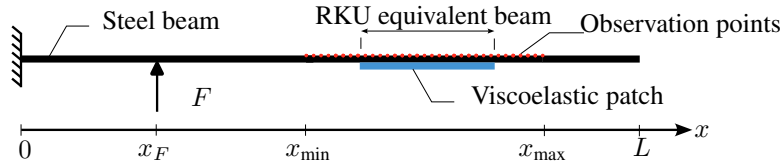


Figure 1: A clamped–free beam with a hidden viscoelastic patch.

Non-trivial identifiability. Since $x_F \notin \Omega_x$, the governing equation enforced on the observation window is homogeneous. Therefore, the restriction of the response to Ω_x depends on unknown boundary traces at $x = x_{\min}$, which encode the upstream excitation and the clamped condition at $x = 0$. Identifying $\tilde{E}(x)$ on Ω_x is consequently underdetermined without additional structure; the proposed approach combines data anchoring on the measured points, weak-form physics regularization on Ω_x , and an explicit low-dimensional prior for $\tilde{E}(x)$ on the window. This adds structure, but the solution remains non-unique.

3. METHOD

3.1 Governing equation and weak residual

In the frequency domain, the transverse displacement amplitude $U(x) \in \mathbb{C}$ of an Euler–Bernoulli beam satisfies (Timoshenko *et al.*, 1974) on the beam domain $x \in (0, L)$,

$$\frac{d^2}{dx^2} \left(\tilde{E}(x) I \frac{d^2 U}{dx^2} \right) - \rho S \omega^2 U(x) = -F \delta(x - x_F), \quad (1)$$

where $\tilde{E}(x) = E_r(x) + iE_i(x)$ is the complex Young’s modulus, I is the second moment of area, ρ the density, S the cross-sectional area, $\omega = 2\pi f$ the angular frequency, and $\delta(\cdot)$ the Dirac distribution associated with the point load F applied at x_F . In the presented study, only measurements on the window $\Omega_x = [x_{\min}, x_{\max}]$ are used, and the weak formulation is enforced on Ω_x . Since $x_F \notin \Omega_x$, the forcing term does not contribute to residuals integrated over Ω_x . Let ϕ be a test function such that

$$\phi(x_{\min}) = \phi(x_{\max}) = 0, \quad \phi'(x_{\min}) = \phi'(x_{\max}) = 0. \quad (2)$$

Multiplying the restricted equation by ϕ and integrating over Ω_x , followed by two integrations by parts, yields the Petrov–Galerkin weak residual (Kharazmi *et al.*, 2019; Berrone *et al.*, 2022):

$$\mathcal{R}(U, \tilde{E}; \phi) := \int_{\Omega_x} \tilde{E}(x) I U''(x) \phi''(x) dx - \int_{\Omega_x} \rho S \omega^2 U(x) \phi(x) dx, \quad (3)$$

where we integrated by parts twice and used the boundary conditions on test functions. In VPINNs, $\mathcal{R}$ is evaluated for a finite set of test functions and driven to zero in a least-squares sense. To construct test functions satisfying Eq. (2), we introduce the normalized coordinate $s \in [0, 1]$ defined by $x = x_{\min} + Js$ with $J = x_{\max} - x_{\min}$, and set

$$\phi(s) = b(s) g(s), \quad b(s) = s^2(1 - s)^2, \quad (4)$$

where $g(s)$ is selected from a prescribed test-family (Fourier, Legendre, or RBF; see Section 3.4), and $b(s)$ enforces $\phi(0) = \phi(1) = \phi'(0) = \phi'(1) = 0$.

With $x = x_{\min} + Js$, we have $dx = J ds$ and, for any differentiable field v , $v_x = \frac{1}{J}v_s$ and $v_{xx} = \frac{1}{J^2}v_{ss}$. The discrete (approximated) residual corresponds to the case where the displacement field and the modulus are given by neural representations U_θ and $\tilde{E}_\eta$, respectively, and the integrals in Eq. (3) are approximated numerically. We therefore define the discrete residual

$$\mathcal{R}_h(U_\theta, \tilde{E}_\eta; \phi) := \int_{\Omega_x} \tilde{E}_\eta(x) I U_\theta''(x) \phi''(x) dx - \int_{\Omega_x} \rho S \omega^2 U_\theta(x) \phi(x) dx, \quad (5)$$

where both integrals are evaluated by Gauss–Legendre quadrature on $s \in [0, 1]$ (Berrone *et al.*, 2022).

3.2 Trainable plateau prior for the complex modulus on the window

The complex displacement is represented by a neural surrogate $U_\theta(x) \in \mathbb{C}$, while the unknown complex modulus on Ω_x is parameterized by three contiguous sigmoid plateaus (“bubbles”). Let

$$0 = e_0 < e_1 < e_2 < e_3 = 1$$

be an ordered set of coordinates in the normalized coordinate $s \in [0, 1]$, inducing the partition of $[0, 1]$ into three subintervals $[e_{j-1}, e_j]$, $1 \leq j \leq 3$, with learnable interior coordinates (e_1, e_2) . The j -th plateau function, $j = 1, 2, 3$, is defined as

$$B_j(s) = \sigma(k(s - e_{j-1})) - \sigma(k(s - e_j)), \quad (6)$$

where $k > 0$ controls the transition sharpness. The real and imaginary parts of the modulus are then expressed as

$$E_r(s) = E_{\text{scale}} \sum_{j=1}^3 a_j^{(r)} B_j(s), \quad E_i(s) = E_{\text{scale}} \sum_{j=1}^3 a_j^{(i)} B_j(s), \quad (7)$$

where E_{scale} is a fixed scaling constant and the plateau amplitudes $a_j^{(r)}$ and $a_j^{(i)}$ are trainable nonnegative coefficients controlling the storage and loss levels on each plateau. Nonnegativity is enforced by setting

$$a_j^{(r)} = 10^{\alpha_j^{(r)}}, \quad a_j^{(i)} = 10^{\alpha_j^{(i)}}, \quad (8)$$

with unconstrained trainable parameters $\alpha_j^{(r)}, \alpha_j^{(i)} \in \mathbb{R}$. The plateau *locations* are determined by the learned coordinates (e_1, e_2) in the normalized coordinate, which correspond to physical positions $x_j = x_{\min} + J e_j$ and satisfy $x_{\min} < x_1 < x_2 < x_{\max}$, while the plateau *levels* are determined by the learned amplitudes $(a_j^{(r)}, a_j^{(i)})$. Both sets of parameters are optimized jointly with the displacement network U_θ .

3.3 Loss and training protocol (high-level)

Let $\{x_i\}_{i=1}^{N_{\text{obs}}} \subset \Omega_x$ denote the observation points and $U^{\text{obs}}(x_i)$ the observed complex displacement. The training objective $\mathcal{L}$ combines a data mismatch term $\mathcal{L}_{\text{data}}$ and a VPINN physics term $\mathcal{L}_{\text{phys}}$:

$$\mathcal{L}(\theta, \eta) = \lambda_{\text{data}} \mathcal{L}_{\text{data}}(\theta) + \mathcal{L}_{\text{phys}}(\theta, \eta), \quad (9)$$

where $\lambda_{\text{data}} > 0$ balances data fitting and physics consistency. The data loss is the mean-squared error on the observed window:

$$\mathcal{L}_{\text{data}}(\theta) = \frac{1}{N_{\text{obs}}} \sum_{i=1}^{N_{\text{obs}}} |U_\theta(x_i) - U^{\text{obs}}(x_i)|^2, \quad (10)$$

and the physics loss is the mean-squared projected residual over a set of test functions $\{\phi_m\}_{m=1}^{N_t}$,

$$\mathcal{L}_{\text{phys}}(\theta, \eta) = \frac{1}{N_t} \sum_{m=1}^{N_t} |\mathcal{R}_h(U_\theta, \tilde{E}_\eta; \phi_m)|^2, \quad (11)$$

with $\mathcal{R}_h$ defined by Eq. (5) and N_t denoting the number of test functions. The displacement is normalized for the data term, while the physics residual is evaluated in physical units.

Network architectures. U_θ is represented by a fully-connected network with input x (mapped to $s \in [0, 1]$) and two outputs (U_r, U_i) , using either three hidden layers of width 64 with tanh activations or a SIREN architecture with sinusoidal activations (Sitzmann *et al.*, 2020). The modulus $\tilde{E}_\eta$ is parameterized by the bubble model in Eqs. (6)–(7).

3.4 Five test-function families

The VPINN residual is evaluated against a finite set of test functions $\{\phi_m\}_{m=1}^{N_t}$ defined on the normalized coordinate $s \in [0, 1]$. As stated in Eq. (4), each test function is constructed as $\phi_m(s) = b(s) g_m(s)$, where the boundary envelope $b(s)$ enforces vanishing values and slopes at $s = 0$ and $s = 1$. Function $g_m(s)$ is chosen from one of the following five families (Tancik *et al.*, 2020; Trefethen, 2000; Fasshauer, 2007; Buhmann, 2003; Wendland, 1995, 2004): Fourier, Legendre, Gaussian RBF, inverse multiquadratic (IMQ) RBF, and compactly supported Wendland C^2 RBF.

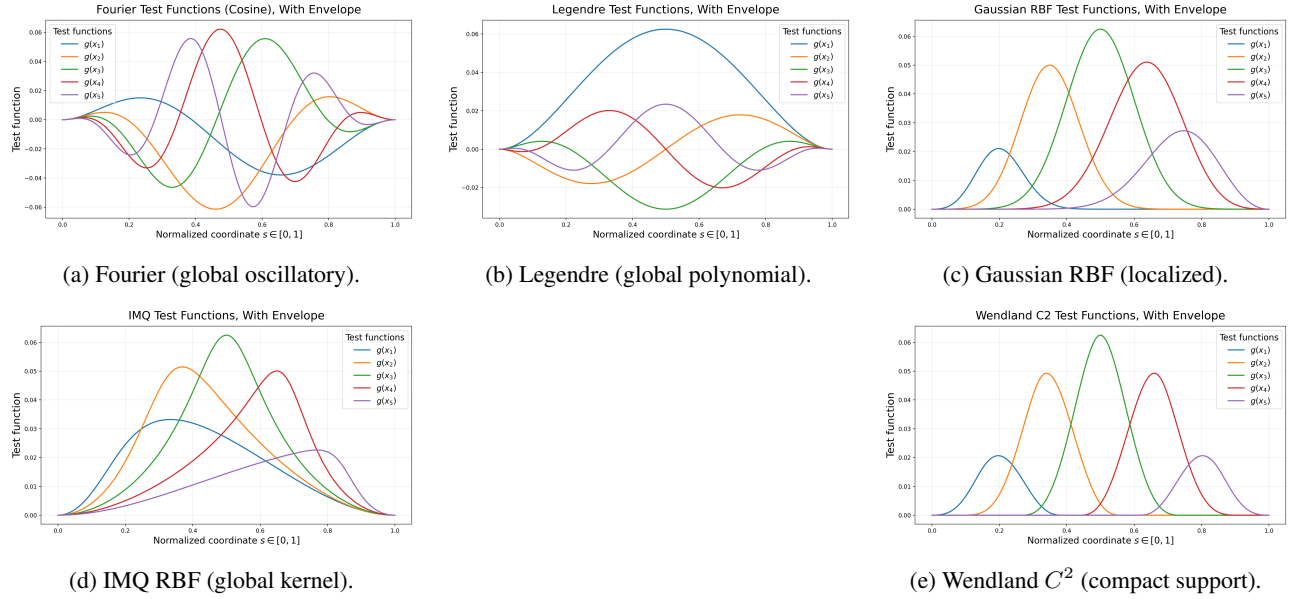


Figure 2: Representative test functions used in the VPINN residual (examples on $s \in [0, 1]$), illustrating the contrast between global bases and localized/compact kernels.

As illustrated in Fig. 2(a,b), Fourier and Legendre define global test functions, the RBF families in Fig. 2(c–e) provide localized (Gaussian), globally supported smooth (IMQ), or compactly supported (Wendland) kernels.

4. NUMERICAL TESTS

4.1 Numerical setup

Physical parameters. The beam is prismatic with a rectangular cross-section of width $b = 0.02$ m and thickness $e = 0.002$ m, hence $S = be = 4 \times 10^{-5}$ m² and $I = 1.3333 \times 10^{-11}$ m⁴. The density is $\rho = 7080$ kg m⁻³. A FE reference model is used to generate data observations. The FE reference model uses the complex moduli

$$\tilde{E}_{\text{steel}} = 2.05 \times 10^{11} + i 1.1275 \times 10^9 \text{ Pa}, \quad \tilde{E}_{\text{visco}} = 2.05 \times 10^{11} + i 1.025 \times 10^{11} \text{ Pa}, \quad (12)$$

where $\tilde{E}_{\text{visco}}$ denotes the equivalent Ross–Kerwin–Ungar (RKU) modulus used in the patched region of the FE model.

Numerical settings. In the bubble model (6)–(7), the scaling factor $E_{\text{scale}} = 10^{11}$ is used for numerical conditioning, and amplitudes are mapped back to physical units through Eq. (7). The observation window $\Omega_x = [x_{\min}, x_{\max}]$, the VPINN weak residual and quadrature, and the test-function construction are fixed as described in Sections 3.23.4. In particular, all test functions are multiplied by the boundary envelope of Eq. (4), and projected residuals are evaluated by Gauss–Legendre quadrature.

Run configuration. All runs use the same dataset at $f = 3490$ Hz, with $N_{\text{obs}} = 100$ complex displacement observations on Ω_x , and are trained for 150 000 iterations under identical optimization settings. The only varying component across runs is the test-function family used in the VPINN residual (Fourier, Legendre, Gaussian RBF, IMQ, Wendland C^2). For the predicted complex modulus $\tilde{E}_\eta$, we observe in the next subsections that E_r remains close to the steel value in the adopted

Table 1: Quantitative comparison at $f = 3490$ Hz — **Best-loss snapshot**. Metrics are evaluated at the iterate achieving the minimum training objective over 150 000 iterations. Best (lowest) values are highlighted in bold.

Family	Best loss	Iter	$\varepsilon_{\text{coordinates}}$ (%)	ε_U (%)	$\varepsilon(E_r)$ (%)	$\varepsilon(E_i^{\text{patch}})$ (%)	$\varepsilon(E_i^{\text{steel}})$ (%)
WendlandC2	9.05e-6	143930	0.0960	0.409	0.928	1.500	61.29
GaussianRBF	8.57e-6	144941	0.103	0.403	1.112	1.451	69.91
Legendre	3.03e-6	146014	0.0669	0.199	0.558	1.718	84.23
Fourier	4.58e-6	143902	0.0347	0.225	1.309	2.497	119.3
IMQ	2.50e-5	149448	0.162	0.700	0.918	1.746	123.4

equivalent model, thus the inverse identification challenge is mostly constituted by the recovery of the loss modulus $E_i(x)$ and its localization on the observation window.

4.2 Quantitative comparison

To support a comparison across test spaces, metrics are reported at two training instants. Firstly, the *best-loss snapshot* selects, for each family, the iterate achieving the minimal training objective over the full 150 000 iterations. This selection is used to retain parameter sets that are most consistent with the implemented VPINN objective and to avoid conclusions based on late-iteration drift. Secondly, the *first-passage* iteration below a reference threshold 10^{-5} provides a simple convergence-speed indicator that can be extracted from logs. Tables 1 and 2 summarize the best-loss metrics and the first-passage information, respectively.

In both tables, ε_U denotes the relative reconstruction error of the complex displacement on the N_{obs} observation points (computed from the ℓ_2 norm of the complex residual normalized by the ℓ_2 norm of the observations). The edge error $\varepsilon_{\text{coordinates}}$ quantifies the average absolute error on the patch entrance/exit locations with respect to the true positions $x = 0.7$ m and $x = 0.8$ m (reported as a percentage of the beam length). The modulus errors $\varepsilon(E_r)$ and $\varepsilon(E_i^{\text{patch}})$ are computed from the relative error between the estimated and reference *mean* storage and loss moduli inside the patched region. Finally, $\varepsilon(E_i^{\text{steel}})$ measures the relative error on the mean background (steel) loss modulus, computed over the observation window excluding the patched interval.

At the best-loss snapshot (Table 1), all families reconstruct the complex displacement on the observation points with relative errors below 0.7%, and recover the patch entrance/exit locations with edge errors below 0.2%. Hence, for detecting the *presence* and *location* of a localized dissipative treatment on Ω_x , the five test-function families perform comparably in this representative case. Differences become more apparent when inspecting constitutive quantities. The storage modulus inside the patch is recovered within about 1% for all families, with Legendre providing the smallest error (about 0.6%). For the loss modulus inside the patch, the relative error remains low across families (from about 1.45% for Gaussian RBF to about 2.5% for Fourier), indicating that the dissipative level in the patched zone is consistently identified. The main discrepancy arises in the background loss modulus (steel region): the relative error on E_i^{steel} remains high for all families, although Wendland and Gaussian RBF yield the most acceptable values among the tested options. This indicates that, while patch localization and patched dissipation are reliably captured, accurately recovering the smaller background loss modulus is still challenging under the present partial-observation setting and modelling assumptions.

Table 2 complements Table 1 by reporting the first iterate at which the loss falls below 10^{-5} , together with synchronized error metrics at that iterate. The ranking in convergence speed differs from the best-loss ranking: Fourier reaches the threshold fastest, while Legendre crosses later and IMQ does not reach the threshold within 150 000 iterations. Importantly, early threshold crossing does not guarantee good constitutive recovery: although Fourier converges rapidly in terms of loss, its loss-modulus errors (notably the background E_i^{steel}) remain high at that stage, whereas localized kernels (Gaussian RBF and Wendland C^2) provide more balanced loss-modulus estimates once the threshold is met. This confirms that both convergence-speed indicators and constitutive metrics are needed to compare test spaces under partial observations. Moreover, this simple stop criterium of the loss threshold shows that a more refined stop criterium is needed to ensure a satisfactory global modulus identification.

In conclusion, Wendland C^2 and Gaussian RBF provide the most balanced compromise between displacement fit, edge localization, and loss-modulus identification in this configuration, even though the background-loss error is not yet satisfactory. Moreover, to complete the analysis, a robustness study with respect to frequency selection is required; preliminary tests on nearby frequencies indicate that RBF-type test spaces are promising, motivating a broader multi-frequency assessment.

Table 2: Quantitative comparison at $f = 3490$ Hz — **First passage below 10^{-5}** . The table reports the first iterate where the loss drops below 10^{-5} (convergence speed). For each column the lowest values are highlighted in bold. Error metrics are evaluated on the synchronized snapshot at that iterate.

Family	Loss ($< 10^{-5}$)	Iter	$\varepsilon_{\text{coordinates}}$ (%)	ε_U (%)	$\varepsilon(E_r)$ (%)	$\varepsilon(E_i^{\text{patch}})$ (%)	$\varepsilon(E_i^{\text{steel}})$ (%)
WendlandC2	9.98e-6	137513	0.17	0.43	1.28	1.71	74.80
GaussianRBF	9.98e-6	133798	0.17	0.43	1.17	1.49	54.72
Legendre	9.87e-6	111880	0.21	0.28	1.46	1.36	107.52
IMQ	—	—	—	—	—	—	—
Fourier	9.91e-6	65941	0.04	0.34	1.65	3.03	114.10

4.3 Qualitative comparison

All test-function families provide a similar displacement reconstruction on Ω_x , with comparable error levels on the observation points. For conciseness, Fig. 3 illustrates a representative reconstruction obtained with Gaussian RBF tests only. Figure 4 compares the recovered modulus profiles for all families. While Table 1 captures performance through scalar metrics evaluated from averaged quantities and selected iterates, the modulus curves provide complementary insight into the *shape* of the recovered fields, which is not fully summarized by averaged errors. In particular, different families may produce similar best-loss indicators while exhibiting distinct transition sharpness, plateau smoothness, or small oscillations in $E_i(x)$ outside the patch, which impacts physical interpretability and model selection. This motivates enriching the stopping and selection criteria beyond the raw VPINN training loss. Since the loss value is test-space dependent, a practical strategy is to evaluate all trained models *a posteriori* on a common fixed reference test bank and quadrature rule, yielding a standardized residual score that is comparable across families. Complementary indicators can also be considered, such as the stability of the learned coordinates (e_j) and amplitudes under test resampling (or random seeds), and the consistency of the recovered loss-modulus baseline in regions expected to be homogeneous. These perspectives guide ongoing work toward robust model-selection criteria for inverse identification under partial observations.

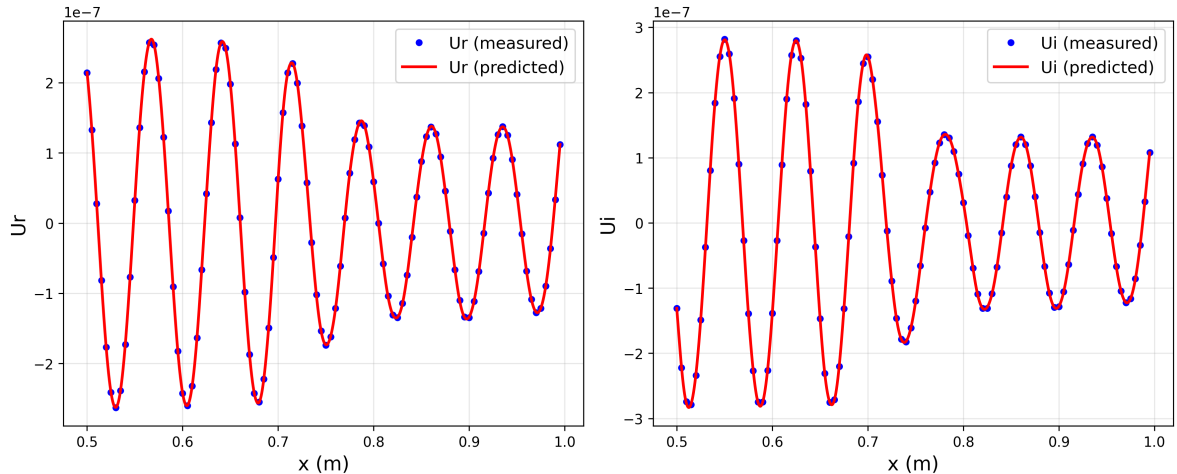
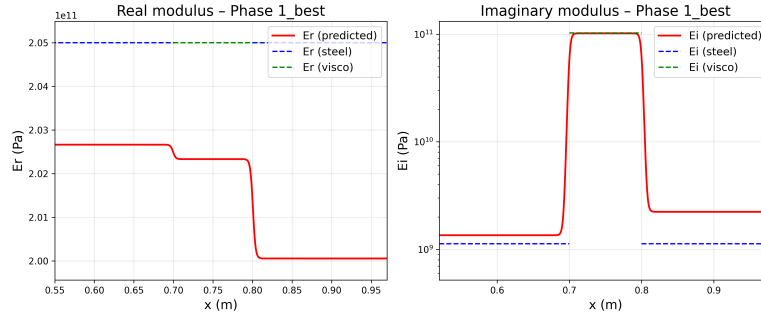
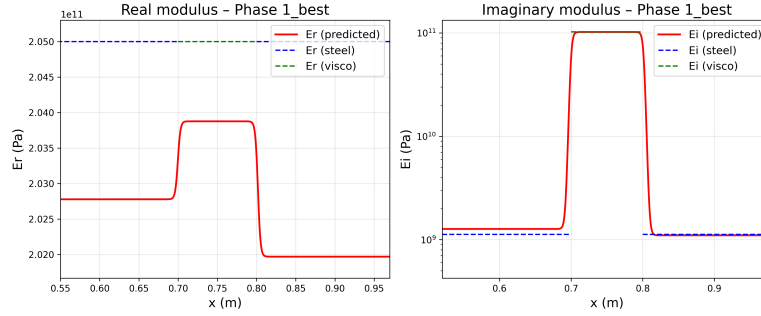


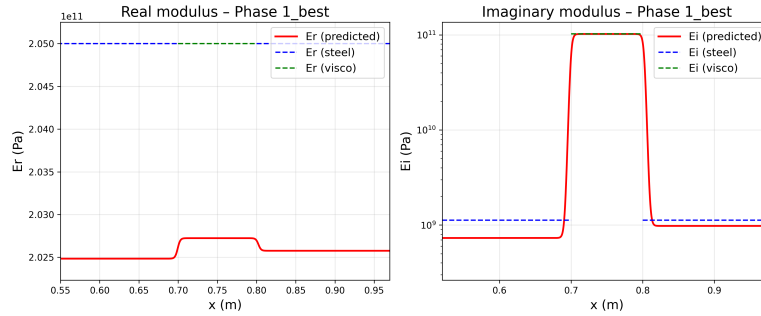
Figure 3: Displacement reconstruction on Ω_x at 3490 Hz (Gaussian RBF tests). Discrete markers: FE observations at the $N_{\text{obs}} = 100$ measurement locations for U_r and U_i . Continuous curves: VPINN predictions $U_{r,\theta}(x)$ and $U_{i,\theta}(x)$ evaluated on a dense grid.



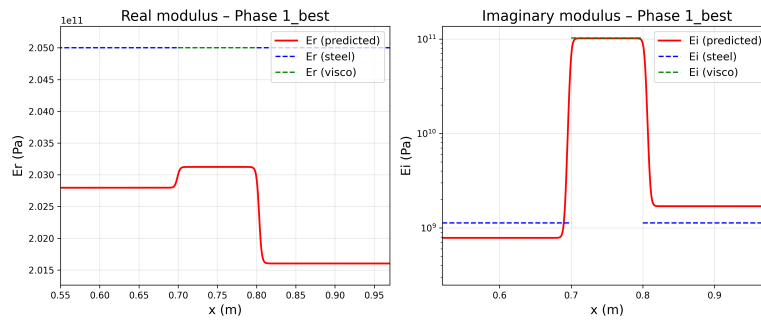
(a) Fourier (global).



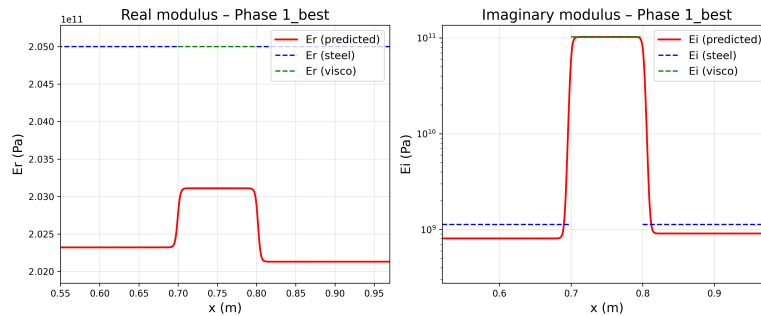
(b) Legendre (global).



(c) Gaussian RBF (local).



(d) IMQ (global RBF).



(e) Wendland C^2 (compact RBF).

Figure 4: Recovered complex modulus $\tilde{E}(x) = E_r(x) + iE_i(x)$ on Ω_x at 3490 Hz. For each family the real modulus E_r (left plot) and the imaginary modulus E_i (right plot) are displayed. The loss modulus E_i is the discriminative quantity for patch localization in the adopted equivalent model.

5. CONCLUSION

This study introduces a VPINN-based inverse strategy to localize and quantify a hidden dissipative treatment in a clamped–free Euler–Bernoulli beam from sparse, complex-valued frequency-response measurements restricted to an observation window. By enforcing the governing equation through a Petrov–Galerkin weak residual on the window, the approach reduces the derivative order acting on the trial network and yields stable training in a partial-observation setting where the excitation lies outside the enforced domain and the physics residual is homogeneous. Identifiability is promoted by the bubble model model (6)–(7) for the complex modulus, which captures a short localized region without prescribing its boundaries.

A focused comparison of five VPINN test-function families highlights that the test space is a not a decisive modelling choice for inverse identification. On the representative configuration at 3490 Hz, all families achieve similarly accurate reconstruction of the complex displacement on the observation points and comparable localization of the patch coordinates, yet they lead to markedly different constitutive estimates, especially for the background loss modulus. Global polynomial/oscillatory tests (Legendre/Fourier) can minimize the training objective and provide excellent displacement fits, whereas localized and compactly supported kernels (Gaussian and Wendland C^2) produce more informative dissipation-localization signatures and a more balanced recovery of loss-modulus levels under the same training protocol. These observations indicate that the values of the different VPINN losses $\mathcal{L}$ with respect to each family of test functions are not directly comparable and that inverse results should be interpreted using test-space-invariant criteria, combining data-fit accuracy with standardized residual evaluations on a common reference test bank and physically motivated localization proxies.

Future work will extend the analysis to multi-frequency identification and multi-seed robustness, and will refine model-selection and stopping criteria based on standardized residuals and parameter stability, with the objective of improving the recovery of low background damping while preserving reliable localization of localized dissipation.

6. ACKNOWLEDGEMENTS

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