



EURODINAME III - An International Symposium on Dynamic Problems of Mechanics
June 8-11, 2026 - Giens Peninsula, FR



EURODINAME-2026-13682

Robust design of shunted and active piezo metamaterial for cloaking against flexural waves

Grégoire Pizon

Morvan Ouisse

Université Marie et Louis Pasteur, Supmicrotech-ENSM, CNRS, Institut FEMTO-ST, F-25000 Besançon, France
gregoire.pizon@femto-st.fr, morvan.ouisse@femto-st.fr

Scott Cogan

Fondation FC'INNOV, F-25000 Besançon, France
scott.cogan@univ-fcomte.fr

Manuel Collet

CNRS, Ecole Centrale de Lyon, ENTPE, LTDS, UMR5513, 69134 Ecully, France
manuel.collet@ec-lyon.fr

Abstract. This paper proposes a robust active cloaking strategy for flexural waves in thin plates using piezoelectric patches combined with negative-capacitance shunts and voltage-controlled actuation. The cloaking domain surrounding a hole is divided into two regions: an absorbing region facing the incident wave, designed to attenuate it, and an emitting region dedicated to reconstructing the missing wavefront. While negative-capacitance shunting enables wave attenuation, its tuning often drives the system close to instability, creating a trade-off between performance and stability. To address this issue, a robustness analysis is performed on the optimized cloaking configuration obtained via a finite element model. Finally, a novel performance-retainment-oriented design framework based on Info-Gap theory is introduced, allowing performance degradation to be explicitly incorporated as a design specification.

Keywords: Uncertainty, metamaterial, cloaking, shunted piezos, flexural waves

1. INTRODUCTION

Cloaking is the ability of a structure to avoid waves interaction with a specific internal subdomain. A classical academic example of such a control strategy consists in tailoring the material properties around a hole in a thin plate so that a plane wave remains planar after passing across the hole.

We propose a cloaking strategy inspired by Li *et al.* (2021), consisting of dividing the cloaking domain surrounding the hole into two distinct regions: an absorbing region located upstream of the incident waves and an emitting region placed downstream to reconstruct the missing wavefront. The absorbing region is implemented using piezoelectric patches shunted with resistive and negative-capacitance circuits for waves total absorption, while the emitting region relies on voltage-controlled piezoelectric patches for waves rebuilding.

Piezoelectric patches shunted with passive or semi-active electrical circuits have been extensively investigated for vibration attenuation and wave control (Thomas *et al.* (2012)). In particular, negative-capacitance (NC) shunts have demonstrated strong potential for broadband attenuation and bandgap enhancement in metamaterials (Yi and Collet (2021); Tateo *et al.* (2016)). However, the tuning process of NC shunts often operates close to unstable modal regions (Berardengo *et al.* (2018); Collet *et al.* (2011)), leading to a trade-off between stability and performance.

To quantify this trade-off, we need to understand the interaction between the operational and manufacturing environments—such as industrial conditions—and the targeted configuration. Thus we have to assess the robustness of the structure. The influence of parametric uncertainties on shunted piezoelectric systems has received limited attention. Although some studies have investigated uncertainty propagation and sensitivity in vibration mitigation problems (Trindade *et al.* (2020); Aloui *et al.* (2019)), robustness issues remain largely unexplored in the specific context of active cloaking strategies combining absorbing and emitting regions.



Therefore, we propose to investigate the influence of uncertainties on the cloaking performance of incident flexural waves within our two-region strategy. The starting point of the robust design approach adopted here is the Info-Gap framework (Ben-Haim (2004)). We also aim to extend this methodology toward a novel framework in which performance degradation is treated explicitly as a design specification.

In this work, we first present the finite element (FEM) model of the structure. We then report the optimization results that serve as the basis for the robustness analysis. Finally, we introduce a novel robust design methodology in which performance loss is explicitly incorporated as a design specification.

2. NUMERICAL MODEL OF THE STRUCTURE

The numerical model is developed using COMSOL 6.2 and symmetrised to reduce computational time. The three-dimensional finite element model (FEM) is presented in Fig. 1:

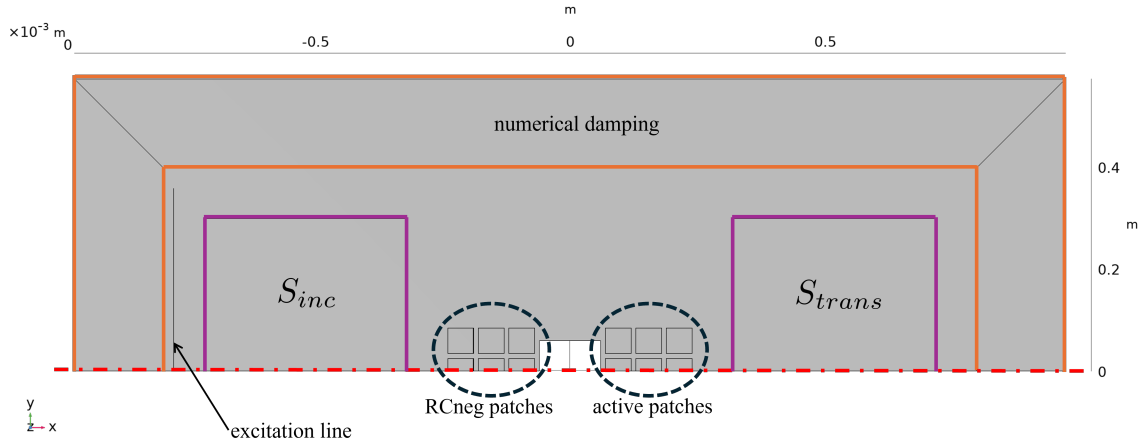


Figure 1. Schematic of the symmetrised numerical FEM model

The main structure consists of a half aluminum plate containing a fixed square hole at its center. Numerical damping is applied at the non-symmetrized boundaries in order to emulate an infinite plate. A 4000 Hz excitation propagates from left to right, originating from the excitation line.

The shunted piezoelectric patches are designed to absorb the incident excitation and thereby prevent wave scattering, while the active patches located on the opposite side of the hole generate the missing portion of the wavefront. Both effects are quantified using a cloaking quality criterion based on the plate's transverse displacement w defined as follows:

$$e = \frac{1}{A_S} \int_S \left| \frac{w - w_{ref}}{w_{ref}} \right| dS \quad (1)$$

with $S = S_{inc} \cup S_{trans}$, A_S its area and w_{ref} a reference field calculated for the same excitation but without patches and without the hole.

The absorbing region is located on one side of the hole, upstream of the incoming flexural waves. It consists of three patches arranged in the propagation direction, allowing the absorbing performance to be tuned through variations of the electrical parameters.

The active region follows the same philosophy. The modeling of both regions is detailed in the following sections.

2.1 Modelling the shunted patches

The shunted patches consist of a pair of piezoelectric ceramic patches bonded to each side of the plate. The effect of the electrical circuit is implemented through weak constraints applied to the electrodes, together with the appropriate Lagrange multiplier associated with the charge flowing in the circuit. Figure 2 presents the electrical circuit associated with each shunt.

The electrical parameters are shared across each column of the patch network. Thus, if we represent the network using a simple matrix coordinate system with m for the row and n for the column, the equation implemented in our FEM model is as follows:

$$V_{mn} = -\left(\frac{1}{C_n} + i\omega R_n\right)Q_{mn} \quad (2)$$

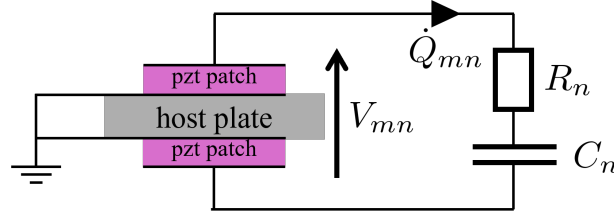


Figure 2. Schematic of the shunted piezo patches

2.2 Modelling the active part

The objective of the active region is to transfer the excitation information from the front of the hole to its back. Under the main assumptions of plane wave excitation and linear mechanical behavior, the effect of the free propagation of waves from one region to the other one is equivalent for waves to avoid interactions with the fixed hole. This simple propagation between regions can be modeled as a time delay τ between the normal incident displacement $w(\mathbf{X}_{inc}, t)$ and the transmitted normal displacement $w(\mathbf{X}_{trans}, t)$, thus:

$$w(\mathbf{X}_{trans}, t) = w(\mathbf{X}_{inc}, t - \tau) \quad (3)$$

However, the available information is limited to the electrical quantities generated by the shunted patches, and actuation can only be achieved through the voltage applied to the patches in the active region. Therefore, in steady state at pulsation ω , we have:

$$V_{trans}^{mn}(t) = A_n e^{-i\omega\tau_n} V_{inc}^{mn}(t) \quad (4)$$

The complex exponential allows the signal to be properly delayed, while the amplification factor A compensates for the amplitude loss induced by the piezoelectric coupling. The active patches are therefore voltage-controlled and bonded to the plate in the same manner as the absorbing ones.

3. NUMERICAL OPTIMIZATION

The optimization (minimization) is performed using the cloaking criterion defined in Eq. (1). The structure is excited with a harmonic force at 4000 Hz. Optimization relies on the twelve parameters introduced previously, namely the six electrical parameters (R_n and C_n) and the six active control parameters (A_n and τ_n).

It should be recalled that the structure is known to become unstable for values of C_n within the range $-2C_p^T = -75.2$ nF to $-2C_p^S = -36.6$ nF (Collet *et al.* (2011)), corresponding to twice the piezoelectric capacitance at constant stress and constant strain of a patch, respectively. Therefore, these stability constraints are explicitly incorporated into the design space.

The material properties and the bounds of the optimization variables are summarized in Tables 1, 2, and 3.

Parameter	Symbol	Value
Density	ρ_{pl}	2700 kg/m ³
Young's modulus	E	70 GPa
Poisson ratio	ν	0.33
Half width	l_{pl}	0.8 m
Length	L_{pl}	1.6 m
Thickness	t_{pl}	3 mm
Hole width	h_{pl}	0.12 m

Table 1. Parameters of the aluminium plate

Parameter	Symbol	Value
Density	ρ_{pz}	7750 kg/m ³
Elasticity matrix	$c_{11}^E = c_{22}^E$	1.2e11 Pa
	c_{33}^E	1.1e11 Pa
	$c_{12}^E = c_{23}^E = c_{13}^E$	7.5e10 Pa
	$c_{44}^E = c_{55}^E$	2.1e10 Pa
	c_{66}^E	2.3e10 Pa
Piezoelectric matrix	$d_{31} = d_{32}$	-1.7e-10 C/N
	d_{33}	3.7e-10 C/N
	$d_{24} = d_{15}$	5.8e-10 C/N
Relative permittivity matrix	$\varepsilon_{11}^E = \varepsilon_{22}^E$	1730
	ε_{33}^E	1700
Patch width	l_{pz}	5 cm
Patch thickness	t_{pz}	1 mm

Table 2. Parameters of the piezo patches

The optimization is performed using in-house developed software, and we obtained an optimized configuration with $e_{opt} = 0.349$. The corresponding displacement field is shown in Figure 3, together with the reference field and the holed plate, for which the criterion is $e_{hole} = 0.517$.

Parameter	Symbol	Design interval
Resistance of the shunts	R_n	[150 ; 6000] Ω
Capacity of the shunts	C_n	[-160 ; -75.2] nF
Amplification	A_n	[0 ; 10]
Phase of amplification	τ_n	[0 ; 2π]

Table 3. Parameters of optimization

We are generally satisfied with the results, particularly regarding the active region, as the missing portion of the displacement field is well reproduced. However, the displacement field in the incident region remains somewhat disturbed by the hole. Indeed, the negative capacitances did not converge to the expected values, which should be close to $-2C_p^T$, as the reduced stiffness allows for increased dissipation (Tateo *et al.* (2016)). We propose to retain this optimized configuration for the remainder of this work, while exploring alternative optimization strategies to investigate this discrepancy with the literature.

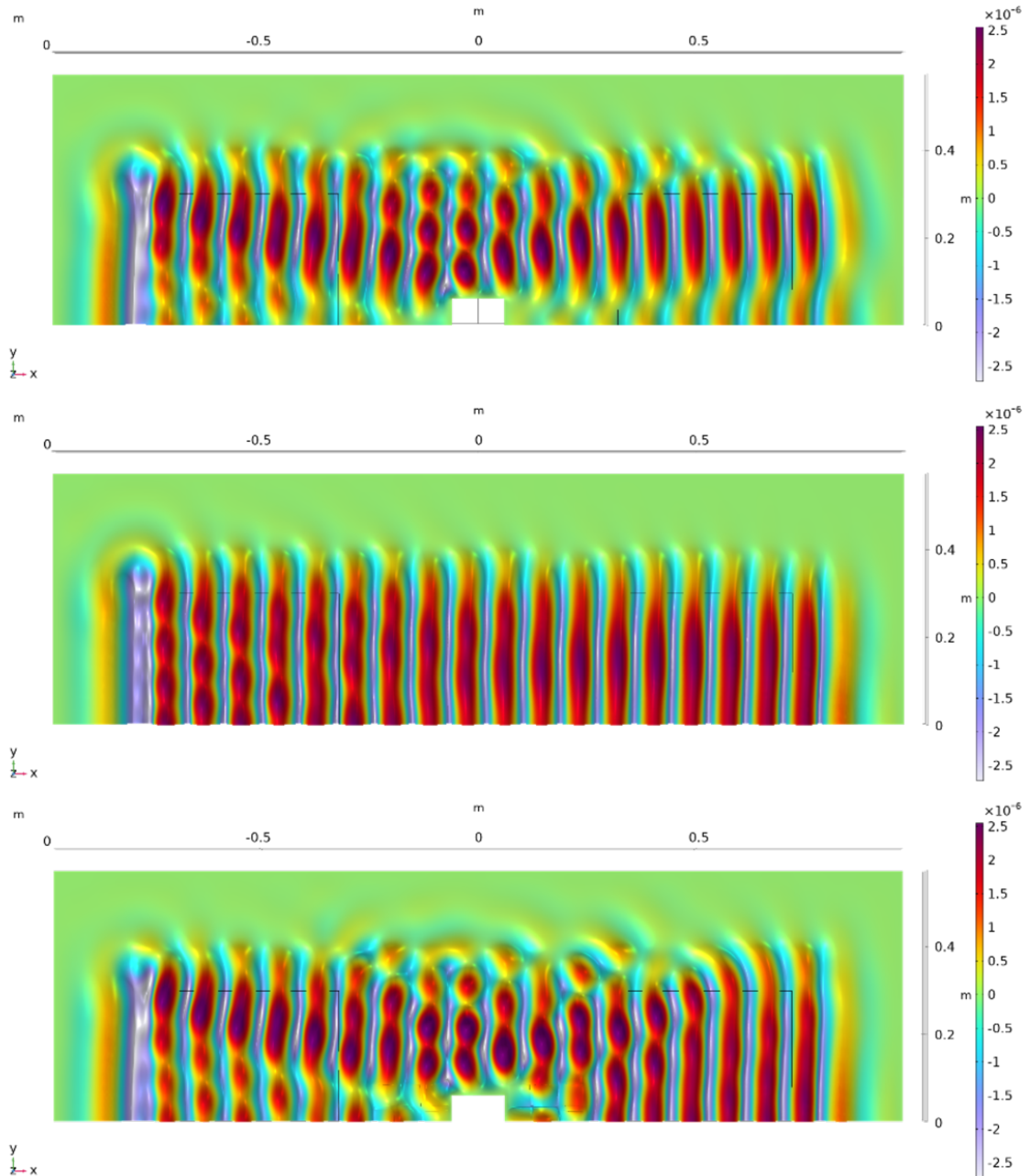


Figure 3. Normal displacement of the holed plate (top), reference (middle) and optimum (bottom)

4. ROBUST DESIGN

Robust design makes it possible to account for and quantify the influence of uncertainties in model input parameters, whether geometrical, material, environmental, or design-related, on system performance. It therefore provides a decisive criterion for selecting an optimal configuration by comparing the robustness of different candidate solutions. Robustness considerations can also be directly integrated into the optimization process.

A robust design methodology is generally structured around three main steps. Since robustness evaluation typically requires significant computational effort, the process begins with a qualitative or quantitative sensitivity analysis of system performance with respect to input variables. If certain parameters are shown to have negligible influence, the number of variables can be reduced, thereby decreasing the computational cost of the subsequent stage. The second step consists in evaluating the robustness of the selected solution using an uncertainty modeling approach appropriate to the system. Finally, a robust optimization of the retained configuration is performed in order to improve robustness or reliability, either by reducing performance sensitivity to input variability or by statistically decreasing the probability of system failure.

4.1 Sensitivity analysis

The sensitivity analysis (SA) to identify influential parameters is performed using the Morris method. Details of the method can be found in Saltelli *et al.* (2008). The main idea of the method is to perform multiple finite-difference evaluations in the parameter space to obtain the so-called elementary effects of each parameter, which approximate the local partial derivatives. The mean and standard deviation of these elementary effects are then analyzed to identify: influential linear parameters (high mean, low standard deviation), influential and potentially coupled parameters (high mean, high standard deviation), and negligible parameters (low mean, low standard deviation).

The sensitivity study is conducted around the previously obtained optimum (Table 4) and considers the geometric and material parameters of the structure, the optimization parameters, and the operating temperature, which affects the permittivity of the patches (modeled as a relative perturbation). The additional parameters and their nominal values are summarized in Table 5.

Parameter	Symbol	Nominal value
Resistance (column 1)	R_1	750 Ω
Capacity (column 1)	C_1	-98.1 nF
Resistance (column 2)	R_2	3680 Ω
Capacity (column 2)	C_2	-98.1 nF
Resistance (column 3)	R_3	4430 Ω
Capacity (column 3)	C_3	-160 nF
Amplification (column 1)	A_1	5.86
Phase (column 1)	τ_1	3.14
Amplification (column 2)	A_2	8.44
Phase (column 2)	τ_2	3.12
Amplification (column 3)	A_3	4.53
Phase (column 3)	τ_3	0

Table 4. Value of optimization parameters at optimum

Based on the nominal values of the parameters, we applied a relative perturbation of $\pm 5\%$ to define the explored parameter space. The results of the Morris analysis are shown in Figure 4, which presents the ranking based on the mean values of the elementary effects.

The Morris analysis highlights the influence of the plate parameters, particularly three of them—thickness, volumetric mass, and Young’s modulus—which are retained as influential parameters for the subsequent robustness analysis.

4.2 Robustness analysis

In this work, robustness modeling is investigated using the Info-Gap (IG) theory (Ben-Haim (2004)). The IG approach is a non-probabilistic method that enables the development of solutions resilient to deviations from a nominal reference model. It is particularly attractive in the early design stages, as it requires limited information about input variability.

Let $\mathbf{d}$ denote the vector that fully defines the state of the system, and let $\mathbf{u}$ be the subvector of $\mathbf{d}$ containing the influential parameters for which uncertainty is considered. The nominal estimate of these uncertain parameters is denoted by $\mathbf{u}_0$. The amount of uncertainty is modeled using the parameter $h \in [0, 1]$, called the horizon of uncertainty, which defines the uncertain space in which $\mathbf{u}$ resides: $U(h, \mathbf{u}_0)$. In our case, U corresponds to three uniform probability density functions (PDFs) associated with the three selected influential parameters. Uniform PDFs are chosen because they make

Parameter	Symbol	Nominal value
Density	ρ_{pl}	2700 kg/m ³
Young's modulus of the plate	E	70 GPa
Poisson ratio of the plate	ν	0.33
Thickness of the plate	t_{pl}	3 mm
Hole width	h_{pl}	0.12 m
Patch width	l_{pz}	5 cm
Patch thickness	t_{pz}	1 mm
Relative perturbation of the permittivity of the patch	$Reps_{pz}$	1

Table 5. Additionnal parameters of the SA

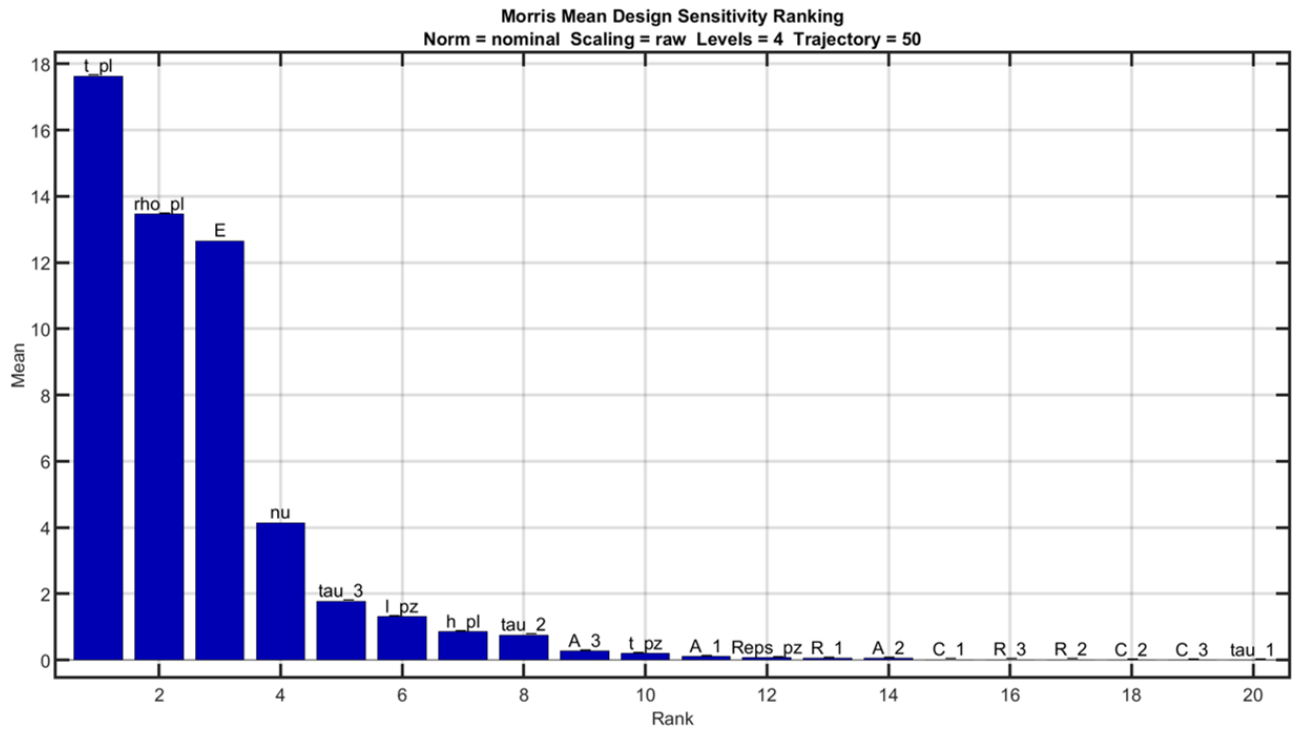


Figure 4. Morris ranking of parameters

minimal assumptions regarding probabilistic properties and can be easily translated to geometrical tolerances and relative variations.

For $h = 0$, $\mathbf{u}$ can only take the nominal values $\mathbf{u}_0$ (i.e., the bounds of the uniform PDFs collapse to the nominal values). At maximum uncertainty $h = 1$, $\mathbf{u}$ can take any value within the full range of the uniform PDFs.

Our optimization is formulated as a minimization of the criterion e , so the IG framework focuses on the quantity $e_w(h)$, defined as the worst-case performance over the horizon of uncertainty, and is defined as follows:

$$e_w(h) = \max_{\mathbf{u} \in U(h, \mathbf{u}_0)} e(\mathbf{d}, \mathbf{u}) \quad (5)$$

The robustness of the configuration defined by $\mathbf{d}$ is then assessed by examining the evolution of e_w with respect to h , which is referred to as the robustness curve. In our case, the uniform PDF of t_{pl} is constructed according to the european norm EN 485-4 concerning cold-rolled aluminum plates, giving $t_{pl} \in [2.85, 3.15]$ mm. For the two other parameters, we assume a relative variation of $\pm 3\%$ for E and $\pm 1\%$ for ρ .

The robustness of the optimized configuration is evaluated for values of $h \in [0; 0.05]$ using 5 increments. The associated e_w is determined using a pattern-search algorithm. The results are shown in Figure 5, together with the evolution of the uncertain parameters (compared to their nominal value).

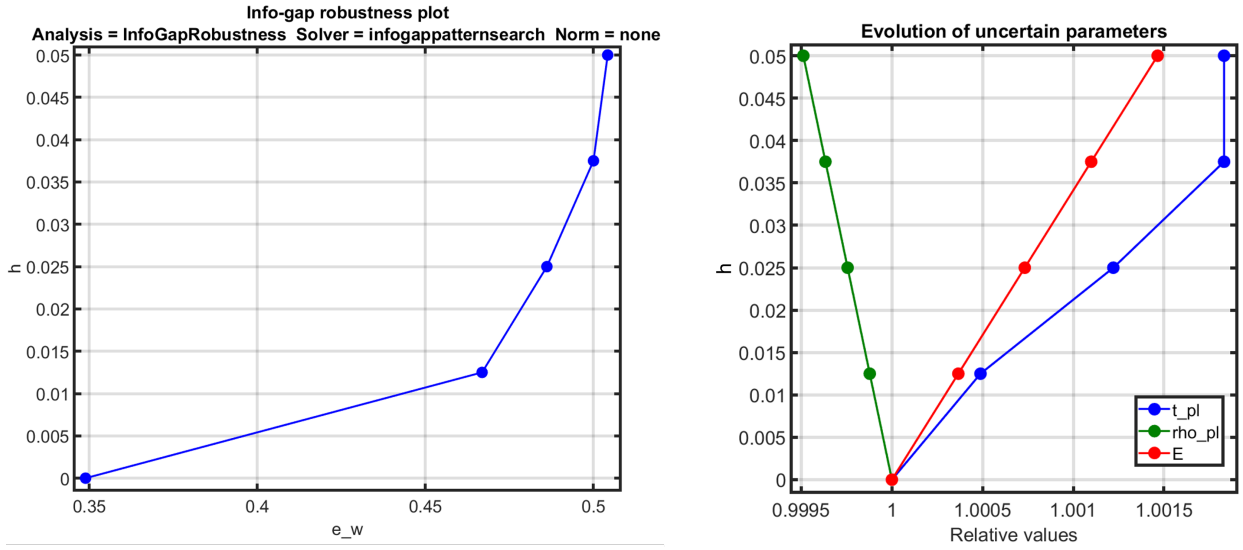


Figure 5. Robustness curve of optimum configuration and evolution of uncertain parameters

It can be observed that the current optimum is not robust to uncertainties, as e_w quickly rises to a value close to $e_{hole} = 0.517$, which corresponds to the criterion value for a simple holed plate. Therefore, a new configuration is required, with a higher nominal e value, that better retains its performance under similar horizons of uncertainty. This would correspond to a robustness curve with a steeper slope.

We aim to formulate this search as an optimization problem, for which the criterion could be defined through a proportion β such that $\beta e_w(h = 0) = \beta e_{opt} = e_w(h = 1)$. The idea is to define a performance-loss criterion by relating the nominal optimized performance to the worst-case performance at maximum uncertainty. This approach defines a set D_β of design vectors $\mathbf{d}_\beta$ satisfying:

$$\forall \mathbf{d}_\beta \in D_\beta, \max_{\mathbf{u} \in U(h=1, \mathbf{u}_0)} e(\mathbf{d}_\beta, \mathbf{u}) \leq \beta e(\mathbf{d}_\beta, \mathbf{u}_0) \quad (6)$$

We then select the best compromise design vector $\mathbf{d}_c \in D_\beta$ such that, for our original minimization problem:

$$\mathbf{d}_c = \arg \min_{\mathbf{d}_\beta \in D_\beta} e(\mathbf{d}_\beta, \mathbf{u}_0) \quad (7)$$

As this optimization can be computationally expensive, we aim to develop this methodology using a metamodel of the criterion e , particularly since not all values of β may admit a feasible solution set D_β .

5. CONCLUSION AND PERSPECTIVES

We have shown that our structure demonstrates cloaking capabilities thanks to our two-region strategy. However, the optimization reveals some discrepancies with the state of the art regarding the configuration of the absorbing region. We plan to implement a two-stage optimization process: the first stage for the absorbing region, focusing on the criterion over the surface S_{inc} , and a second stage for the active region, focusing on S_{trans} .

Using a standard robust design (RD) methodology, we demonstrated the sensitivity of this metamaterial to uncertainties and formulated a novel RD approach that prioritizes performance retention. Additionally, with the aid of a metamodel, we aim to adopt a probabilistic perspective on the uncertain quantities, in order to reduce the conservatism of the performance reduction criterion.

6. ACKNOWLEDGEMENTS

The authors would like to thank l'Agence Nationale de la Recherche (ANR) for funding the ASTRIA research project under the grant number ANR-21-CE46-001-05.

7. REFERENCES

- Aloui, R., Larbi, W. and Chouchane, M., 2019. “Global sensitivity analysis of piezoelectric energy harvesters”. *Composite Structures*, Vol. 228, p. 111317. ISSN 0263-8223. doi:<https://doi.org/10.1016/j.compstruct.2019.111317>. URL <https://www.sciencedirect.com/science/article/pii/S0263822319321038>.
- Ben-Haim, Y., 2004. “Uncertainty, probability and information-gaps”. *Reliability Engineering & System Safety*, Vol. 85, No. 1, pp. 249–266. ISSN 0951-8320. doi:10.1016/j.ress.2004.03.015. URL <https://www.sciencedirect.com/science/article/pii/S095183200400064X>.
- Berardengo, M., Manzoni, S., Thomas, O. and Vanali, M., 2018. “Piezoelectric resonant shunt enhancement by negative capacitances: Optimisation, performance and resonance cancellation”. *Journal of Intelligent Material Systems and Structures*, Vol. 29, No. 12, pp. 2581–2606. URL <https://hal.science/hal-02159249>.
- Collet, M., Ouisse, M., Cunefare, K., Ruzzene, M., Beck, B., Airolidi, L. and Casadei, F., 2011. *Beams and Plates Vibroacoustic Energy Diffusion Optimization by Mean of Distributed Shunted Piezoelectric Patches*, pp. 265–302. ISBN 978-94-007-1702-2. doi:10.1007/978-94-007-1703-9_10.
- Li, X., Chen, Y., Zhu, R. and Huang, G., 2021. “An active meta-layer for optimal flexural wave absorption and cloaking”. *Mechanical Systems and Signal Processing*, Vol. 149, p. 107324. ISSN 0888-3270. doi:<https://doi.org/10.1016/j.ymsp.2020.107324>. URL <https://www.sciencedirect.com/science/article/pii/S088832702030710X>.
- Saltelli, A., Ratto, M., Andres, T., Campolongo, F., Cariboni, J., Gatelli, D., Saisana, M. and Tarantola, S., 2008. *Global sensitivity analysis: the primer*. John Wiley & Sons.
- Tateo, F., Collet, M., Ouisse, M. and Cunefare, K., 2016. “Design variables for optimizing adaptive metacomposite made of shunted piezoelectric patches distribution”. *Journal of Vibration and Control*, Vol. 22, No. 7, pp. 1838 – 1854. URL <https://hal.science/hal-02392825>.
- Thomas, O., Ducarne, J. and Deü, J.F., 2012. “Performance of piezoelectric shunts for vibration reduction”. *Smart Materials and Structures*, Vol. 21, No. 1, p. 015008. doi:10.1088/0964-1726/21/1/015008. URL <https://hal.science/hal-01082925>.
- Trindade, M.A., Lossouarn, B. and Deü, J.F., 2020. “Effect of parametric uncertainties on vibration mitigation with periodically distributed and interconnected piezoelectric patches”. *Journal of Intelligent Material Systems and Structures*, Vol. 32, No. 9, pp. 971–985. ISSN 1530-8138. doi:10.1177/1045389x20942847.
- Yi, K. and Collet, M., 2021. “Broadening low-frequency bandgaps in locally resonant piezoelectric metamaterials by negative capacitance”. *Journal of Sound and Vibration*, Vol. 493, p. 115837. ISSN 0022-460X. doi:10.1016/j.jsv.2020.115837. URL <https://www.sciencedirect.com/science/article/pii/S0022460X20306660>.

8. RESPONSIBILITY NOTICE

The authors are solely responsible for the printed material included in this paper.