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HIGH-FREQUENCY CHARACTERIZATION OF VISCOELASTICITY: DEVELOPMENT OF A SANDWICH BEAM EXPERIMENTAL SETUP

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Abstract. Accurate characterization of viscoelastic materials at high frequencies is critical for the design of lightweight structures and vibration-damping applications. Conventional techniques, such as Dynamic Mechanical Analysis (DMA), are limited to low frequencies, and the time-temperature superposition principle (TTSP) is not universally applicable. This work presents a numerical framework for the inverse identification of viscoelastic parameters in sandwich beam structures up to 10 kHz without relying on TTSP. A finite element model of a sandwich beam with elastic skins and a viscoelastic core, represented by a Zener model, is used to generate simulated dynamic response data. Bayesian optimization is employed to identify the model parameters from the simulated measurements, leveraging a force residual-based cost function. Results demonstrate that the identified parameters closely reproduce the true frequency response of the system, confirming the effectiveness of the methodology. The framework is flexible and can be extended to cases where the viscoelastic behavior does not follow a predefined constitutive law. Future work will focus on application to experimental data and high-frequency characterization of complex materials without assuming a specific viscoelastic model.

Keywords: Viscoelasticity, High-frequency characterization, Inverse identification, Bayesian optimization

1. INTRODUCTION

Viscoelastic materials exhibit a combination of viscous and elastic properties (Shabani *et al.* (2019)), and are characterized by a complex viscoelastic modulus. As a result, they display time- and temperature-dependent mechanical behavior and are suitable for applications requiring materials that can undergo deformation while simultaneously dissipating energy, such as cyclic loading (Zhang and Zhang (2024); He *et al.* (2025)), large strains (Kaliske *et al.* (2001)), and high-frequency applications (Esmaeeli *et al.* (2019)).

For the proper design of systems employing viscoelastic materials, accurate identification of their mechanical properties over a wide range of frequencies and temperatures is essential (Butaud *et al.* (2018)). However, the characterization of these materials at high frequencies remains a challenge, since these frequencies are difficult to achieve in conventional experimental setups. For instance, commonly used technique for characterizing viscoelastic materials as a function of temperature and frequency is Dynamic Mechanical Analysis (DMA). In DMA, a sinusoidal force is applied to a sample and the resulting deformation is measured, or alternatively, a sinusoidal deformation is imposed and the corresponding force response is measured, allowing the complex viscoelastic modulus to be estimated from these measurements (Huayamares *et al.* (2020)). Nevertheless, most commercial DMA devices operate over a limited frequency range, typically reaching only frequencies below 1000 Hz (Esmaeeli and Farhad (2020)).

To overcome this limitation, the time-temperature superposition principle (TTSP) is commonly employed (Ferry and



Myers (1961)). TTSP allows data obtained at different temperatures to be shifted along the frequency axis to construct a master curve representing the material's behavior over a wide frequency range at a chosen reference temperature. Typically, material properties are measured at elevated temperatures and low frequencies, and then shifted to higher frequencies using the Williams–Landel–Ferry (WLF) relation (Williams *et al.* (1955)). However, the WLF relation is applicable only to thermorheologically simple polymers (Shoyama and Fujimoto (2018)), requires measurements over a wide temperature range (Yan *et al.* (2015)), and TTSP itself is not valid for certain polymers, such as polymeric blends (Van Gurp and Palmen (1998)).

To address the constraints imposed by DMA measurements and the application of TTSP, it is important to develop characterization techniques capable of exploring a broad range of frequencies without relying on TTSP. In this context, recent studies have investigated vibration-based experimental approaches to identify viscoelastic parameters at high frequencies. For instance, Esmaeeli *et al.* (2020) reported the development and validation of a device capable of performing simple shear tests of viscoelastic materials up to 5 kHz, employing a piezoelectric actuator and a double-sandwich design. However, the maximum strain level is limited to 0.05%, which does not allow the evaluation of strain-level effects in viscoelastic behavior, such as the Payne effect (Jaboviste *et al.* (2024)). Similarly, Shoyama and Fujimoto (2018) developed a paired base-excitation resonant-mass method capable of measuring shear properties up to 2 kHz, while also avoiding high amplitudes. In addition, Ege *et al.* (2018) characterized a sandwich plate up to 20 kHz, although the identification was restricted to equivalent material properties.

In parallel, another strategy adopted for the characterization of viscoelastic materials consists of fitting a constitutive viscoelastic model directly to experimental measurements. In Henriques *et al.* (2020), a fractional derivative model is calibrated to describe the behavior of polymeric foams based on experiments conducted using a torsional rheometer. Similarly, Rouleau *et al.* (2016) employ a fractional derivative model for the inverse characterization of an adhesive layer in a sandwich beam, based on experimental results going up to 800 Hz. In addition, Rouleau *et al.* (2015) identify the parameters of both a generalized Maxwell model and a fractional derivative model from master curves obtained via DMA experiments using the TTSP.

To combine high-frequency characterization with the inverse identification of viscoelastic models, the present study focuses on the design of a sandwich beam setup capable of characterizing a viscoelastic material up to 10 kHz in different strain levels. The objective is to establish a framework for the inverse identification of the parameters of a viscoelastic Zener model at high frequencies using full-field measurements. In this work, the methodology is developed using simulated data. First, a finite element (FE) model of the sandwich beam is constructed. The sandwich beam configuration is adopted because it is well established in the literature for viscoelastic characterization, particularly through the Oberst beam test (Wojtowicki *et al.* (2004); Jaboviste *et al.* (2024)), which has become a standard experimental method for identifying properties of viscoelastic materials. Then, the model is used to generate simulated response data. Finally, Bayesian optimization (BO) is employed to identify the Zener model parameters from these simulated measurements.

Based on the above, the paper is organized as follows: Section 2 presents the numerical modeling framework, including the viscoelastic Zener model for the core material and the finite element formulation of the sandwich beam. Section 3 details the inverse characterization methodology, describing the Bayesian optimization approach and the definition of the cost function used to identify the viscoelastic parameters. Section 4 reports the results obtained from simulated data, including the identified parameter values and the comparison between predicted and true frequency responses. Finally, Section 5 summarizes the main conclusions of the study and discuss potential directions for future work.

2. NUMERICAL MODEL

The proposed methodology aims to identify the parameters of a viscoelastic model from the dynamic response of a sandwich beam structure composed of two elastic skin layers and a viscoelastic core. The approach requires modeling both the viscoelastic behavior of the core material and the structural response of the sandwich beam. The core is described using a viscoelastic constitutive law, while the sandwich beam itself is modeled using a FE formulation. This section presents the formulation of both components.

2.1 Viscoelastic model

Different rheological models have been developed to describe the variation of viscoelastic parameters with frequency. These models are typically combinations of springs, which represent the purely elastic response (i.e., the proportionality between force and displacement), and dashpots, which represent the viscous response (i.e., the proportionality between stress and strain rate) (Butaud (2015)).

In this work, the Zener model is employed due to its simplicity, since it requires only three parameters, and its ability to capture the viscoelastic behavior of a wide range of materials (Carniel (2025)). This model, illustrated in Fig. 1, is described by the following equation:

$$G^*(\omega) = \frac{G_0 + G_\infty j\omega\tau}{1 + j\omega\tau}, \quad (1)$$

where $G^*(\omega)$ is the complex shear modulus, G_0 and G_∞ are the relaxed and unrelaxed moduli, τ is the relaxation time, ω is the angular frequency, and j is the imaginary unit. This model captures both the elastic (G') and viscous (G'') behavior of the material.

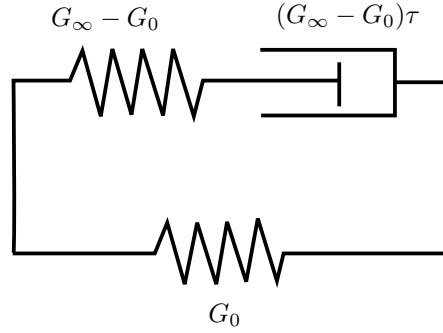


Figure 1. Schematic of the Zener model, showing the configuration of the springs and dashpot.

To simulate the data used in this study, the parameters of the Zener model defined in Eq. 1 were selected according to the values reported in Table 1. Based on these parameters, the frequency-dependent complex shear modulus was computed over the range from 1 Hz to 10 kHz. Figure 2 presents the frequency evolution of the magnitude of the complex shear modulus, $|G^*|$, together with the corresponding loss factor, defined as $\eta = G''/G'$.

Table 1. Chosen values of the viscoelastic parameters.

Parameter	Value
G_0	0.25 MPa
G_∞	1 MPa
τ	5×10^{-5} s

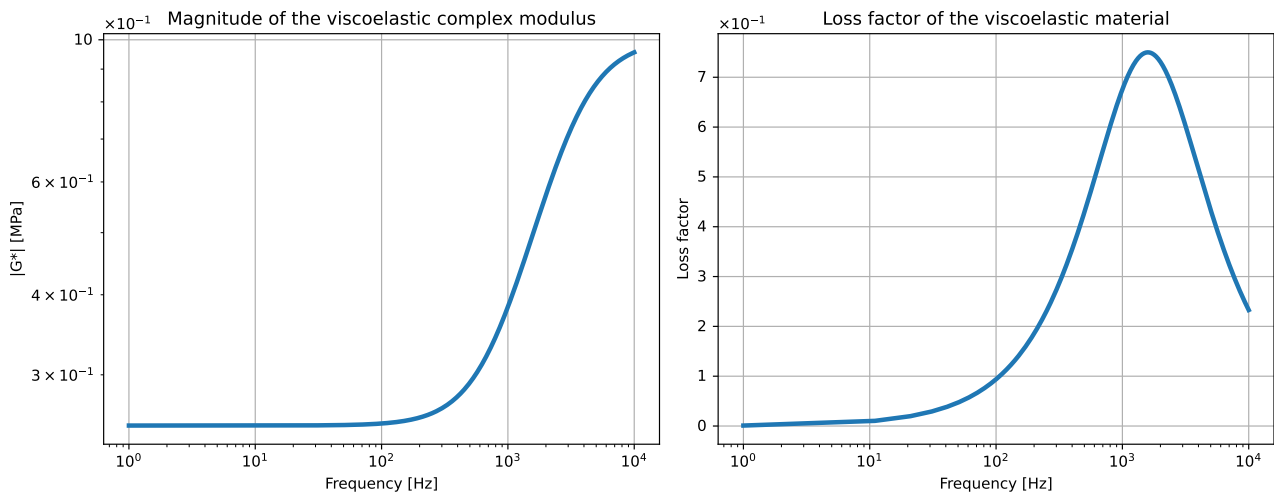


Figure 2. Magnitude of the viscoelastic modulus (left) and loss factor (right) described by the Zener model, evaluated over the frequency range from 1 Hz to 10 kHz.

2.2 Finite Elements formulation of the sandwich beam

The FE method approximates continuous fields by discretizing them into a mesh of elements, using nodal values and basis functions (Pagnacco *et al.* (2007)). In the frequency domain, the equation of motion is:

$$(\mathbf{K}(\omega) - \omega^2 \mathbf{M})\mathbf{U}(\omega) = \mathbf{F}(\omega), \quad (2)$$

where ω is the angular frequency, $\mathbf{K}$ is the complex stiffness matrix, $\mathbf{M}$ is the mass matrix, $\mathbf{F}$ is the external load vector and $\mathbf{U}$ is the displacement vector at nodal points. The stiffness matrix $\mathbf{K}$ can be decomposed as:

$$\mathbf{K}(\omega) = \mathbf{K}_b + G^*(\omega)\mathbf{K}_v^0, \quad (3)$$

where $\mathbf{K}_b$ is the stiffness matrix of the elastic skins, $\mathbf{K}_v^0$ is the stiffness matrix of the viscoelastic layer evaluated for a unit shear modulus, and G^* is given by Eq. 1.

The two-dimensional geometry of the sandwich beam, shown on Fig. 3, consists of two outer linear elastic aluminum skins and a viscoelastic core. The aluminum skins are modeled as linear elastic materials with a Young's modulus of 200 GPa, a Poisson's ratio of 0.3, and a density of 8000 kg/m³. The viscoelastic core is characterized by a frequency-dependent complex modulus, as illustrated in Fig. 2, a Poisson's ratio of 0.49, and a density of 1380 kg/m³.

A harmonic point force with an amplitude of 1 N is applied to the structure, which is clamped at its base. To model the response, a two-dimensional formulation is implemented in a FE software. The computational mesh consists of 30 rectangular elements, resulting in a total of 228 degrees of freedom. Regarding the geometric parameters, the beam has a length of 100 mm, and both the elastic skins and the viscoelastic core have a thickness of 5 mm. Based on this configuration, the resulting displacement frequency response function (FRF) for the first degree of freedom (DOF) is shown in Fig. 4.

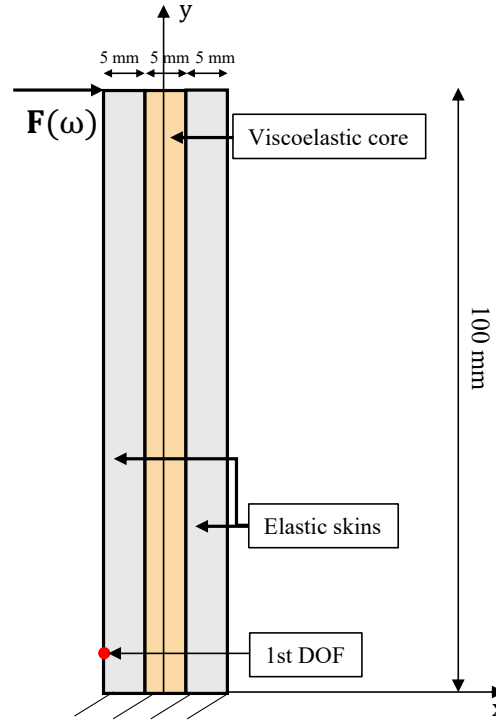


Figure 3. Two-dimensional representation of the sandwich beam. The structure consists of two outer elastic skins separated by a viscoelastic core. A harmonic point force $\mathbf{F}(\omega)$ is applied. In red, the first DOF is highlighted.

3. INVERSE CHARACTERIZATION FRAMEWORK

The inverse characterization procedure aims to identify the parameters of the viscoelastic layer from vibration tests performed on the sandwich beam, in which a sinusoidal excitation force is applied and the displacement response is measured over a range of frequencies. The flowchart of the procedure is presented in Fig.5. This section details the chosen optimization algorithm and the definition of the cost function.

3.1 Bayesian optimization

Bayesian optimization is a global optimization framework particularly suited for objective functions that are noisy, computationally expensive to evaluate, or lack closed-form expressions and gradient information (Wang *et al.* (2023)). Its main advantage lies in its high data efficiency, as it seeks near-optimal solutions with a limited number of function evaluations while maintaining an effective trade-off between exploration and exploitation.

The method is built upon two key components (Frazier (2018)): a probabilistic surrogate model of the objective function, usually a Gaussian process (GP), and an acquisition function that determines where to evaluate the function

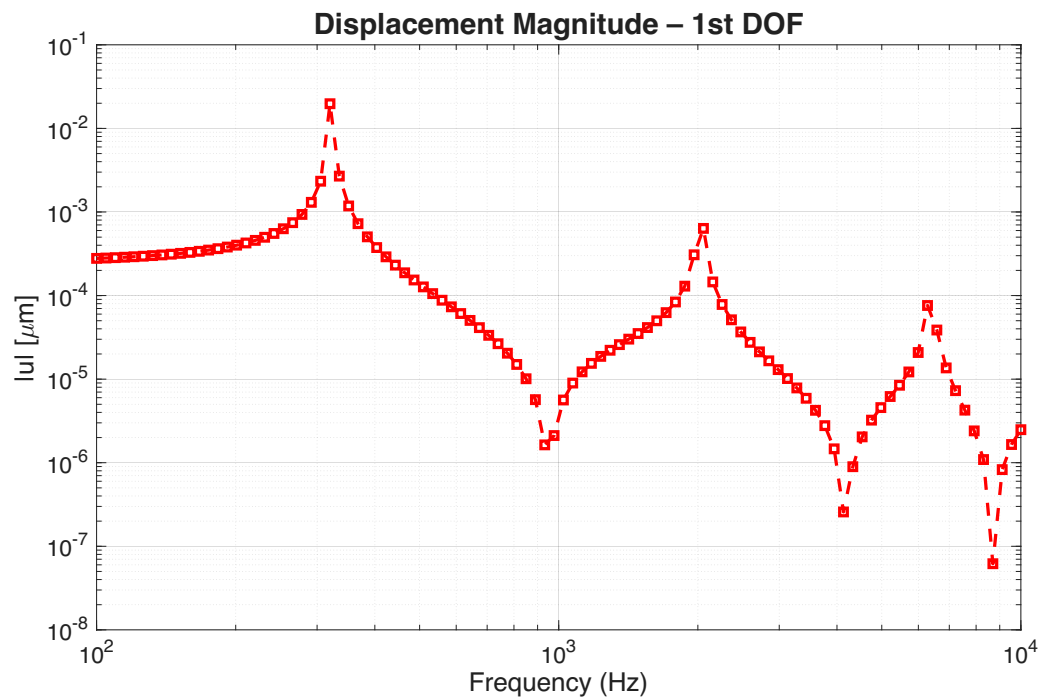


Figure 4. Frequency response of the displacement at the first DOF of the beam.

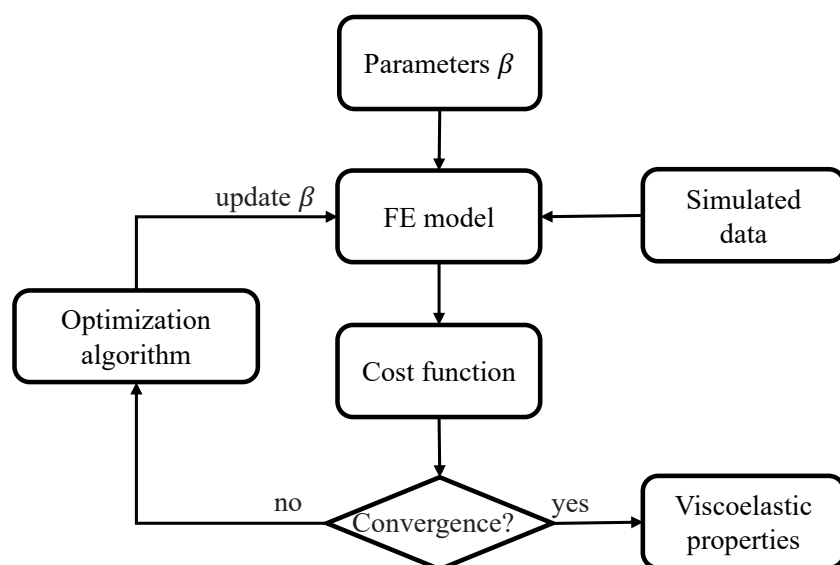


Figure 5. Flowchart of the proposed optimization procedure.

next. Starting from an initial space-filling experimental design, the GP posterior distribution is sequentially updated as new observations are acquired. At each iteration, the acquisition function is maximized to select the next sampling point, favoring regions that are either promising in terms of predicted performance or characterized by high uncertainty. This iterative procedure continues until a predefined number of function evaluations is reached, and the final solution is typically chosen as either the best observed point or the one with the highest posterior mean.

In the present work, BO is adopted even though the viscoelastic constitutive law is known and only its parameters are to be identified. This choice is motivated by its suitability for more general identification problems, in which no specific viscoelastic model is assumed and the frequency-dependent evolution of the complex modulus and loss factor must be directly estimated. In such cases, gradient information is not explicitly available and the cost function is expensive to evaluate, as it requires repeated calls to a FE model at each iteration. Therefore, BO constitutes a robust and flexible strategy for this class of inverse problems.

3.2 Cost function

The proposed identification strategy is based on a force residual formulation, to avoid the appearance of discontinuities near resonance (Pagnacco *et al.* (2007)). Let $\beta = \{G_0, G_\infty, \tau\}$ denote the set of viscoelastic parameters to be identified. For a given angular frequency ω_i , the predicted force vector is computed from the FE model as:

$$\tilde{\mathbf{F}}(\omega_i; \beta) = \left(\mathbf{K}_b + \tilde{G}^*(\omega_i; \beta) \mathbf{K}_v^0 - \omega_i^2 \mathbf{M} \right) \mathbf{U}_{\text{meas}}(\omega_i), \quad (4)$$

where $\tilde{G}^*(\omega_i; \beta)$ is defined in Eq. 1 and $\mathbf{U}_{\text{meas}}(\omega_i)$ is the measured displacement field at frequency ω_i .

The identification procedure consists of minimizing the discrepancy between the predicted and measured force responses over the selected frequency range. The objective function is defined as:

$$J(\beta) = \sum_{i=1}^{N_f} \left\| \frac{\mathbf{F}_{\text{pred}}(\omega_i; \beta) - \mathbf{F}_{\text{meas}}(\omega_i)}{\max |\mathbf{F}_{\text{meas}}(\omega_i)|} \right\|_2, \quad (5)$$

where N_f denotes the number of frequency points and $\| \cdot \|_2$ represents the Euclidean norm. The normalization term ensures that each frequency contributes comparably to the objective function, preventing the optimization from being dominated by frequencies associated with larger force amplitudes.

4. RESULTS

Based on the previous discussion, the optimization procedure is carried out using a Python library for Bayesian optimization (Nogueira (2014)). The search bounds for each parameter β are defined as

$$\beta_0 = [\beta(1 - \Delta), \beta(1 + \Delta)], \quad (6)$$

where β is the nominal value and $\Delta = 0.5$ specifies the relative variation around the nominal value, as listed in Table 1.

Since the parameters span different orders of magnitude, they are normalized to the $[0, 1]$ range prior to optimization to improve numerical stability and convergence, and the optimization is performed in logarithmic scale. To account for experimental variability, a Gaussian noise of 5% is added to the measured force, $\mathbf{F}_{\text{meas}}$. The optimizer is first initialized with 30 points sampled from a space-filling design, after which it performs 300 iterative evaluations to refine the parameter estimates. The final identified values obtained through this procedure are summarized in Table 2, together with their percentual differences relative to the nominal values.

Table 2. Identified values of the viscoelastic parameters after the optimization process.

Parameter	Identified Value	Percentual difference
G_0	0.2505 MPa	0.2 %
G_∞	1.05 MPa	5%
τ	4.7×10^{-5} s	-6%

The displacement frequency response at the first DOF was calculated using the identified parameters and is shown in Fig. 6. As can be seen, the simulated response obtained from the identified parameters closely matches the true displacement values. In addition, the complex modulus derived from the identified parameters is compared with the corresponding true values in Fig. 7. The correspondence between the identified and reference results demonstrates that the proposed method is effective for the inverse identification of viscoelastic parameters in sandwich structures.

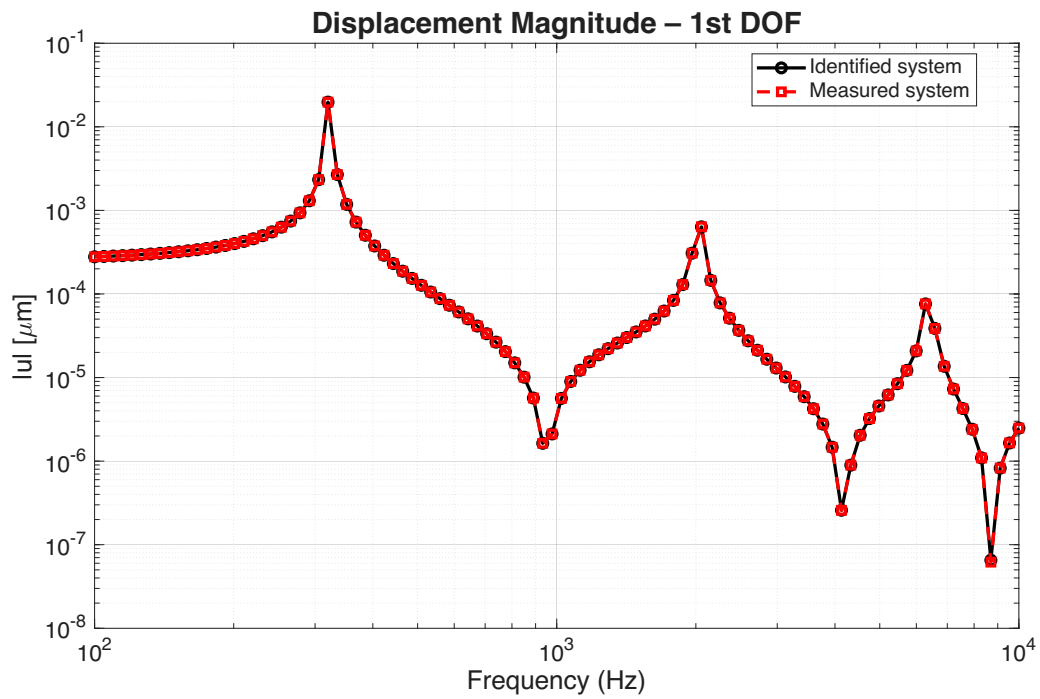


Figure 6. Frequency response of the displacement at the first DOF of the beam for the true values (red) and the response calculated using the identified parameters (black). The two curves are overlapping, indicating good agreement.

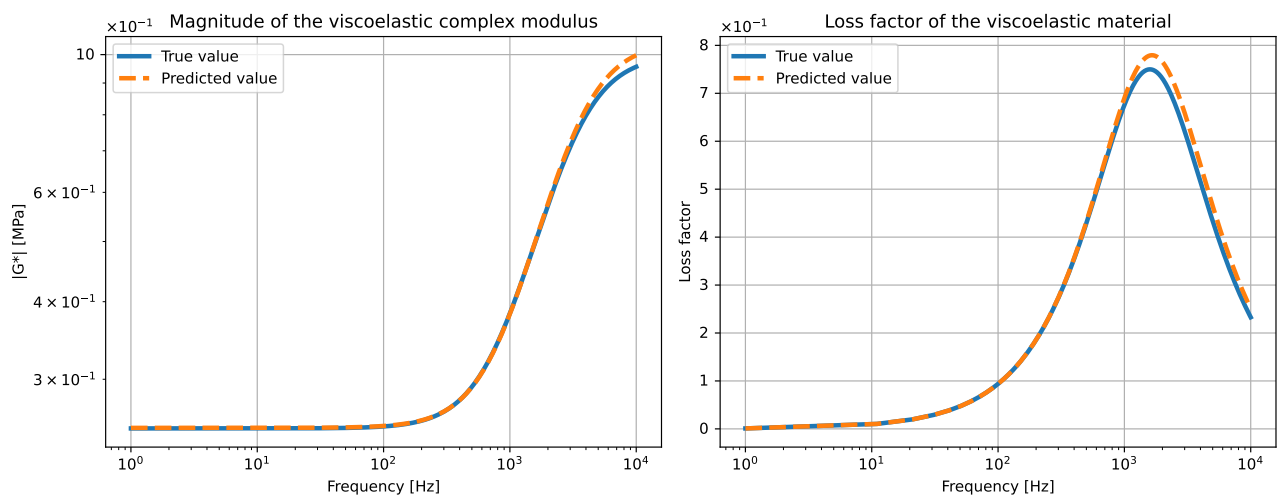


Figure 7. Comparison between the magnitude of the viscoelastic modulus (left) and loss factor (right) for the true values (blue) and the one calculated using the identified parameters (orange).

5. CONCLUSIONS

This work presented a numerical framework for the inverse identification of viscoelastic parameters in sandwich beam structures at high frequencies, up to 10 kHz, without relying on extrapolations from the time–temperature superposition principle. A finite element model of the beam, coupled with a Zener viscoelastic representation of the core, was used to generate simulated dynamic response data, and Bayesian optimization was employed to identify the model parameters from these measurements.

The results demonstrate that the proposed methodology accurately recovers the viscoelastic parameters, as evidenced by the close agreement between the frequency response of the true system and that predicted using the identified parameters. The framework is suitable for more general cases in which the viscoelastic behavior may not follow a predefined constitutive law.

Future work will address two directions: applying this methodology to experimental data, and extending it to scenarios where no explicit viscoelastic model is assumed. This will enable comprehensive high-frequency characterization of complex materials without relying on assumptions such as TTSP.

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