

# **Recalage stochastique d'un modèle de propagation des ondes ultrasonores à l'aide d'un modèle de substitution à apprentissage automatique**

## ***Stochastic model updating for ultrasonic waves propagation using a machine learning surrogate model***

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### **Résumé**

Ce travail présente une approche de réajustement d'un modèle stochastique dans le cadre de la vérification et de la validation d'un modèle à éléments finis (FE) d'une plaque composite, en tenant compte de la propagation des ondes de Lamb, grâce à l'inférence bayésienne. L'étude s'appuie sur une campagne expérimentale menée sur une plaque en polymère renforcé de fibres de carbone, où une onde guidée (Lamb Wave) est générée à l'aide de transducteurs PZT. La méthode proposée est divisée en deux étapes : une identification déterministe initiale, suivie d'une procédure de recalage de modèle stochastique ainsi qu'un processus d'inférence bayésienne qui utilise des simulations de Monte Carlo par chaîne de Markov (MCMC) et l'algorithme d'échantillonnage de Metropolis-Hastings. Pour accélérer les itérations MCMC, un modèle de substitution basé sur un réseau neuronal artificiel (ANN) a été développé et intégré. Grâce à ce processus, les fonctions de densité de probabilité des paramètres du modèle les plus sensibles aux variations physiques sont obtenues.

### **Abstract**

This work presents a stochastic model updating framework for verification and validation of a finite element (FE)-based model of a composite plate using Bayesian inference, considering the Lamb wave propagation. The study involved an experimental campaign on a carbon fiber reinforced polymer plate where guided waves were induced using PZT sensors. The proposed method is divided into two steps : an initial deterministic updating followed by a stochastic updating procedure consisting of a sensitivity analysis using Sobol Indices and a Bayesian inference process using Markov-chain Monte Carlo (MCMC) simulations and the Metropolis-Hastings sampling algorithm. The updating process retrieves the probability density functions of the most sensitive model parameters. A surrogate model based on an artificial neural network (ANN) is implemented to speed up MCMC rounds.

**Mots Clés :** Ondes de Lamb, Structures composites, Inférence Bayésienne, Modèle de substitution, variations environnementales

**Keywords :** Lamb wave, composite structures, Bayesian calibration, surrogate model, environmental variations

## 1. Introduction

Lamb waves are ultrasonic guided elastic waves that are commonly used for structural health monitoring and assessing operational conditions of plate-like structures. They are capable of traveling long distances and can be used for local inspections as well as large-scale applications [1]. However, simulating Lamb wave propagation in composite structures can be challenging due to their natural heterogeneity, different layout possibilities, and environmental-dependent physical properties [2]. The Finite Element method stands out as a candidate for simulating guided wave propagation due to its relative ease of implementation and ability to accurately capture wave front propagation in complex structures [3]. Unfortunately, high-frequency signals or small features on the structure require a very fine mesh density, resulting in computationally expensive simulations.

To ensure consistency with experiments and account for environmental variations, these models must be updated with experimental data using deterministic [4] or stochastic approaches [5]. Typically, deterministic updating involves formulating a minimization problem to obtain optimal values or point estimates for physical parameters [6], whereas stochastic approaches account for experimental and model uncertainties during the calibration steps and incorporate them into the model [7]. The Bayesian updating framework is a widely employed stochastic approach for parameter estimation [8]. It leverages Bayesian inference to obtain information about the posterior probability distribution of model parameters by combining a prior probability distribution, experimental data, and a likelihood function. Bayesian inference has been utilized in various studies, such as those involving bolted joints [9], vibration of beams with varying boundary conditions [10], and geometric nonlinearities [11].

This paper presents a Bayesian framework for stochastically updating a numerical model for Lamb wave propagation in composite structures. The updating procedure involves a modified least-squares method as initial step to identify values for mechanical properties. Then, a sensitivity analysis based on Sobol indices is used to assess the influence of identified parameters on the model response. Parameters with low contributions are considered as determined quantities, whereas parameters with high contributions are updated using Bayesian inference and non-informative *a priori* distributions derived from the deterministic optimization scheme. The Markov-Chain Monte Carlo (MCMC) simulations and Metropolis-Hastings sampling algorithm are used to perform the Bayesian inference. To reduce computational time, an artificial neural network (ANN) surrogate model that can be trained using parallelized Monte Carlo simulations is proposed. This approach significantly reduces the time required for updating and maintains the accuracy of the resulting probability density functions for model parameters.

The paper is organized as follows : Section 2 presents the proposed framework. The experimental setup and numerical model are described in Section 3. The application of the methodology for adjusting Lamb wave signals is demonstrated in Section 4. Concluding remarks and suggestions for future research are provided in Section 5.

## 2. Problem statement and methodology framework

The proposed approach employs Bayesian inference to establish a stochastic numerical model for Lamb wave propagation. The method begins with a deterministic updating process that uses a modified version of the least-squares technique to obtain a set of material properties from a range of options. The deterministic version of the model parameters provides the necessary information for the stochastic updating process. A sensitivity analysis using Sobol indices identifies the most crucial parameters that significantly impact the model's outcomes and should, therefore, be included in the Bayesian updating process. A complete theoretical discussion about Sobol indices is presented in reference [12]. A Bayesian inference process utilizing MCMC simulations and the Metropolis-Hastings sampling algorithm is utilized to obtain posterior probability functions from uniform prior distributions. A complete discussion about Bayesian inference can be found in Jin et al. [13] and Lye et al. [8] presents an implementation of the Metropolis-Hastings sampling algorithm. Along the process, the numerical

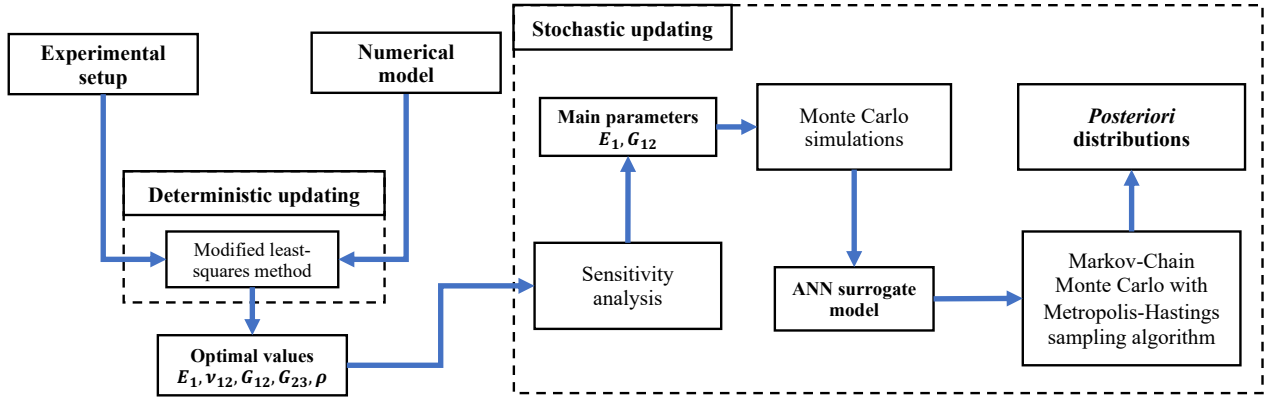


FIG. 1. – Proposed framework structure with deterministic and stochastic updating schemes.

model is replaced by a surrogate model based on neural networks to reduce simulation time. The entire approach is illustrated in Fig. 1.

### 3. Experimental application and numerical model

#### 3.1. Experimental Setup

The carbon fiber reinforced polymer (CFRP) plate used in the experimental setup has dimensions of  $500 \times 500 \times 2 \text{ mm}^3$  and is composed of 10 layers of plain weave fibers oriented along the plate borders directions. To detect the behavior of the plate, four PbZrTi (Lead Zirconate Titanate - PZT) SMART Layers with a diameter of 6.35 mm are bonded to the plate with epoxy resin. PZT 1 acts as the actuator, and the remaining PZTs, namely PZT 2, PZT 3 and PZT 4, serve as sensors. The excitation signal consists of a 5-cycle sinusoidal wave with an amplitude of 35V and a frequency of 250 kHz, which is modulated by a Hanning window. The output signal is then recorded over a  $100 \mu\text{s}$  window using a NI USB 63533 from National Instruments and an oscilloscope DSO7034B from Keysight with a sampling frequency of 5 MHz. Fig. 2a presents the experimental setup. The acquisition of data is conducted under controlled temperatures, which are incrementally raised in steps of  $10^\circ\text{C}$  from  $0^\circ\text{C}$  to  $60^\circ\text{C}$ , using a thermal chamber manufactured by Thermotron. Statistical characterization of the data is carried out by performing 100 repeated tests at each temperature. The dataset is publicly available in the GitHub repository CONCEPT : CarbON-epoxy Composite Plate [14].

#### 3.2. Numerical model

A finite element (FE) model using continuum shell elements (SCR8) is implemented in ABAQUS/Explicit to model the composite plate with ply-based properties and an orthotropic laminate with 3 integration points per lamina. To ensure stability of the numerical simulation, a spatial resolution of at least 20 nodes per wavelength and a time step based on the maximum expected frequency in the simulation are used. For an input frequency of 250 kHz, this resulted in an element size of approximately 2 mm and a maximum time increment of  $0.2 \mu\text{s}$ . However, further stability investigations of the model found that a time increment of  $0.1 \mu\text{s}$  is needed. The simulation window is set to  $100 \mu\text{s}$  to capture the experimental window. To generate a structured mesh, the plate is divided into several smaller rectangular regions, and the area where the PZT actuators and sensors are attached is specifically modeled with a circular mesh. The integrated strain results within the sensor region are used and transformed to voltage as suggested by Sirohi and Chopra [15]. Fig. 2b presents the numerical model.

1. [https://github.com/shm-unesp/DATASET\\_PLATEUN01](https://github.com/shm-unesp/DATASET_PLATEUN01)

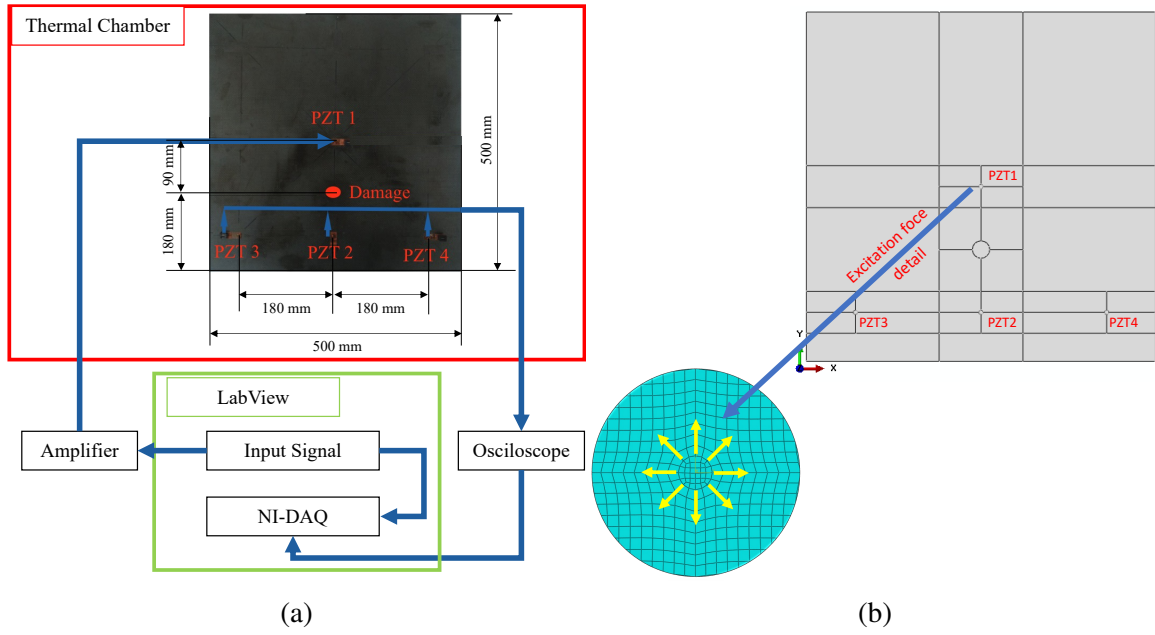


FIG. 2. – Experimental setup and numerical model.

| Property            | Min  | Max  | Initial Value | Optimal Value |
|---------------------|------|------|---------------|---------------|
| $E_1$ [GPa]         | 60.0 | 80.0 | 70.0          | 61.1          |
| $E_2$ [GPa]         | 60.0 | 80.0 | 70.0          | 61.1          |
| $\nu_{12}$          | 0.05 | 0.15 | 0.10          | 0.075         |
| $\rho$ [ $Kg/m^3$ ] | 1550 | 2250 | 1800          | 1620          |
| $G_{12}$ [GPa]      | 5.00 | 15.0 | 7.50          | 10.1          |
| $G_{23}$ [GPa]      | 4.00 | 6.00 | 5.00          | 5.00          |

TAB. 1. – Properties limits used at deterministic updating procedure

## 4. Stochastic Model Updating Framework

### 4.1. Initial deterministic updating

The aim of the deterministic updating process is to modify the mechanical properties of the materials in order to make the response signal of the model ( $y(t)$ ) for sensors 2 and 3 as similar as possible to the experimental results ( $y_{EXP}(t)$ ). The initial deterministic updating of input parameters is based on a modified version of the least-squares method [16]. The squared norm of the difference between the experimental and numerical signals and the cross-correlation between the signals are used as loss metrics. The impact of each input on the loss functions is linearized and the partial derivatives of the loss function with respect to each parameter are numerically calculated. Then, a new set of parameters is obtained and the process repeated until the loss function converges.

Only PZT-2 and PZT-3 signals are used to adjust the mechanical properties because the numerical model is symmetric with respect to the horizontal and vertical axes, resulting in identical signals from PZTs 3 and 4. The classical laminate theory is used to model the mechanical behavior of the materials, where each layer is considered an orthotropic material. This required determining Young's modulus in two perpendicular directions in the plane ( $E_1$  and  $E_2$ ), Poisson's ratio ( $\nu_{12}$ ), shear moduli in and out of the plane ( $G_{12}$  and  $G_{23}$ , respectively), and density ( $\rho$ ). The initial range of values is determined based on the work of Paiva et al. [17]. It is assumed that the values of  $E_1$  and  $E_2$  for plain weave fibers are close, and thus considered equal as a simplification. Tab. 1 presents the initial values, the search range and the optimal values for the mechanical properties.

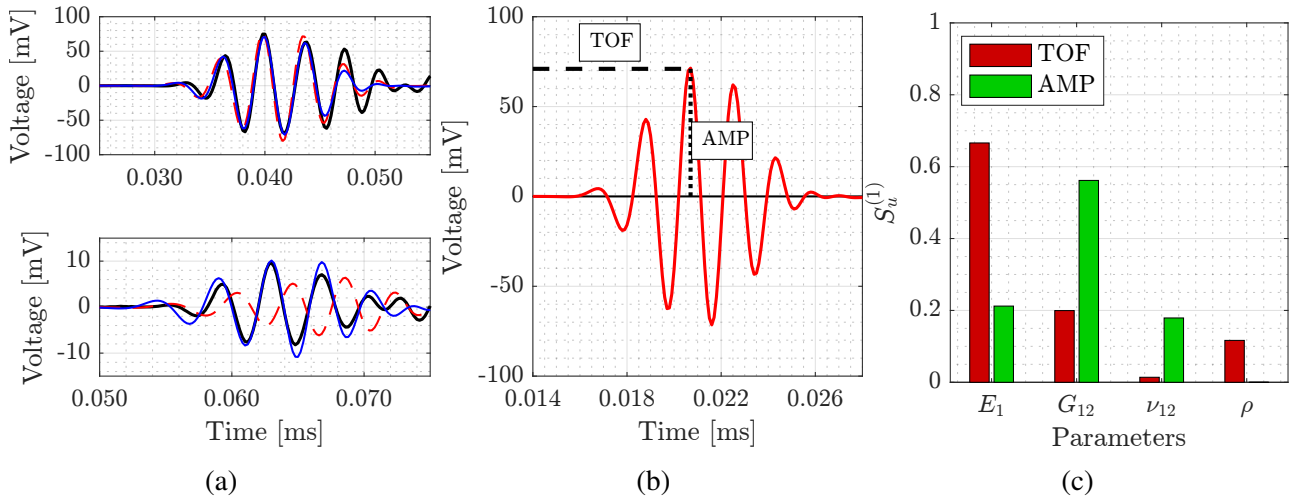


FIG. 3. – Deterministic updating and sensitivity analysis results. (a) comparison between initial (—) and optimal models (—) and experimental (—) results for PZTs 2 (above) and 3 (below); metrics used in sensitivity analysis; and (c) first-order Sobol indices

The updating procedure is carried out with experimental data collected for 20°C and all parameters are updated within the prescribed ranges. The shear modulus  $G_{23}$ , which had negligible impact on the model's performance. Thus,  $G_{23}$  is held constant at 5 GPa throughout the analysis. Fig. 3a shows a comparison of the model response using initial and optimized properties against experimental results.

## 4.2. Stochastic updating

### 4.2.1. Sensitivity analysis

After the deterministic adjustment, a sensitivity analysis is conducted to evaluate the impact of each parameter on the model's behavior. The parameters  $E_1$ ,  $\nu_{12}$ ,  $G_{12}$ , and  $\rho$  are assessed using Sobol indices, with each parameter perturbed within  $\pm 20\%$  of its deterministic values. The sensitivity of the model's response is quantified using time of flight (TOF) and maximum envelope value (AMP) of the first wave packet, as shown in Fig. 3b.

The first-order Sobol indices obtained from the sensitivity analysis, as depicted in Fig. 3c, indicate that  $E_1$  and  $G_{12}$  have the greatest influence on the model response, affecting the time of flight (TOF) and the maximum value of the envelope (AMP), respectively. Intrinsic variations must be considered since they can cause slight changes in the response pattern for the same test conditions. The parameters  $\nu$  and  $\rho$  are determined based on the optimization results presented in Table 1. However, the remaining parameters are considered undetermined, and their posterior probability distributions need to be updated to incorporate the uncertainties present in the experimental data. Therefore, deterministic estimation of these parameters cannot be generalized when there are uncertainties in the experimental data, and they must be treated as random variables.

### 4.2.2. ANN-based surrogate model for Lamb wave propagation

In order to accelerate the random-walking process, a surrogate model utilizing a neural network is proposed as a substitute for the FE-based model. The surrogate model employed in this study is a multilayer perceptron artificial neural network with two fully connected hidden layers. The input layer consists of two neurons representing the random variables  $E_1$  and  $G_{12}$ . The second and third hidden layers consist of 100 and 250 neurons, respectively, while the output layer contains 140 neurons representing the time series output. To reduce training time and data requirements for network convergence, the model response is truncated after the first wave package. Extensive simulations were conducted on the FE model using properties sampled from a bounded uniform distribution, specifically

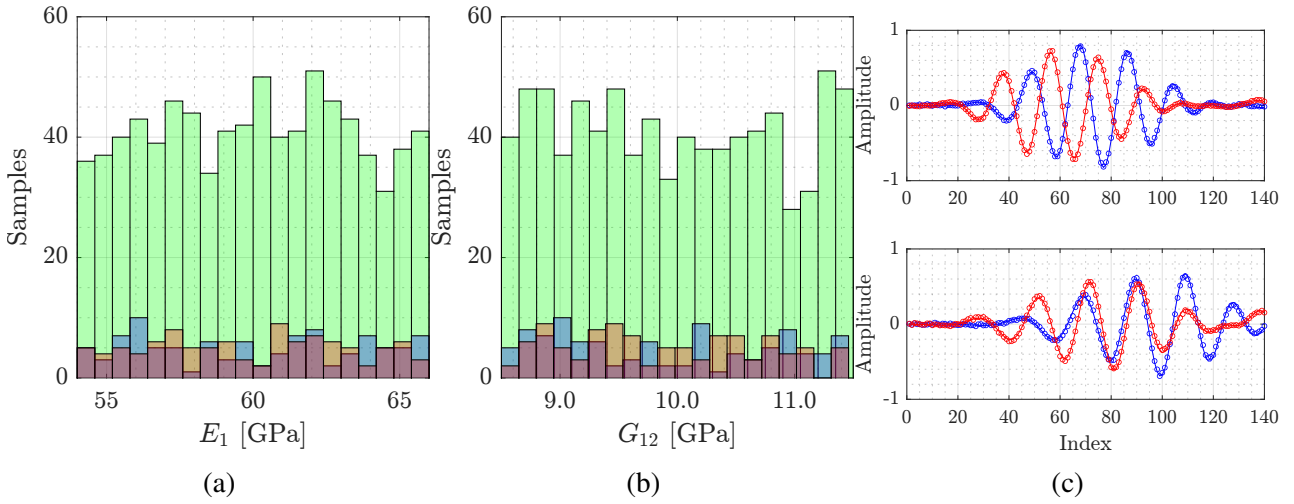


FIG. 4. – Surrogate model evaluation. (a)  $E_1$  Histogram for training (■), validation(■) and test(■); (b)  $G_{12}$  Histogram; and (c) Comparison between surrogate model (○) and FE model (—) for different values of test data

$E_1 \sim \mathcal{U}(55, 65)$  [MPa] and  $G_{12} \sim \mathcal{U}(8.5, 11.5)$  [MPa]. This generated the necessary training data, as depicted in Fig. 4a. To enhance convergence, the input data is normalized between 0 and 1, while the output signal is normalized between -1 and 1. Training of the model is performed using the Stochastic Gradient Descent with Momentum (SGDM) algorithm, with a leave-out strategy implemented to prevent overfitting on the training data. The neural networks are trained using a dataset of 1024 samples, divided into 80/10/10 for training, validation, and testing sets, respectively, as shown in Fig. 4 a-b. The loss on the validation data served as the stopping criterion for training, and a decreasing learning rate strategy is employed to further enhance the model's performance.

Fig. 4c presents the results from the surrogate model compared to the FE model in three values of  $E_1$  and  $G_{12}$  randomly selected from the test data. In these three samples, the response of the surrogate model replicated the response of the numerical model without noticeable differences. Additional tests showed that this behavior is repeated throughout the entire training range of the model. The surrogate model is much faster than the FE-based model, each simulation using the FE model takes between two to three minutes on a typical PC, whereas the surrogate model takes less than a second to be evaluated. The 1024 simulations used at the training step took around 30 hours to perform and could be performed in multiple machines to speed up development.

#### 4.2.3. Bayesian updating

The Bayesian inference process depicted in Fig. 1 is applied to experimental data obtained at 20°C. Both numerical and surrogate models are utilized to validate the methodology. The parameter vector  $\Theta = \{E_1, G_{12}\}^T$  had search limits of  $\Theta_{min} = \{55, 8.5\}^T$  and  $\Theta_{max} = \{65, 11.5\}^T$ , centered at  $\Theta_i = \{60, 10\}^T$ . For the Markov Chain estimation, the variance of the likelihood function is defined iteratively through rounds of the fitting algorithm, trying to ensure a stable acceptance rate between 40 and 50%.

The random walk algorithm began at the center of the parameter search space and utilized the Metropolis-Hastings sampling method to generate new samples. These samples are accepted or rejected based on the likelihood comparison using the acceptance criteria of the Metropolis-Hastings algorithm. Through repeated iterations, the algorithm converged towards an optimal parameter region, as demonstrated in Figure 5. To discard the initial part of the sampling process, a burn-in period equivalent to 20% of the samples is implemented, guided by the acceptance rate curve.

Figures 6a and 6b illustrate a comparison of the probability distributions obtained for  $E_1$  and  $G_{12}$ , respectively. Tab 2 provides a concise summary of the obtained results. Notably, the parameters for

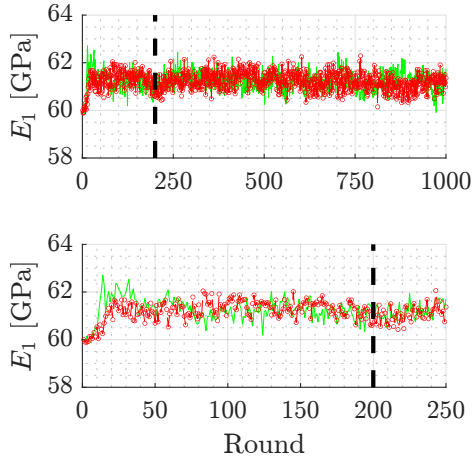


FIG. 5. – Random walk for  $E_1$  and  $G_{12}$  using the FE model (■) and the ANN (■), and detail at burn-in region (---).

|                | RESULTS            | MODEL    |       |
|----------------|--------------------|----------|-------|
|                |                    | FEM      | ANN   |
| $E_1$ [GPa]    | Mean               | 61.25    | 61.24 |
|                | Standard deviation | 0.29     | 0.23  |
|                | CV                 | 0.47     | 0.37  |
|                | 1% Percentile      | 60.26    | 60.10 |
|                | 99% Percentile     | 61.97    | 61.62 |
| $G_{12}$ [GPa] | Mean               | 10.29    | 10.32 |
|                | Variance           | 0.14     | 0.09  |
|                | CV                 | 1.41     | 0.9   |
|                | 1% Percentile      | 9.92     | 10.11 |
|                | 99% Percentile     | 10.52    | 10.47 |
|                | Time               | 115000 s | 240 s |

TAB. 2. – Comparison of Bayesian updating results for  $E_1$  and  $G_{12}$

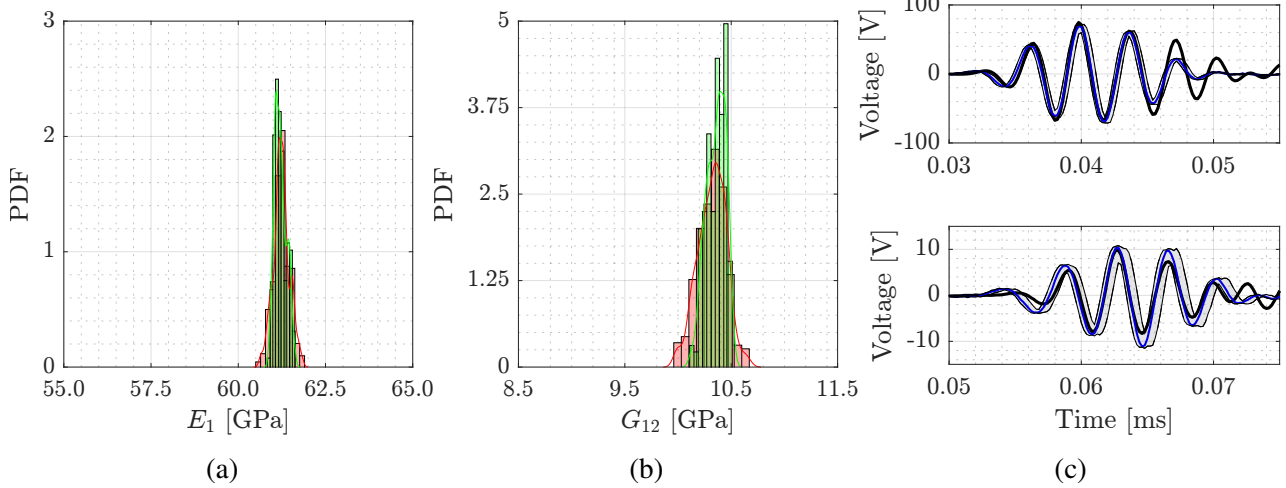


FIG. 6. – Results for the stochastic updating procedure : (a)  $E_1$  posterior histograms and PDFs using the FE model (■) and the ANN (■), (b)  $G_{12}$  posterior histograms and PDFs , and (c) Experimental (—), model mean result (—) and model confidence interval (■) signal comparison for 20°C in PZT 2 and PZT 3

$E_1$  and  $G_{12}$  exhibit similarity regardless of whether the numerical model or the surrogate model is used. Evaluating the numerical model or the surrogate model with the values delimited by the 99% percentile of the results in Fig. 6c gives the confidence interval of the models output.

Moreover, the surrogate model-based approach demonstrates computational efficiency and substantially reduces the time required for analyzing the Markov chains. Using the FE model, a complete round of Markov-chain Monte Carlo with 1000 samples takes at least 115,000 seconds (around 32 hours), excluding initial parameter tuning. However, employing the surrogate model reduces this time to approximately 240 seconds, nearly 500 times faster. The main computational cost of the surrogate model is the training data, but it can be distributed across multiple machines. In contrast, Markov-chain Monte Carlo simulations must be performed sequentially due to step dependency.

## 5. Final remarks

This work presents a stochastic framework for adjusting a FE model to simulate Lamb waves in composite material plates with uncertainties in the fiber/resin system. The framework consists of an

initial adjustment, sensitivity analysis using Sobol indices, and a stochastic adjustment using Bayesian inference and Markov Chain Monte Carlo simulations with the Metropolis-Hastings algorithm. A surrogate model based on neural networks is developed to accelerate simulations and improve efficiency without compromising accuracy.

The presented framework can be applied to multiple temperatures, resulting in a FE model capable of simulating temperature variations and providing results within a confidence interval that includes experimental data uncertainty. Due to its stochastic nature, the model can be evaluated multiple times by sampling the PDFs of the relevant parameters, building a dataset of Lamb waves signals. These signals can compose a hybrid database for training machine learning classifiers.

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