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# MODE SCALING ERRORS DUE TO MODAL TESTS UNCERTAINTIES: A NUMERICAL STUDY

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**Abstract.** *Experimental modal analysis results are heavily impacted by the uncertainty in their input data, as the modal identification process is an inverse problem. These results are used to assess the ability of Finite Element Models (FEM) to represent the low-frequency dynamic behaviour of mechanical structures. It is therefore necessary to study how experimental uncertainties influence the identified eigenfrequencies and eigenvectors that are compared to the FEM. The Modal Assurance Criterion (MAC) is generally employed for this comparison, but it fails to take into account the norms of the eigenvectors, although some applications depend on this information. This numerical study on a simple plate focuses on the effect of measurement uncertainties on the norms of the identified eigenvectors. The influence of parameters such as the noise level, the frequency resolution and the maximum order of identification are investigated in detail. The results demonstrate that these parameters have a significant impact on the eigenvector norm. Additionally, a theoretically constructed mass matrix is used to renormalize the modes. This approach reduces the dispersion of the norm error but introduces a fixed error attributable to the simplistic nature of the matrix.*

**Keywords:** *Modal Identification, Normalization, Uncertainties*

## 1. INTRODUCTION

Experimental modal identification is a complex process and is thus impacted by the presence of uncertainties. During a vibration experiment, a structure is excited, typically by an impact hammer or a shaker, while its response is measured using force sensors and accelerometers. From these measurements, the Frequency Response Functions (FRF) are computed.

An inverse identification method is subsequently used, such as p-LSCF (poly-reference Least Squares Complex Frequency) also referred to as PolyMAX, to identify poles and residues. Many works revolve around this method of identification, such as this one (Peeters *et al.*, 2004). PolyMAX searches a model's poles and residues by fitting a polynomial to the FRF. Contrary to many other modal extraction methods, this one takes place in the z-domain which is a frequency-domain model derived from a discrete-time model. The polynomial order is progressively increased, leading to an increasing number of identified poles. Only those meeting certain stability criteria are retained and considered to be real poles while the others are treated as numerical artifacts. The selection criteria include frequency and damping deviation, mode shape stability, and complexity of the modes. Once a predefined maximum order of identification is reached, a stabilization diagram, which is a model order versus eigenfrequency diagram, is constructed. Usually the poles are chosen by hand by the user but this work (Reynders *et al.*, 2012) presents an automated protocol. The residues and eigenvectors are subsequently computed based on the selected poles. The residues show the influence of the modes lower and higher in frequency than the identified modes. The eigenvectors represents the shapes of the modes. Because these vectors are defined up to a scaling constant, they must be normalized to fix that constant and obtain unique modal vectors. The most common choice is the mass-normalization, where the modal mass of the modes is equal to 1.

As usual in inverse problems, uncertainties in the input data can significantly affect the identified modal parameters.



There are many sources of uncertainty in an experimental modal identification. Uncertainties can arise from an incorrectly calibrated sensor or an incorrectly applied force. References (Tondreau and Deraemaeker, 2014) and more recently (Steffensen *et al.*, 2025) focus on the definition and propagation of these uncertainties. Other papers focus on the impact of these uncertainties on the output, which is the case in this work. Many of them examine thoroughly the influence on frequency, damping and sometimes modal shapes, for example (Melero *et al.*, 2022). Reference (Govers *et al.*, 2006) investigates the sensitivity of frequencies, damping ratio and modal mass, to variations in experimental configurations, such as excitation techniques or analysis methods. That is why the typical uncertainties on these modal parameters are well-known, contrary to uncertainties on modal mass. This is largely because experimental and numerical eigenvectors are compared using the Modal Assurance Criterion (MAC) which only takes into account the modal shapes of the vectors and does not depend on their norms. However, experimental eigenvectors are often used for further analyses such as response predictions. In this case, the validity of the eigenvectors' norm is essential as they directly influence the prediction results.

The present work explores how uncertainties in input data derived from the FRF influence the norm of the identified real eigenmodes. For this purpose, a finite element model of a simple plate is used to simulate experimental data. This approach offers the advantage of providing a modal reference basis for comparison and allows the generation of as much experimental data as needed. First, the input parameters in the simulated experimental data are varied to examine their effects on the eigenvector norms. Then, a mass matrix is constructed to normalize these vectors and to observe how this affects their norms.

## 2. Influence of uncertain parameters on the eigenvector norms

### 2.1 Case study

This study is conducted on an aluminium plate of dimensions 980x260x3mm and of mass 2.195kg that can be seen Fig. 1. Twelve nodes are defined as the sensor locations, three of which also serve as excitation points. The modal base between 5 and 500 Hz is studied, resulting in nine modes of interest.

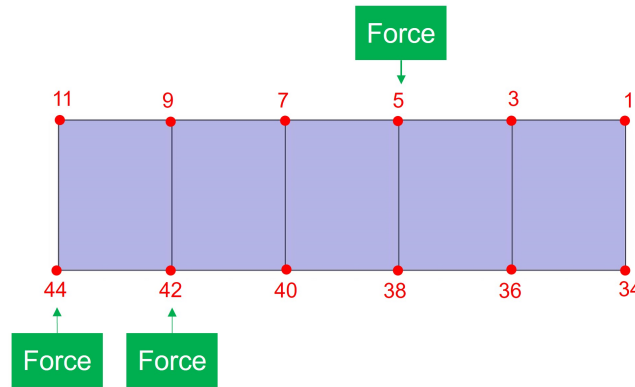


Figure 1. Position of the Excitators and Forces on the Plate.

The corresponding FEM is made using shell elements where the twelve points are the element nodes. Then the numerical modal base is extracted using the MSC-NASTRAN SOL103 solver. Although the resulting modal base is not converged, it does not affect the present study, since parameter variations are evaluated relative to the same reference model. This modal base will be the starting point of the study. The experimental data are numerically simulated from the NASTRAN modal basis by constructing FRF and adding noise. This approach provides a known reference, allowing direct comparison between the identified and original modal bases. This advantage is not possible with a real experiment as it is never known what the correct results are.

The same methodology is followed for every input parameter study : first the initial NASTRAN modal basis from is transformed into a set of FRF - each node with each force having one FRF on the Z axis, as can be seen in Fig. 2. Instead of deducing the modal basis from a set of experimental FRF, the experimental FRF is deduced from the numerical modal basis using MATLAB. Noise is added to the FRF afterwards to simulate experimental error. This noise is modeled in the frequency domain as a complex-valued random variable which is added to unity before being multiplied by the exact FRF. Its magnitude is scaled such that high-amplitude regions of the FRF contain the specified noise level, while low-amplitude regions exhibit a higher noise contribution, reflecting realistic experimental conditions.

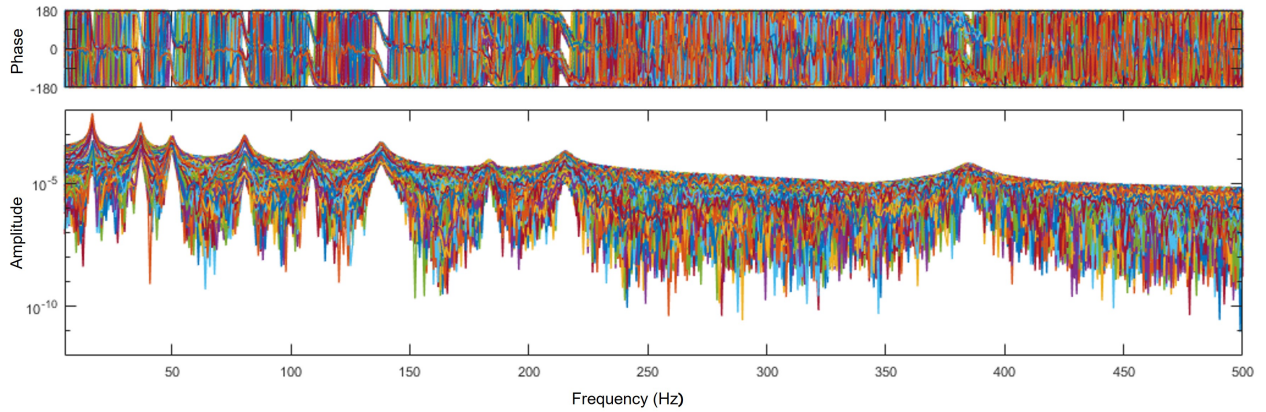


Figure 2. Superposition of the 100 FRF simulations with a 5% Noise Level.

Then the complex eigenmodes are identified automatically using a PolyMAX algorithm. The stabilization diagram is calculated until a specified maximum order of identification, an example can be seen in Fig. 3. Every stable pole is chosen automatically on the final order of identification so there is no human intervention.

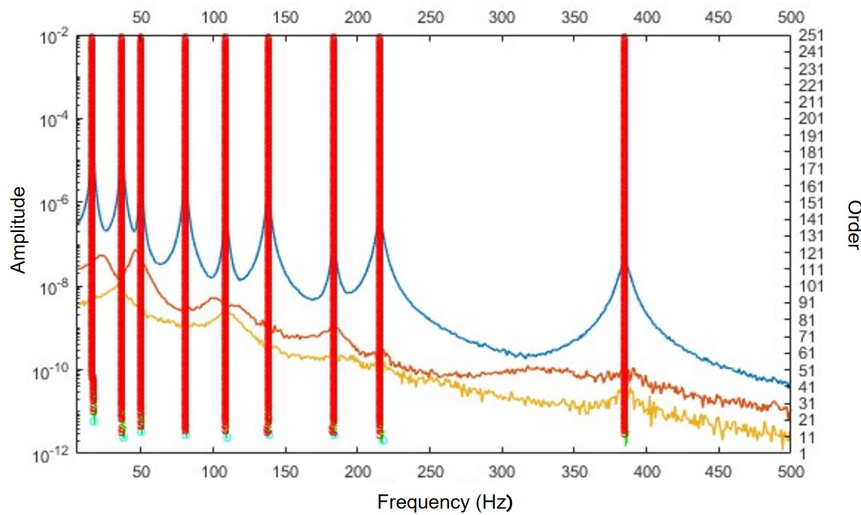


Figure 3. Stabilisation Diagram with a Maximum Order of Identification of 250.

The identified eigenmodes are complex as the effects of damping are taken into account. The real modes are extracted from them as described in the paper (Ahmadian *et al.*, 1995).

At the end of these operations, the eigenvectors should be mass-normalized, meaning that they satisfy Eq. 1 :

$$\{\varphi^T\}[M]\{\varphi\} = I \quad (1)$$

With  $\{\varphi\}$  the modal base and  $[M]$  the mass matrix of the structure.

These operations are repeated a certain number of times to establish statistics on the output data. This way the bias that can be found in incoherent data is avoided. The identified modal basis is organized using the MAC values with respect to the numerical reference basis. Only the modes with a MAC value greater than 0.7 are retained.

## 2.2 Influence of parameters on eigenvector norms

The influence of noise on the eigenvector norms is investigated by considering nine noise levels : 1%, 5%, 10%, 20%, 30%, 50%, 70%, 90% and 100%. These noise levels are introduced into the FRF while constructing them from the NASTRAN modal basis. The maximum order of identification is set to 250, the frequency resolution to 512 sampling points and 100 simulations are performed for each noise level. After classifying the different modes identified by MAC values compared with the numerical model, various statistical indicators are evaluated.

For the following plots, the relative norm error is defined as Eq. (2):

$$\epsilon_{\varphi} = \frac{\| \{\varphi\} \| - \| \{\varphi_0\} \|}{\| \{\varphi_0\} \|} \quad (2)$$

where  $\| \{\varphi\} \|$  represents the Euclidean norm of the identified eigenvectors and  $\| \{\varphi_0\} \|$  the Euclidean norm of the FEM modal basis.

The quadratic mean relative norm error of the 5th mode for each noise level is presented in Fig. 4.

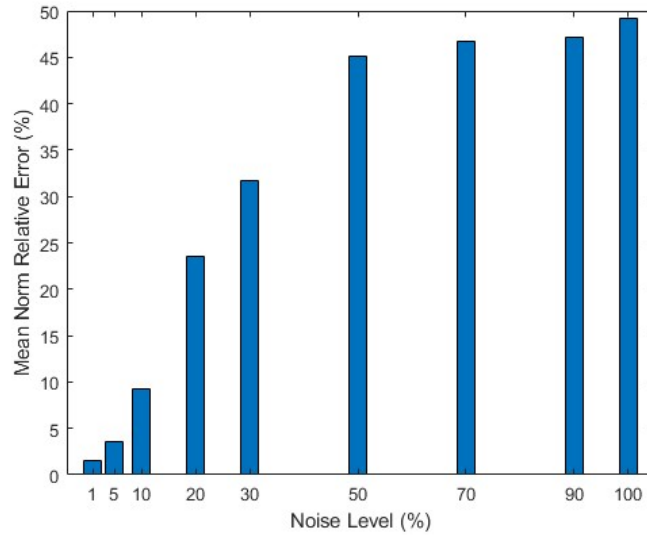


Figure 4. Quadratic Mean Norm Relative Errors on Mode 5 for different Noise Levels.

The noise level has a direct influence on the norm of the identified eigenmodes. Notably the quadratic mean of the norm error goes from 1% for a noise level of 1% to 49% for a 100% noise level.

To have a better understanding of the relationship between the input and output parameters, the medians and quantiles of the relative norm error are investigated. This analysis is illustrated in two representations : a box plot seen Fig. 5 and a conventional plot Fig. 6.

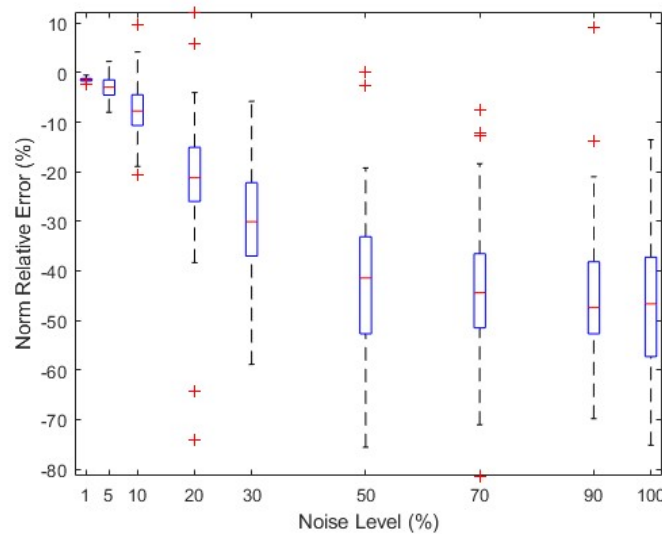


Figure 5. Median, Quantiles and Outliers Norm Relative Errors for Mode 5 for different Noise Levels: A Box Plot Representation.

The median has a similar evolution to the quadratic mean, increasing from 1.5% to 49%. The fluctuation in the eigenvector norms increases with the noise level, such that the mode 5 can have the same norm error with a FRF at a 10%

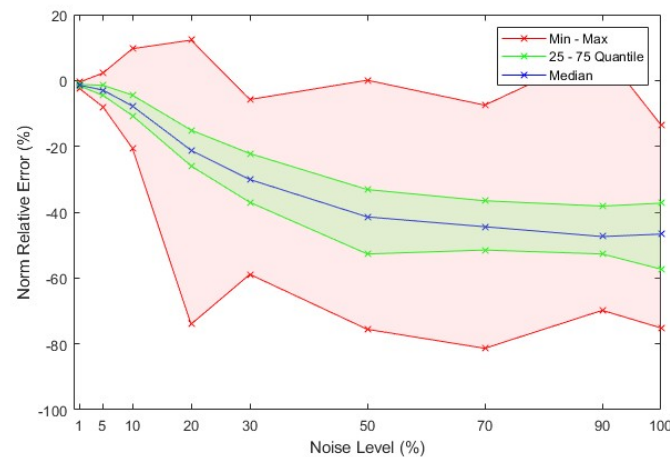


Figure 6. Median, Quantiles and Min-Max Norm Relative Errors for Mode 5 for different Noise Levels.

or at a 100% noise level. Most errors are negative, meaning that the identified eigenvector norms tend to be lower than the real eigenvector norms. It should be noted that not all modes are identified at every iteration for large noise levels. For example, mode 5 is found in 64 iterations for 100 iterations in total for a 100% noise level whereas it is found for the 100 iterations for a 1% noise level. This is the result of an entirely automated process. The statistics are calculated based on the number of iterations where the mode is found and the others are not taken into account. This means that the statistics of mode 5 is calculated on a fewer number of iterations for the 100% noise level than the 1%. So there are sometimes better results in higher levels of noise.

The influence of frequency resolution on the norm of the identified eigenvectors is investigated by varying the number of sampling points in the FRF. As the number of sampling points increases, the computational cost and time required for eigenmode extraction also increase. Accordingly, the study is limited to 50 simulations, with a maximum order of identification at 250 and a 20% level of noise. The frequency resolutions considered are 512, 1024, 2048, 4096 and 8192 sampling points.

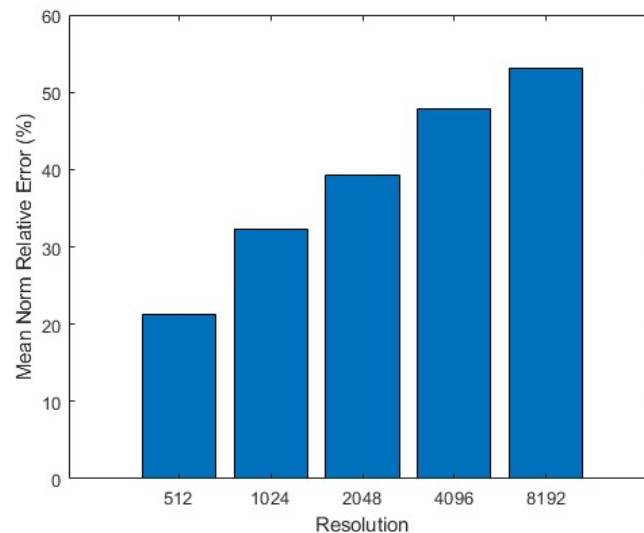


Figure 7. Quadratic Mean Norm Relative Errors for Mode 5 as a function of Resolution.

As shown in Fig. 7, Fig. 8 and Fig. 9, increasing the resolution leads to an increase of the quadratic mean and median norm relative error, while the standard deviation decreases. The results appear counter-intuitive as one would expect a higher resolution to improve the quality of the identified modal basis. However, in the present case increasing the resolution also increases the amount of noise introduced into the FRF. This effect is an artifact of the specific noise modeling approach used in the FRF simulations.

Finally, the impact of the maximum order of identification of PolyMAX on the normalization of the identified eigenvectors is explored. The maximum order of identification orders considered are 100, 150, 200, 250, 300, 400 and 500. For this study the noise level is set to 20%, the sampling point 512 and a total of 100 FRF are computed. The maximum

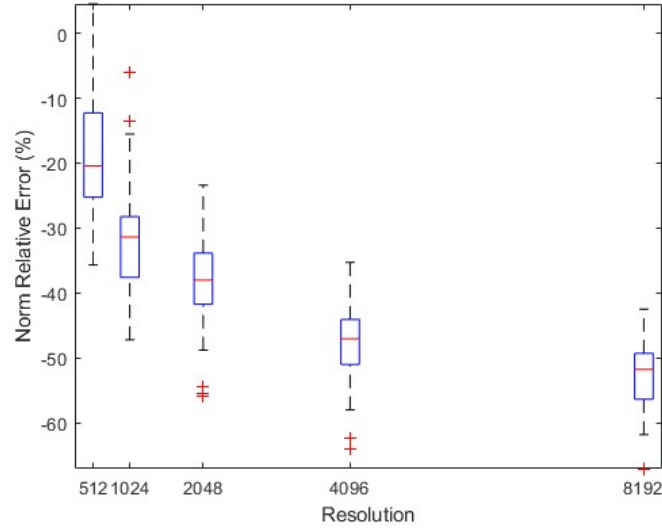


Figure 8. Box Plot of Norm Relative Errors for Mode 5 as a function of Resolution.

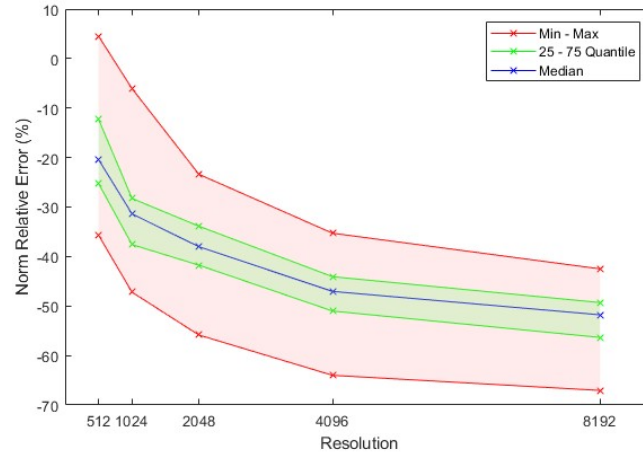


Figure 9. Median, Quantiles and Min-Max Norm Relative Errors for Mode 5 as a function of Resolution.

order of identification is commonly selected as 10 times the number of expected modes. In this present case this guideline would correspond to a maximum order of 90.

However, as can be seen in the Fig. 10, Fig. 11 and Fig. 12, the mode 5 is not identified with a maximum order of identification at 100. As the maximum order of identification increases, the quadratic mean and median relative norm error approach 0%. In contrast, the quantiles remain unchanged.

These three parameters exhibit a clear influence on the norm of the identified eigenvectors.

### 3. Renormalization with a theoretically constructed mass matrix

#### 3.1 The theoretically constructed mass matrix

From these results, one can wonder if there could be an easy way to reduce these uncertainties. Here the method chosen is a theoretically constructed mass matrix used to renormalize the eigenvectors. A mass matrix of size 12x12 containing 12 nodes and 5 shell elements is built. Equation (3) represents the mass for a single shell element :

$$M_{i,j}^e = \frac{m_e}{36} \begin{pmatrix} 4 & 2 & 1 & 2 \\ 2 & 4 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 1 & 2 & 4 \end{pmatrix} \quad (3)$$

This equation is duplicated for the 5 elements.

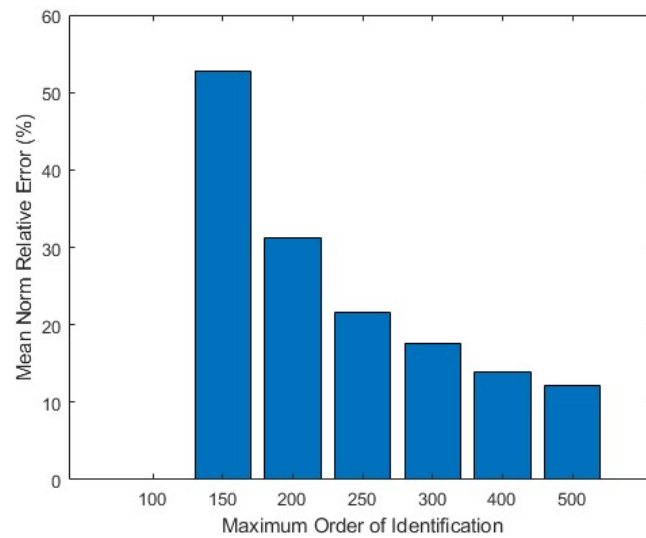


Figure 10. Quadratic Mean Norm Relative Errors for Mode 5 as a function of the Maximum Order of Identification.

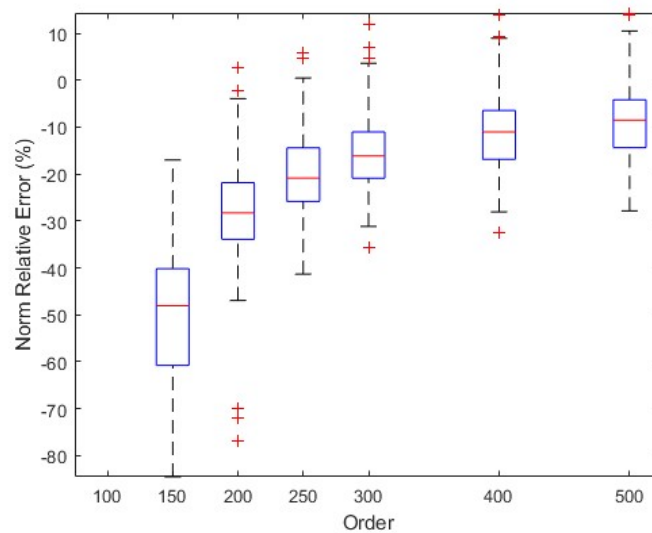


Figure 11. Box Plot of Norm Relative Errors for Mode 5 as a function of the Maximum Order of Identification.

### 3.2 Results of the renormalization

For this analysis, a mesh-converged plate model is necessary as the goal is to simulate the results of an experiment. The reference modal basis is extracted from a 460 nodes FEM. The same protocol is applied on this new modal basis: exact FRF are polluted with noise before identifying a noisy modal basis. This time the eigenvector data is extracted only on the 12 nodes of interest. The identified eigenvectors are then renormalized with the constructed mass matrix and their Euclidean norms are compared to the Euclidean norms of the FEM.

On the left of Fig. 13 is a box plot of the identified modes for a 20% noise level and on the right the same plot with the renormalized modes. Comparing them shows that no matter the noise level, the dispersion of the norm error is greatly reduced thus indicating consistent results. However a systematic error remains. All the other noise levels not shown here yield the exact same systematic error. This error may be due to the static error coming from a non-converged mesh. So to have a perfect renormalization the mass matrix should have the dimension of a converged mesh - that however defeats the purpose of having a simple solution to renormalize modes.

## 4. Conclusion and remarks

This study demonstrates that several input parameters significantly affect the identified eigenvector normalization. Noise levels in the FRF have a direct impact on the norm error: as the noise level increases, both the median and deviation

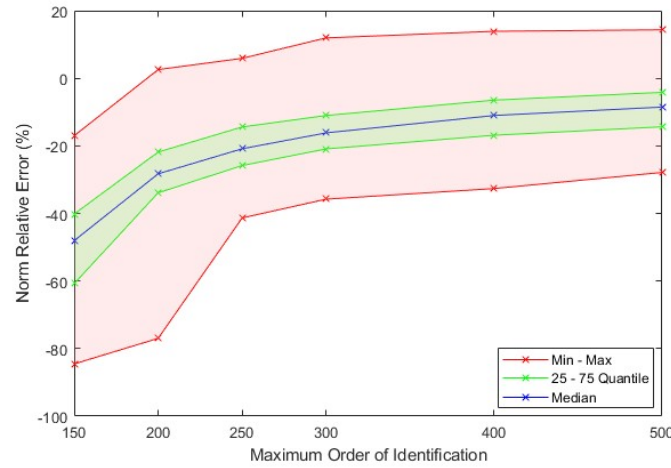


Figure 12. Median, Quantiles and Min-Max Norm Relative Errors for Mode 5 as a function of the Maximum Order of Identification.

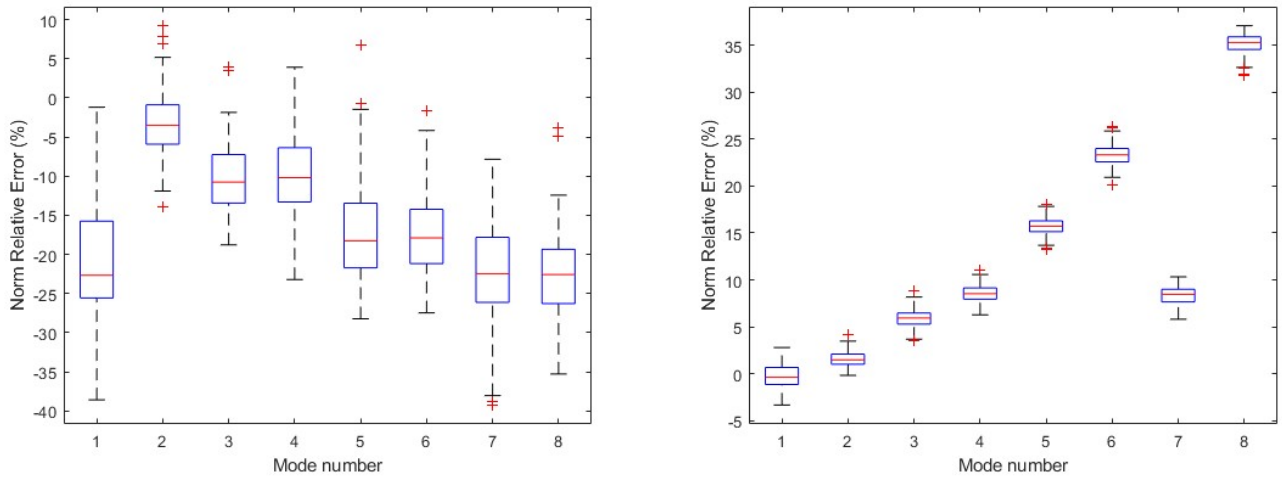


Figure 13. Box Plot of Norm Relative Errors for a 20% Noise Level of All Modes (left) and of these Modes Corrected (right).

of the error increase. The frequency resolution of the FRF also influences the norm: higher resolutions lead to larger median errors and greater variability. Finally, increasing the maximum order of identification of PolyMAX identifies reduces the norm error. There is still many more input parameters that influences the norm that could be explored.

A renormalization strategy based on the construction of a simplified mass matrix was evaluated. The results show a consistent but faulty renormalization, erasing the deviations of the norm but introducing a static error deriving from the non-convergence of the constructed mesh. Meanwhile, this renormalization procedure could be used if the static error between the converged mesh and the simplified meshes were quantified. The mass matrix formulation could be improved with a frequency-dependent matrix that renormalizes each mode independently. Another possible approach is to expand the modes identified from the non-converged mesh on a finer mesh in order to improve the identified norm.

## 5. ACKNOWLEDGEMENTS

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