

Self-induced Vector Geometric Parametric Instability in a Birefringent Fiber

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Abstract: We report a new form of vector geometric parametric instability (GPI) in a birefringent optical fiber, driven by self-induced polarization beating between orthogonal eigenmodes. Unlike conventional GPI in multimode fibers, the longitudinal modulation arises intrinsically from Kerr-induced vector dynamics, enabling discrete Floquet parametric resonances in a nominally single-spatial-mode system. The observed resonances are accurately described by a generalized quasi-phase-matching theory and confirmed by numerical simulations and experiments.

Nonlinear wave propagation in dispersive media exhibits a wide variety of parametric instabilities arising from periodic modulation of the system dynamics, enabling resonant energy transfer between spectral components [1, 2]. In optical fibers, modulation instability (MI) is the paradigmatic example, but it is governed by continuous phase matching and does not generally produce discrete Floquet resonances. By contrast, geometric parametric instability (GPI), first observed in graded-index multimode fibers, originates from the longitudinal modulation induced by periodic spatial self-imaging, leading to a discrete set of parametric sidebands [3, 4].

Here we demonstrate that Floquet parametric resonances can emerge in a nominally single-mode fiber without multimode propagation or externally imposed modulation. In a birefringent fiber, Kerr-induced coupling between orthogonal polarization eigenmodes produces self-induced polarization beating, which acts as an intrinsic longitudinal modulation of the nonlinear interaction. This mechanism gives rise to a previously unexplored form of vector geometric parametric instability characterized by multiple polarization-dependent resonances.

The dynamics are described by the coupled nonlinear Schrödinger equations for the two polarization components E_x and E_y ,

$$\partial_z E_{x,y} = \pm i \frac{\Delta\beta}{2} E_{x,y} - v \partial_t E_{x,y} - i \sum_{n \geq 2} \frac{\beta_{n,x,y}}{n!} \partial_t^n E_{x,y} + i\gamma \left(|E_{x,y}|^2 E_{x,y} + \frac{2}{3} |E_{y,x}|^2 E_{x,y} + \frac{1}{3} E_{x,y}^2 E_{y,x}^* \right), \quad (1)$$

where $\Delta\beta = \beta_x - \beta_y = 2\pi\Delta n/\lambda_p$ denotes the propagation-constant mismatch between the polarization eigenmodes at the pump wavelength λ_p , and $v = (\beta_{1x} - \beta_{1y})/2 = \Delta n_g/(2c)$ is the inverse group-velocity mismatch. Here Δn and Δn_g are the phase and group birefringence, $\beta_{n,x}$ and $\beta_{n,y}$ are the n th-order dispersion coefficients of the slow and fast polarization axes, respectively, and γ is the Kerr nonlinear coefficient. Throughout this work we assume $\Delta\beta > 0$, such that the x and y axes correspond to the slow and fast axes of the fiber.

To characterize the nonlinear polarization dynamics, we introduce the dimensionless power parameter $p = \frac{2\gamma P}{3\Delta\beta}$, together with the effective polarization $\epsilon = \frac{1}{p} \left[\sqrt{1 + 2p \cos(2\alpha)} + p^2 - 1 \right]$, where P is the pump power and α is the input polarization angle. The nonlinear polarization beating generates an intrinsic longitudinal modulation with period L , corresponding to an effective quasi-phase-matching wavevector $k_{\text{qpm}} = \frac{2\pi}{L} \simeq \Delta\beta \left(1 + \frac{p\epsilon}{2} \right)$ [5].

Parametric gain is obtained by linearizing Eq. (1) around the periodically evolving pump solution and applying Floquet theory to the resulting linear system with periodic coefficients [5]. Resonant amplification occurs when the generalized phase-matching condition, $\kappa_i(\Omega) + \kappa_j(-\Omega) = 2q_p + nk_{\text{qpm}}$, is satisfied.

Figure 1(a) shows the calculated Stokes gain spectrum as a function of the input polarization angle for $p = 0.11$, using the parameters of the photonic crystal fiber employed in the experiments. For excitation along the slow axis ($\alpha = 0^\circ$), only the conventional polarization modulation instability (PMI) sideband at 1383 nm is observed. As α increases, Kerr-induced polarization beating progressively strengthens the intrinsic longitudinal modulation, causing the PMI resonance to split into several branches that shift toward lower wavelengths. This sideband splitting is the hallmark of vector GPI and marks the transition from conventional vector MI to a Floquet instability.

In the low-power regime ($p \ll 1$), the four principal resonances ($n = 0$) satisfy the generalized phase-matching condition with the hybridized propagation constants

$$\kappa_{1,2} = \frac{1}{2} \left[q_x - k_{\text{qpm}} + q_y \pm \sqrt{(q_x - k_{\text{qpm}} - q_y)^2 + \Delta\beta^2 p^2 (1 - \epsilon^2)} \right], \quad (2)$$

where $q_p = \Delta\beta/2 [1 + p(5 + \epsilon)/2]$ is the pump propagation constant, while q_x and q_y denote the effective propagation

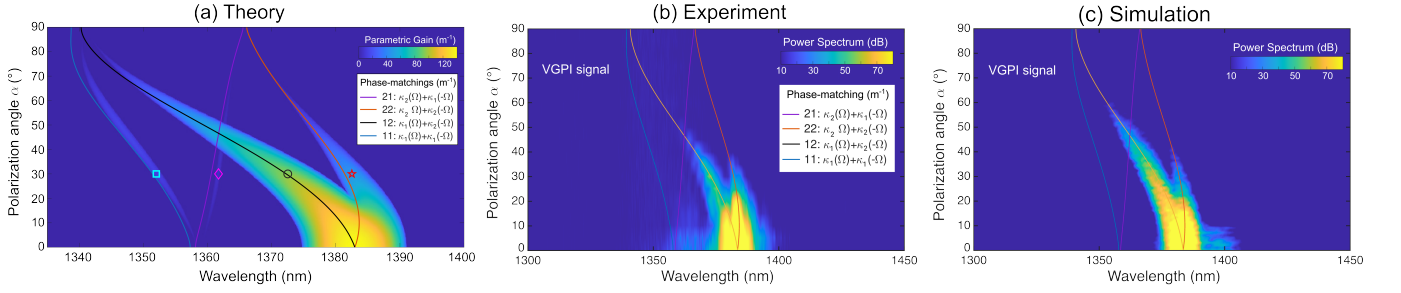


FIG. 1. (a) Theoretical gain analysis (Stokes sidebands) for the dimensionless power parameter $p = 0.11$, pump wavelength $\lambda_p = 1064$ nm: (a) gain spectrum and corresponding phase-matching curves, illustrating the emergence of multiple parametric vector resonances as a function of input pump polarization angle α ; (b) Experimental and (c) numerical spectra (colormap) illustrating vector GPI in a birefringent optical fiber as a function of the input pump polarization angle α . Parameters are $\Delta n = 2.15 \times 10^{-4}$, $\Delta n_g = -1.54 \times 10^{-4}$, $P_0 = 7$ kW, $\gamma = 30$ W⁻¹km⁻¹, $\beta_2 = 0.0109$ ps²m⁻¹, $\beta_3 = -4.2 \times 10^{-5}$ ps³m⁻¹, $\beta_4 = 3.7 \times 10^{-7}$ ps⁴m⁻¹, $L = 50$ cm.

constants of the slow and fast polarization modes, including cross-phase modulation. The corresponding phase-matching curves are shown by the solid color lines in Fig. 1(a), in excellent agreement with the parametric gain spectrum. Higher-order resonances ($n \geq 1$) exhibit significantly lower gain and become observable only at much higher pump powers.

The theoretical predictions are validated experimentally and through direct numerical integration of the coupled NLSEs. Experiments were performed in a highly birefringent, all-normal-dispersion photonic crystal fiber (PCF) originally designed for femtosecond supercontinuum (SC) generation [6]. The fiber was pumped at 1064 nm using 42-ps pulses. The fiber parameters were chosen to suppress scalar modulation instability, ensuring that the observed dynamics are purely vectorial. Figures 1(b,c) compare the measured and simulated spectra as a function of the input polarization angle. As the pump polarization is rotated away from the slow axis, the conventional PMI sideband splits into multiple branches whose evolution agrees remarkably well with the theoretical predictions, providing clear evidence of vector GPI driven by a self-induced polarization grating. The agreement between theory, simulations, and experiments is both qualitative and quantitative.

In conclusion, we have demonstrated a new form of vector geometric parametric instability in birefringent optical fibers. Unlike conventional GPI, the longitudinal forcing is generated spontaneously by Kerr-induced polarization dynamics, requiring neither multimode propagation nor externally imposed modulation. This work extends the concept of geometric parametric instability to vector nonlinear systems and establishes a unified Floquet framework connecting polarization modulation instability and geometric parametric instability in single-mode fibers.

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- [1] V. E. Zakharov and L. A. Ostrovsky, Modulation instability: The beginning, *Physica D* **238**, 540 (2009).
- [2] L. D. Landau and E. M. Lifshitz, *Mechanics*, 3rd ed. (Elsevier, 1976).
- [3] S. Longhi, Modulational instability and space-time dynamics in nonlinear parabolic-index optical fibers, *Opt. Lett.* **28**, 2363 (2003).
- [4] K. Krupa, A. Tonello, A. Barthélemy, V. Couderc, S. Wabnitz, and G. Millot, Observation of geometric parametric instability induced by the periodic spatial self-imaging of multimode waves, *Phys. Rev. Lett.* **116**, 183901 (2016).
- [5] A. Gorbach, Geometric parametric instabilities induced by beatings of two modes, *Optics Express* **33**, 32406 (2025).
- [6] R. Morel, J. Millo, N. Forget, V. Thibaut, M. Marcadier, A. Jullien, Y. Pertot, A. Cassez, V. Andrieux, D. Labat, O. Vanvincq, A. Kudlinski, J. M. Dudley, and T. Sylvestre, Novel photonic crystal fibre for low-noise coherent supercontinuum generation, *Scientific Reports* 10.1038/s41598-026-43460-5 (2026).