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On the use of Gaussian Process Regression of transmissibility functions for loosening detection in bolted structures under excitation level variations

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ABSTRACT

Bolted joints are widely used in industrial structures, but they require regular inspections to prevent failures such as bolt loosening. Conventional inspections are costly and time-consuming, as they involve manual checks and downtime. Therefore, indirect detection of loosening is a major challenge, further complicated by variability and nonlinear effects. This paper proposes a damage detection approach that avoids feature extraction by using the raw frequency response signal. A Gaussian Process Regression (GPR) model is directly trained on transmissibility functions to build a baseline model in the frequency domain. A GPR-based damage index is then formulated to detect outliers relative to this baseline. To improve sensitivity, a Global Sensitivity Analysis (GSA) with Sobol's indices is applied as a feature selection method, identifying frequency ranges most affected by loosening. The paper also extends the GPR-based model to a three-dimensional formulation that captures the evolution of nonlinear behavior as input levels increase. In this way, the GPR-based damage index can distinguish different motion regimes and separate changes caused by nonlinear regime transitions from those caused by loosening. In summary, with proper feature selection, the proposed probabilistic frameworks reliably detect bolt loosening while minimizing false diagnoses in a clear way.

1. Introduction

As addressed in the literature, although applications of SHM in systems with inherent nonlinear dynamics have been increasingly covered in the specialized literature [1], most approaches considers a single input level to represent the nonlinear structural healthy state [2]. In linear systems, it is easier to state a cause-and-effect relationship by employing, for instance, orthogonalization techniques that can distinguish between multiple causes of changing the structural behavior [3,4]. As nonlinear systems, by definition, do not follow the principles of homogeneity and superposition, a different input signal amplitude or wave type is sufficient to generate completely different behavior. Besides, it is not feasible to ensure that the selected methodological framework can cover every regime of structural motion; it is essential to verify that it accurately interprets the system's dynamics within a relevant range of interest. This precaution is essential to prevent, for instance, misidentifying a healthy state characterized by increased nonlinearity resulting from varying input levels, as opposed to nonlinearity due to structural damage [5–7]. This scenario is readily observable in practical applications concerning structures under various operational conditions.

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One of the cases where such situations occur involves bolted joint connections, in which the combined effects of nonlinear damping associated to friction and hysteresis give rise to complex nonlinear behavior [8–10]. These phenomena make it particularly challenging to implement structural health monitoring (SHM) using data-driven models based solely on output data, a well-known weakness recognized for a long time [11]. Several approaches have been proposed in recent years to address this issue [12,13]. A major difficulty lies in the generalization of nonlinear identification, which often requires heavily supervised learning models and extensive datasets covering the relevant range, typically obtained from controlled yet sufficiently realistic test rigs [14].

In this context, the present paper extends the Gaussian Process Regression (GPR)-based classifier proposed by [13] for detecting loosening in a bolted joint structure, namely the Orion beam [15,16]. The method is generalized to a three-dimensional transmissibility regressor by introducing a third axis corresponding to the RMS of the measured output velocity signal under different excitation levels. Miguel et al. [13] illustrated that there is a clear relationship between tightening torque loss and modal features, and consequently that loosening directly affects transmissibility signals. However, as also discussed in [13], damping estimation remains highly challenging due to noise, signal processing, and transmissibility resolution, leading to errors often larger than the variability generated by damage. This makes damping unsuitable as a damage feature and results in significant information loss in the frequency domain.

To overcome these limitations, the present work proposes an alternative strategy that simplifies the feature extraction process. Instead of relying on modal parameters or hand-crafted features derived from the signals, transmissibility functions are directly employed as inputs to the probabilistic model. In this sense, no intermediate modal identification or computation of metrics (e.g., natural frequencies, damping ratios, or distance-based indicators) is performed. The frequency-domain transmissibility signals are used in their original form, preserving the physical information content while avoiding errors associated with modal parameter estimation, especially in scenarios where the system of interest exhibits nonlinear behavior. A GPR model is first identified for the undamaged state using the full transmissibility information in the frequency domain. This model then serves as a baseline to statistically assess the deviation of new data from the healthy condition, enabling the reliable detection of loosening.

Notably, the effects of loosening in the Orion beam are more pronounced in higher-frequency vibration modes [13,17,18]. This behavior is commonly observed in bolted connections and is attributed to the geometrical complexity surrounding the joint in high-order modes, as discussed by [19]. Consequently, employing the full range of the transmissibility function up to this point could generate an excessive volume of data. This abundance of data might negatively impact computational efficiency without enhancing the model's ability to detect loosening within the machine learning framework. This observation becomes apparent following a detailed examination of the modal parameters. However, it is essential to note that, in numerous applications, identifying the most relevant set of data or features to consider in a statistical pattern recognition algorithm is often a complex and non-trivial task. Thus, a Global Sensitivity Analysis (GSA) is also performed to ensure that the machine learning model and analysis only consider relevant information in the context of loosening detection. For this, Sobol's indices are calculated to quantify the influence of a selected frequency range on the structural state. The objective is to assess whether variations in the transmissibility spectrum are directly associated with changes in the structural condition. Based on this analysis, frequency regions with negligible contribution are excluded in order to reduce computational complexity and improve the classifier's performance. It is important to emphasize that this selection of frequency regions constitutes a dimensionality reduction step aimed at computational efficiency; no modal parameters or derived scalar indicators are extracted from the signals. The transmissibility functions are used directly as inputs to the probabilistic model.

The paper is organized as follows. Section 2 provides a brief review of the Orion Beam benchmark, describing the dataset, testing conditions, and the data used to evaluate the proposed approach. Section 3 outlines the framework for directly employing transmissibility functions modeled by GPR for damage detection, highlighting the challenges of indiscriminately using the entire frequency spectrum. As part of the solution, Section 4 presents the concept of GSA and Sobol's indices, along with their practical application in identifying the most relevant frequency ranges for discriminating between the structural states. In Section 5, the performance and influence of each frequency range are analyzed to select those that enable the construction of an optimized classifier, demonstrating the effectiveness of the methodology through three-dimensional GPR damage detection results. Finally, Section 6 provides concluding remarks, summarizing the main findings and implications of the study.

2. Orion beam dataset

Fig. 1 presents the Orion beam benchmark [16]. The experimental setup consists of two duraluminium beams with dimensions $200 \times 30 \times 2$ mm, connected by three M4 bolts distributed along a length of 30 mm. A washer is placed under each bolt and nut. On one of the beams, contact patches are located at each bolt connection, each consisting of a square of 12×12 mm with an additional thickness of 1 mm. In the monolithic condition, the three bolts are assembled with a tightening torque of 80 cNm. To preserve structural integrity and represent the dynamics of well designed bolted joints, only the outer bolts have their tightening torque modified. In this study, such variations are associated with damage induced by torque loss. The central bolt remains fully tightened at 80 cNm throughout all experimental measurements. Five torque levels are applied to the external bolts, namely 80, 60, 30, 20, and 10 cNm. The selection criteria for these values are detailed in [17]. The applied torque is set by using a Lindstrom MA500-1 torque wrench. The assembly procedure is defined as follows. First, to ensure proper alignment between the beams, two pins are inserted into the external holes. The central bolt is then tightened to 80 cNm. Finally, the pins are removed and the external bolts are tightened according to the selected torque level.

The structure is positioned vertically with its neutral axis perpendicular to the ground in order to minimize the influence of gravity and to reduce the contribution of torsional modes to the vibrational response of the beam. The beam with contact patches is

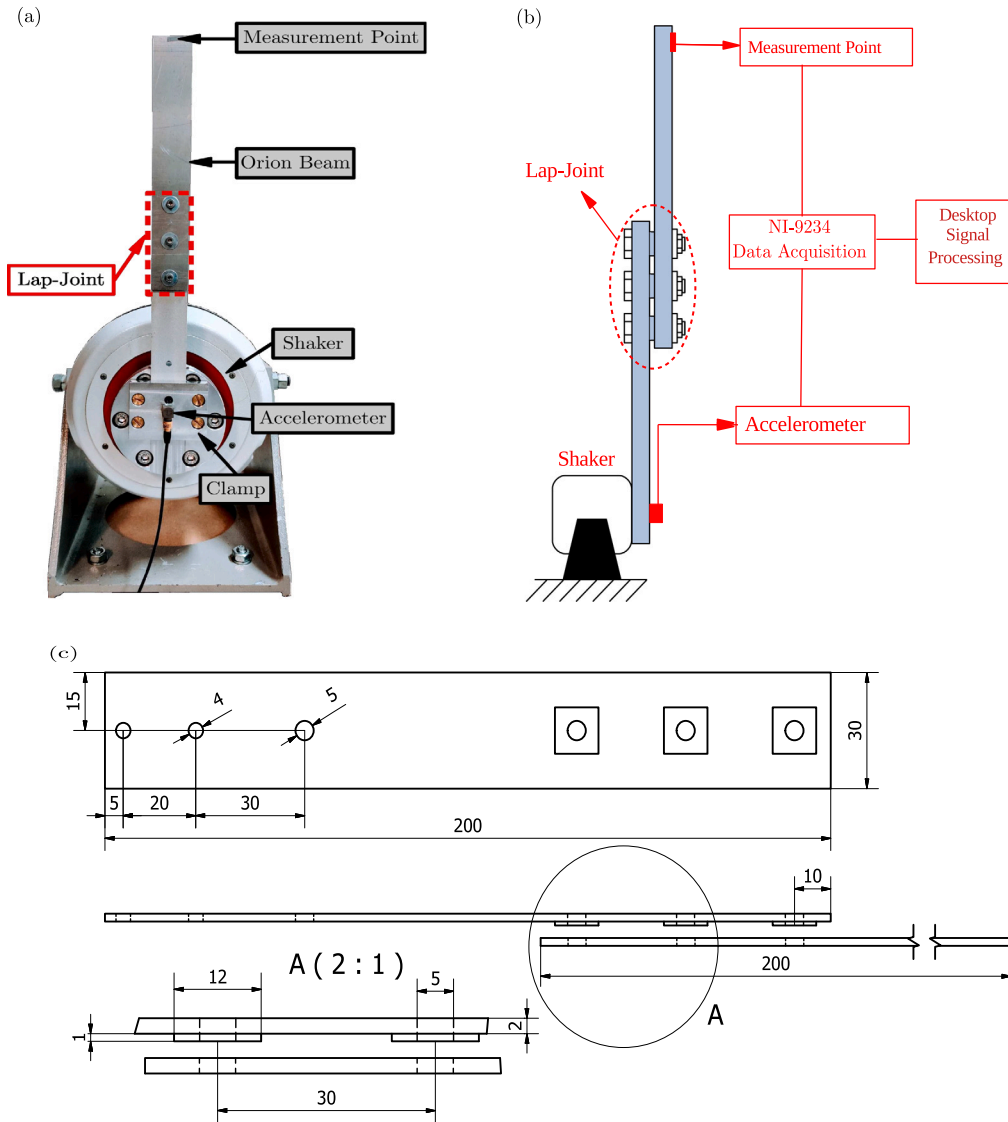


Fig. 1. Orion beam benchmark [16]. (a) illustrates the experimental setup; (b) Schematic representation; (c) Orion beam geometry with specified dimensions.

fixed by a 40 mm segment to a solid aluminum block that serves as the structural base. The base is excited by a permanent magnetic shaker TIRA TV Model 51120 driven by a Gaussian white noise input. The frequency bandwidth is limited between 0 Hz and 2000 Hz. Control of the root mean square level of the base acceleration is performed using the real time feedback system described in [17]. Three different levels are measured, a low (4 m/s^2 RMS), medium (8 m/s^2 RMS) and high (12 m/s^2 RMS) excitation level. Velocity at the free end of the Orion beam is measured with a Polytec PSV-500 scanning laser vibrometer operating in three dimensions. The base acceleration is monitored by a triaxial accelerometer PCB Model 356A4. These two signals, velocity at the beam end and acceleration at the base, are combined to estimate the transmissibility spectrum, considering only the bending motion. Data acquisition is carried out with a National Instruments system composed of a CompactDAQ Chassis NI cDAQ 9134, a C Series Sound and Vibration Input Module NI 9263, and a C Series Voltage Output Module NI 9234.

To account for data variability, 48 experimental realizations were carried out for each excitation level under every tightening torque condition considering assembly-disassembly process among realizations, leading to a dataset of 720 measurements. All signals were acquired with a sampling frequency of 25.6 kHz and 262 144 samples, corresponding to a time duration of 10.24 s. Additionally, white noise with a 35 dB signal-to-noise ratio (SNR) was added to each velocity signal to increase uncertainty and variability. The cases with outer bolts tightened to 80 cNm and 60 cNm represent the healthy condition of the structure, whereas the other torque levels are considered as damaged conditions. The assumption on the healthy condition is based on the dynamic behavior of the structure: since the connected area is relatively small, the microslip occurring at 60 cNm remains limited, leading

to a dynamic response very close to that obtained at 80 cNm. Previous studies on the same benchmark reported and analyzed this behavior [13,15].

3. Loosening detection using a Gaussian Process Regression from transmissibility functions

The proposed methodology is based on transmissibility functions similar to those presented in [13]. However, the dataset is different, despite this paper still dealing with the Orion beam.

First, a GPR-based model is learned from the healthy transmissibility and is taken as a reference model

$$\mathbf{y}^{(i)} = f(\mathbf{x}^{(i)}) + \epsilon_S^{(i)}, \quad i = 1, 2, \dots, N_t, \quad (1)$$

in which $\mathbf{y}^{(i)} \in \mathbb{R}^{(N_f) \times 1}$ is the i th transmissibility function from a dataset with N_t measures, $\mathbf{x}^{(i)} \in \mathbb{R}^{(N_f) \times 1}$ is the respective frequency vector and $\epsilon_S^{(i)} \in \mathbb{R}^{(N_f) \times 1}$ a Gaussian error of zero mean along the frequency range. The function $f(\cdot)$ is then formed by the assumption of a multivariate Gaussian prior distribution of zero mean [20]:

$$f = f(\mathbf{x}) \sim \mathcal{N}(f|0, \mathbf{K}), \quad (2)$$

where $\mathbf{K} \in \mathbb{R}^{N \times N}$ is the covariance (kernel) matrix with $K_{ij} = k(\mathbf{x}_i, \mathbf{x}_j)$, and $k(\cdot, \cdot)$ is the *kernel* function, also named as covariance function. The kernel function can assume several classes depending on the type of application.

The healthy condition, defined by outer bolt torques of 80 cNm and 60 cNm, corresponds to the dataset obtained under low excitation with a RMS acceleration of 4 m/s². This results in $N_t = 96$ transmissibility measurements, with 48 realizations for each torque level. For this application, the learning data set $\mathcal{D}$ has the components

$$\mathcal{D} = \left(\mathbf{y}^{(i)}, \mathbf{x}^{(i)} \right)_{i=1}^{N_t, N_f} \equiv (\mathbf{Y}, \mathbf{X}), \quad (3)$$

where $\mathbf{Y} \in \mathbb{R}^{(N_t, N_f) \times 1}$ is the vector of N_t transmissibility amplitudes measured in N_f frequency values within the frequency ranges of interest for the undamaged condition. Similarly, $\mathbf{X} \in \mathbb{R}^{(N_f) \times 1}$ is the vector of frequencies. In this work, considering the available data, the best results were achieved considering a squared exponential kernel as covariance function:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 \exp \left[-\frac{1}{2} \left(\frac{\mathbf{x}_i - \mathbf{x}_j}{\ell} \right)^2 \right], \quad (4)$$

where σ^2 is the hyperparameter that controls the model's covariance and ℓ is the lengthscale. This choice is also in line with what is observed in the literature, since the squared exponential is the most commonly used in the kernel machines and nonlinear modeling fields [20]. The hyperparameters $\boldsymbol{\beta} = [\sigma^2, \ell]$ are determined by solving an optimization problem of the marginal log-likelihood of the observed data [21]:

$$\begin{aligned} \log \pi(\mathbf{Y}|\mathbf{X}, \boldsymbol{\beta}) &= -\frac{1}{2} \log |\mathbf{K} + \sigma_S^2 \mathbf{I}| + \\ &\quad - \frac{1}{2} \mathbf{Y}^T (\mathbf{K} + \sigma_S^2 \mathbf{I})^{-1} \mathbf{Y} - \frac{N}{2} \log(2\pi), \end{aligned} \quad (5)$$

where $\mathbf{I} \in \mathbb{R}^{N \times N}$ is the identity matrix. In this work, such a maximization procedure is performed using a Hybrid Genetic Algorithm (HGA). The combination of global and local optimization strategies provided by HGA improves both the results and efficiency of the hyperparameter estimation.

The Bayesian inference is used to condition a posterior predictive distribution $\pi(f_*|\mathbf{Y}, \mathbf{X}, \mathbf{x}_*)$ over the predicted function f_* based on the new input samples $\mathbf{x}_*$, which yields the main relationship for the GP regression [20]:

$$\pi(f_*|\mathbf{x}_*, \mathbf{Y}, \mathbf{T}) = \mathcal{N}(f_*|\mu_*, \sigma_*^2), \quad (6)$$

where the posterior predictive mean is given by:

$$\mu_* = \mathbf{k}_{*N} (\mathbf{K} + \sigma_S^2 \mathbf{I})^{-1} \mathbf{Y}, \quad (7)$$

and the posterior predictive variance is given by:

$$\sigma_*^2 = \mathbf{k}_{**} - \mathbf{k}_{*N} (\mathbf{K} + \sigma_S^2 \mathbf{I})^{-1} \mathbf{k}_{N*}, \quad (8)$$

where

$$\mathbf{k}_{*N} = [k(\mathbf{x}_*, \mathbf{x}_1), \dots, k(\mathbf{x}_*, \mathbf{x}_N)],$$

$$\mathbf{k}_{N*} = \mathbf{k}_{*N}^T,$$

$$\mathbf{k}_{**} = k(\mathbf{x}_*, \mathbf{x}_*).$$

A predictive distribution of $\mathbf{Y}$ is similar to f_* . Using the whole spectrum indiscriminately from 0 to 2000 Hz as a first example, the GPR from Fig. 2 is obtained. Only 15 realizations of 96 were taken for learning the model due to computational limitations. To avoid selection bias, the learning samples were randomly selected from the healthy samples to perform the hyperparameter estimation and built the model. The remaining healthy data is used to test the classification performance in terms of Type I Error.

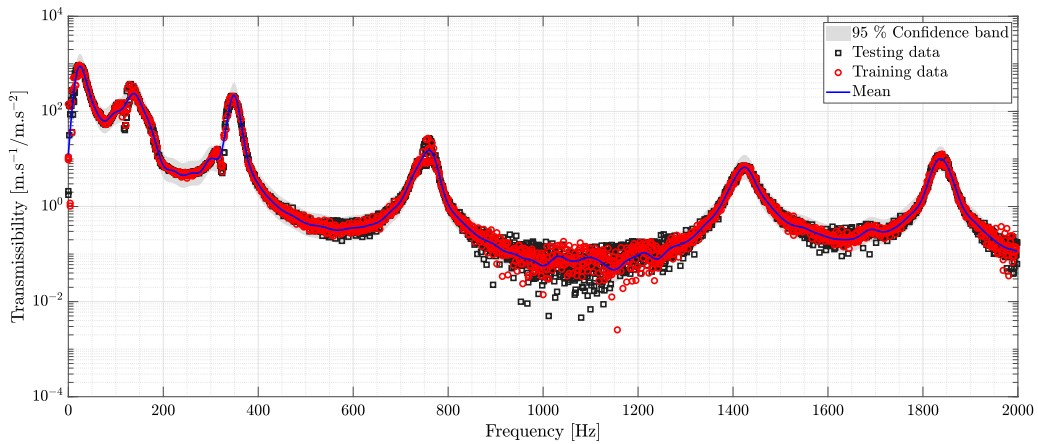


Fig. 2. GPR model identified for the whole transmissibility spectrum.

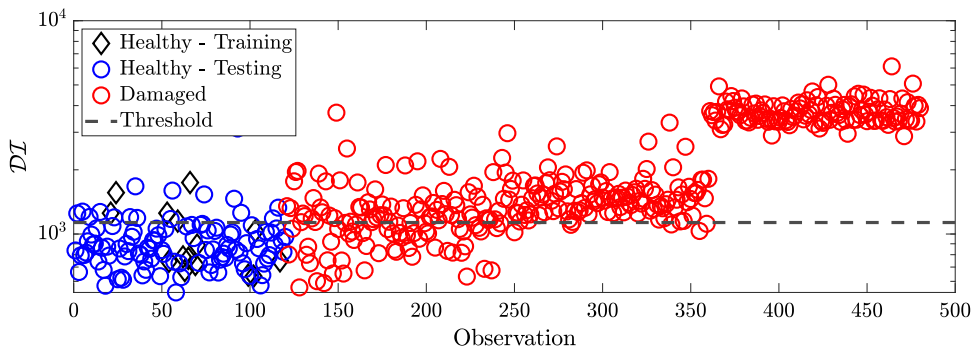


Fig. 3. DI computed by GPR-based MSD using the whole transmissibility. Type I Error are $\approx 17.5\%$, Type II Error are $\approx 44.2\%$.

The main idea of the proposed method is to assess whether unlabeled transmissibility is inside the identified confidence bands and use this information to indicate the structural state. This way, all relevant spectral information is taken into account for the classification procedure. Outlier detection was carried out to quantify the performance of this GPR model and identify damage. At this point, a convenient advantage of identifying the non-parametric stochastic GPR model is highlighted as a mean vector μ_* and covariance matrix σ_*^2 , as they allow seamless integration between the information provided by the stochastic regression and the statistical knowledge required to train a Mahalanobis squared distance (MSD) classifier. In this way, a GPR-based MSD is proposed to compute the distance between a new transmissibility curve $\mathcal{Y}_n$ and the identified multivariate Gaussian distribution. An essential advantage of using this GPR-based version instead of applying MSD directly to the data is that too many spectral lines are taken as features when applying it to a frequency curve. Consequently, the problem may have more features than realizations, which generate a singular covariance function when covariance is applied to the data alone. Thus, the proposed damage index is given by

$$DI_{\text{GPR}}(\mathcal{Y}_n) = (\mathcal{Y}_n - \mu_*)^\top \Sigma_*^{-1} (\mathcal{Y}_n - \mu_*). \quad (9)$$

Fig. 3 illustrates a bad performance of this first attempt at damage identification when applying the new damage index DI_{GPR} . The threshold value is obtained by using a Chi-squared hypothesis test considering only the healthy training-set. The presented result raises some initial considerations about the approach. This first model shows the contrast between using all available data and restricting the analysis to relevant information. Fig. 2 indicates that the frequency ranges outside the resonances do not provide information about the structural state, since the response in these regions remains similar for all conditions. Moreover, the amplitudes in these intervals are small, particularly for higher order modes, which increases the relative influence of background noise and hinders the identification capability of the GPR model. Note that, as the model is identified along the entire spectrum, i.e., the entire transmissibility curve is considered as a feature, every spectral line is treated as an independent measurement. The inclusion of frequency ranges outside resonances therefore inflates the dimensionality of the feature space without adding relevant information. This indiscriminate use of the full transmissibility confuses the classifier and reduces the effectiveness of the model.

To overcome these issues, the frequency ranges outside the resonant regions were disregarded in both the learning dataset and the GPR-based MSD. Fig. 4 shows the improved model, while the corresponding classifier is presented in Fig. 5. With the reduced

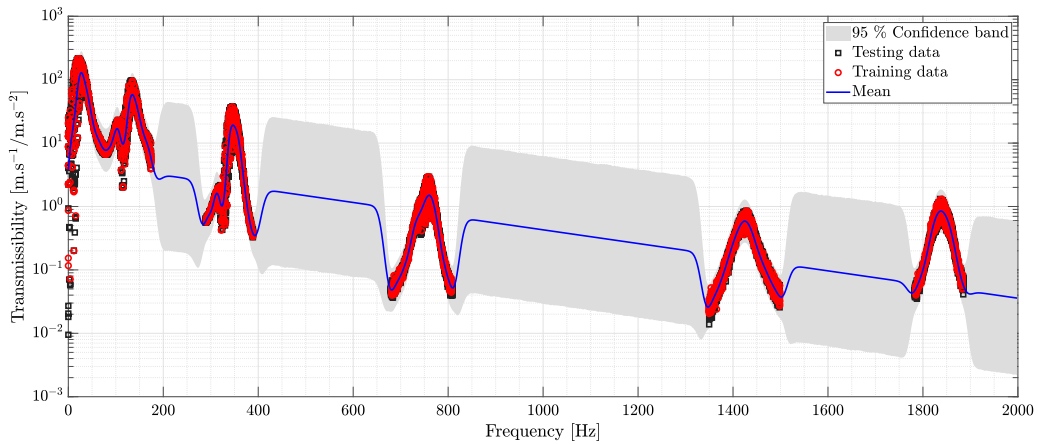


Fig. 4. GPR model identified using only the frequency range around the vibration modes.

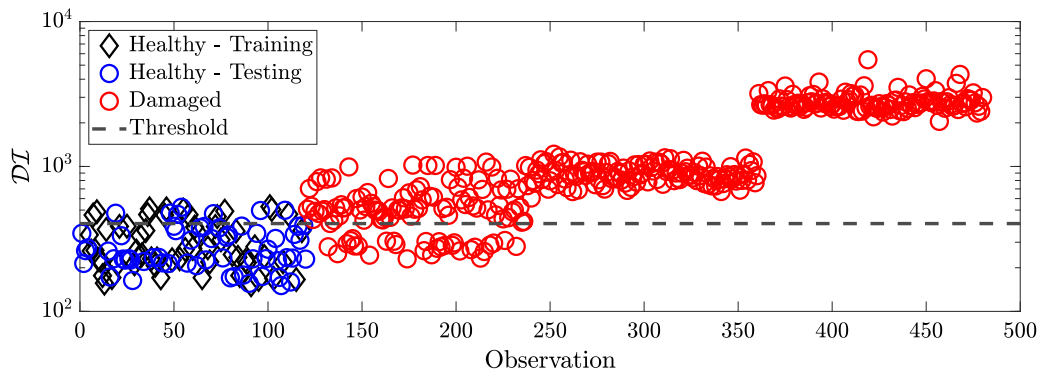


Fig. 5. *DI* computed by GPR-based MSD using only the frequency range around the vibration modes. Type I Error are $\approx 16.67\%$, Type II Error are $\approx 25.00\%$.

feature space, the computational cost decreased and it became possible to train the GPR model using half of the healthy realizations randomly selected, corresponding to 48 transmissibility curves that now compose the dataset D .

The identified model in Fig. 5 depicts important aspects of the GPR-based classifier. First, the model is a robust method for uncertainty quantification and for making stochastic predictions interpolated between learning data. However, GPR cannot make accurate predictions in regions where information is scarce. When moving away from the end of a resonance region, the confidence bands expand rapidly, which shows that the model loses its ability to capture the shape of the response and instead accepts any trend consistent with the decay of the transmissibility data. These limitations, although expected, do not compromise the classification, since only the amplitudes within the ranges used in training contribute to the damage index, leading to a significant improvement in identification.

The performance of the hypothesis test based on the damage index further illustrates the capabilities of the methodology. Type I (false positives) and Type II (false negative) error rates are approximately 16.67% and 25.00% respectively (Fig. 5), compared with Type I $\approx 17.5\%$ and Type II $\approx 44.2\%$ (Fig. 3) obtained when the entire frequency spectrum is considered. These results show a clear reduction in false negatives, although further improvements remain possible, particularly to lower their occurrence, which is of utmost importance for practical applications. The following section addresses these limitations and presents strategies to enhance the reliability of the methodology.

4. Sensitivity-based assessment of structural state

A GSA is employed to perform further improvements. The main idea is to select the frequency ranges that correspond to their respective vibration modes associated with the structural state change. This is a highly relevant analysis that utilizes all (and only) pertinent features of a classifier. Additionally, in the context of increasing big data and deep learning approaches, it is common for algorithms to use more and more information that may be redundant or irrelevant for a specific analysis, which unnecessarily burdens the solution of simple problems.

In general, the sensitivity analysis applied to any given system controlled by a set of parameters aims to correlate the variability of its parameters with the output of the system, thereby verifying which parameters propagate the most uncertainty in the system [22]. This information makes it possible to determine which parameters require more rigorous characterization through stochastic analysis and which settings can be adjusted to achieve specific system characteristics or improvements, while variations in the remaining parameters have limited influence on the response. In global analysis, the effect of each parameter is estimated as an average over the possible values of the others, whereas local sensitivity analysis investigates the influence of parameters individually by varying them one at a time. A brief but sufficiently detailed overview of GSA, including the concepts and mathematical background, can be found in [23]. In contrast, [22] provides a comprehensive guide to its application.

4.1. Variance-based global sensitivity analysis

Among the various GSA methods, such as the Morris method [24] and the Borgonovo indices [25], the variance-based global sensitivity analysis method (Sobol's Indices) [26] is notable. Homma and Saltelli [27] highlight Sobol's indices as the most general and flexible approach to dealing with GSA in different scenarios (linear, nonlinear, analytical, experimental, etc.), besides obtaining superior results. These indices are based on the decomposition of the system response Y in terms of the contribution of a set of N_p parameters $\mathbf{X} = \{X_1, X_2, \dots, X_{N_p}\}$, which is known as the Hoeffding-Sobol decomposition [26,28]:

$$Y = \mathcal{M}(\mathbf{X}) = \mathcal{M}_0 + \sum_{i=1}^{N_p} \mathcal{M}_i(X_i) + \sum_{i<j} \mathcal{M}_{ij}(X_i, X_j) + \dots + \mathcal{M}_{1\dots N_p}(X_1 \dots X_{N_p}), \quad (10)$$

where $\mathcal{M}$ indicates mathematical operators over the parameters that obtain their influence in the response Y . Note, also, that no restrictions are made in the considered response Y , i.e., it can be any quantity of interest from the system that is a function of $\mathbf{X}$. On this decomposition, the first term $\mathcal{M}_0$ is taken as the expected value of Y over the considered distribution of parameters. It is, therefore, not a function of their value in a specific realization. Considering that the set variables $\mathbf{X}$ follows an uniform distribution with support $D_X = [0, 1]^{N_p}$, $\mathcal{M}_0$ can be written as

$$\mathcal{M}_0 = \int_{D_X} \mathcal{M}(\mathbf{X}) d\mathbf{X}. \quad (11)$$

The first order terms $\mathcal{M}_i(X_i)$, in turn, are the conditional expected value related to the parameter X_i :

$$\mathcal{M}_i(X_i) = \int_0^1 \dots \int_0^1 \mathcal{M}(\mathbf{X}) d\mathbf{X}_{\sim i} - \mathcal{M}_0, \quad (12)$$

where the notation $\sim$ indicates an excluded parameter. The higher order components are obtained similarly, as can be seen in the second order $\mathcal{M}_{ij}(X_i, X_j)$ terms, that are the conditional expected value for the parameters X_i, X_j ($i \neq j$):

$$\mathcal{M}_{ij}(X_i, X_j) = \int_0^1 \dots \int_0^1 \mathcal{M}(\mathbf{X}) d\mathbf{X}_{\sim ij} - \mathcal{M}_0 - \mathcal{M}_i(X_i) - \mathcal{M}_j(X_j). \quad (13)$$

Due to orthogonal decomposition properties [27], the total variance of Y can be defined as

$$D = \int_{D_X} \mathcal{M}^2(\mathbf{X}) d\mathbf{X} - \mathcal{M}_0^2, \quad (14)$$

as well as the partial variances, given by

$$D_{1,2,\dots,N_p} = \int_0^1 \dots \int_0^1 \mathcal{M}_{1,2,\dots,N_p}^2(x_1, x_2, \dots, x_{N_p}) dx_1 dx_2 \dots dx_{N_p}. \quad (15)$$

Thus, the Hoeffding-Sobol decomposition can be rewritten conveniently in terms of a variance decomposition:

$$D = \sum_{i=1}^{N_p} D_i + \sum_{i<j} D_{ij} + \dots + D_{1,2,\dots,N_p}. \quad (16)$$

Finally, dividing both sides by the total variance D leads to the sum of Sobol's indices:

$$\sum_{i=1}^{N_p} S_i + \sum_{i<j} S_{ij} + \dots + S_{1,2,\dots,N_p} = 1, \quad (17)$$

defined for any order as

$$S_{i\dots N_p} = \frac{D_{i\dots N_p}}{D}. \quad (18)$$

Thus, it can be seen that Sobol's indices quantify the portion of the total variance of the quantity of interest generated by the variability of each parameter (for the first-order index) and the combined variability of parameters (for the higher-order index). It can be inferred from the integrals over the probability distributions of Eqs. (11) to (15) that the most general way to compute Sobol's indices is through Monte Carlo simulations [29]. However, even though and Saltelli [27] discussed many advantages of Sobol's approach, they also mentioned the computational cost to reach convergence through Monte Carlo as the main drawback, mainly

when the number of parameters N_p increases. Furthermore, this kind of methodology impedes practical applications. An alternative approach is to reduce the computational cost by identifying a surrogate model that can represent the relationship $Y = \mathcal{M}(\mathbf{X})$. An approach that can be conveniently integrated with obtaining Sobol's indices is the use of the polynomial chaos expansion (PCE) surrogate model.

4.2. Computing Sobol' indices via PCE

PCE was introduced by [30] as a surrogate model capable of representing a finite-variance random variable through the sum of multivariate orthonormal polynomials. Over the years, PCE has caught the attention of the uncertainty quantification community as an efficient method for computing nonlinear behavior in stochastic applications [31]. In addition, as it can analytically compute statistical moments without the need for Monte Carlo methods, Sudret [32] noted the convenience of using a PCE surrogate model to compute Sobol's indices analytically. Taking the quantity of interest from the Hoeffding-Sobol decomposition $Y = \mathcal{M}(\mathbf{X})$ (Eq. (10)) as a finite-variance random variable, it can be expanded as

$$Y \approx \sum_{\alpha \in \mathcal{A}} y_{\alpha} \Psi_{\alpha}(\mathbf{X}), \quad (19)$$

where, y_{α} are unknown deterministic coefficients, $\mathcal{A} \subset \mathbb{N}^M$ is the truncation set selected based on all possible multi-indices polynomials, and Ψ_{α} is the polynomial basis composed of multivariate orthonormal polynomials $\Phi_j^{(i)}$, concerning the joint probability density function (PDF) $f_{\mathbf{X}}$ of $\mathbf{X}$ that satisfy the condition

$$\left\langle \Phi_j^{(i)}(x_i), \Phi_k^{(i)}(x_i) \right\rangle = \int_{D_{X_i}} \Phi_j^{(i)}(x_i) \Phi_k^{(i)}(x_i) f_{X_i}(x_i) dx_i = \delta_{jk}, \quad (20)$$

where i identifies the input variable for which they are orthogonal, as well as the corresponding polynomial family, j and k correspond to the degree of the polynomial, $f_{X_i}(x_i)$ represents the input distribution, and δ_{jk} is the Kronecker delta. The literature proposes some classical families of univariate orthonormal polynomials that satisfy (20) depending on the distributions of the input variables. At the same time, the remaining are just defined as the coefficients y_{α} . For this, a Least-Squares minimization problem is carried out, in which Eq. (19) is truncated as follows:

$$Y = \sum_{j=0}^{P-1} y_j \Psi_j(\mathbf{X}) \equiv \mathbf{y}^T \Psi(\mathbf{X}) + \epsilon_p, \quad (21)$$

in which ϵ_p is the truncation error, $y_{\alpha} = \{y_0, \dots, y_{P-1}\}^T$ is the coefficients vector, $\Psi(\mathbf{X}) = \{\Psi_0, \dots, \Psi_{P-1}\}^T$ is the polynomial matrix and

$$P \equiv \frac{(k+p)!}{(k!p!)}, \quad (22)$$

where k is the number of input variables and p is the maximum degree of the polynomials. Thus, the least-square minimization problem is stated as

$$\hat{\mathbf{y}} = \arg \min \mathbb{E} \left[(\mathbf{y}^T \Psi(\mathbf{X}) - \mathcal{M}(\mathbf{X}))^2 \right], \quad (23)$$

that can be solved to a given experimental design dataset $(\mathcal{X}, \mathcal{Y})$

$$\hat{\mathbf{y}} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathcal{Y}. \quad (24)$$

Here, $A_{ij} = \Psi_j(x^{(i)})$, $i = 1, \dots, n$, $j = 0, \dots, P-1$ and $\mathcal{Y} = \{y^{(1)}, \dots, y^{(N)}\}$.

From an identified PCE model that satisfies the orthonormality condition given in Eq. (20), it is known that it has $\Psi_0 \equiv 1$ (constant term) and $\mathbb{E}[\Psi_{\alpha}(\mathbf{X})] = 0 \forall \alpha \neq 0$. Then, statistical moments and Sobol's indices can be analytically computed. This way, the mean value $\hat{\mu}$ can be estimated as

$$\hat{\mu}(Y) = \mathbb{E}[\mathcal{M}(\mathbf{X})] \approx y_0, \quad (25)$$

as well as the variance $\widehat{Var}$ of the system response

$$\widehat{Var}(Y) = \mathbb{E}[(\mathcal{M}(\mathbf{X}) - \mu)^2] \approx \sum_{\substack{\alpha \neq 0 \\ \alpha \in \mathcal{A}}} y_{\alpha}^2. \quad (26)$$

Consequently, the first and second-order Sobol's indices are given by

$$S_i \approx \sum_{\substack{\alpha \neq 0 \\ \alpha \in \mathcal{A}_i}} y_{\alpha}^2 / \sum_{\substack{\alpha \neq 0 \\ \alpha \in \mathcal{A}}} y_{\alpha}^2, \quad (27)$$

$$S_{ij} \approx \sum_{\substack{\alpha \neq 0 \\ \alpha \in \mathcal{A}_{ij}}} y_{\alpha}^2 / \sum_{\substack{\alpha \neq 0 \\ \alpha \in \mathcal{A}}} y_{\alpha}^2. \quad (28)$$

The analysis is the same for higher-order indices. However, most works deal only with the second-order term, as the influence usually decreases with the order.

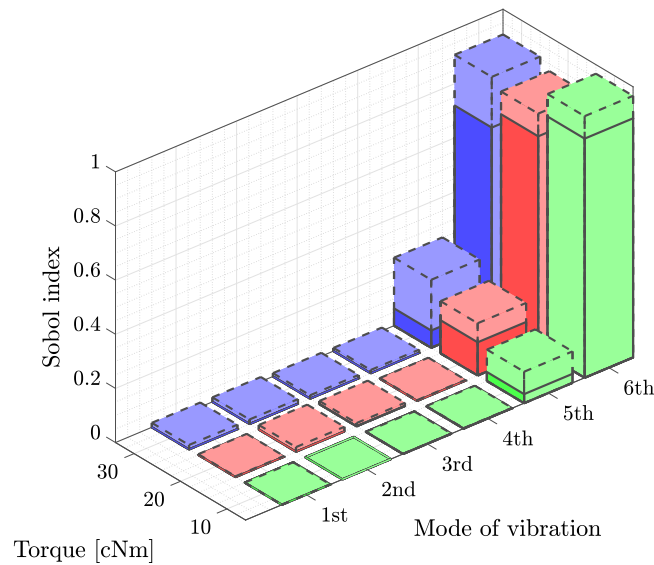


Fig. 6. Sobol' indices computed for all damage levels, evaluated over frequency regions associated with each vibration mode. Solid lines denote first-order Sobol indices, while the dashed line represents the second-order index.

4.3. The use of Sobol's indices to feature selection

In the context of using Sobol's indices to aid feature selection, the label information is treated as a quantity of interest, whereas the GPR-based MSD damage index computed around each mode of vibration is considered a parameter; that is, the structural state is expressed as a function of the damage index. This way, a high value of Sobol's index, verified by the damage index calculated around a vibration mode, would indicate that when the structural state varies, there is a corresponding variation in the damage index. Therefore, it indicates that this mode is sensitive to damage. Otherwise, a low value of Sobol's index indicates that variations in the structural state are not correlated with variations in the damage index, suggesting that this mode is insensitive to damage. Towards this background, the Sobol indices were computed for each damage condition considering all the vibration modes, resulting in Fig. 6. The dashed line represents the second-order Sobol index, indicating that most of the contribution arises from first-order effects (solid lines).

As depicted in Fig. 6, the frequency region involving the fifth and mainly sixth modes of vibration demonstrated the highest values of sensitivity to damage. In addition to that, a considerable influence of the combination of the fifth and sixth modes can be observed. Despite obtaining the same results, GSA allows for a more objective analysis without relying too heavily on user interpretation of the dynamics to which the structure is subjected. Additionally, it provides a very general analysis. It can select the most different kinds of features for other problems. Physical visualization may be more complex, especially with the extensive available features.

Another metric is employed to corroborate the results obtained from the GSA, namely the MacroF1 score, defined as

$$MacroF1 = 2 \frac{precision \times recall}{precision + recall}, \quad precision = \frac{TP}{TP + FP}, \quad recall = \frac{TP}{TP + FN}, \quad (29)$$

where TP , FP , and FN denote true positives, false positives, and false negatives, respectively. The performance of all vibration modes was also calculated in all structural states, resulting in Fig. 7.

Fig. 7 clearly shows which vibration modes can detect each structural state. Values close to 1 indicate nearly perfect detection. Most modes can identify the most severe damages, except modes 1 and 3, which consistently show the lowest performance across all conditions. Early-stage damage requires more sensitive modes for detection. For an initial damage of 30 cNm, only modes 5th and 6th achieve satisfactory classification. This behavior is physically consistent with the findings reported in [17], where these modes were shown to induce larger relative displacements at the bolted joint interface, thereby enhancing contact nonlinearity and hysteretic dissipation effects associated with loosening.

Based on these findings, new classifiers were trained using only the 1st, 5th, 6th, and both 5th and 6th vibrating modes, with the corresponding results shown in Figs. 8, 9, 10, and 11, respectively. Once again, the threshold value is obtained by using a Chi-squared hypothesis test considering only the healthy training-set.

It can be seen that there is much variability in the values of the damage index; however, the GSA did not detect any relation between these variations and the structural state. As predicted by the Sobol index, the classifier trained using the first vibration mode in Fig. 8 fails to detect damage in any structural condition, yielding the worst result. On the other hand, when using modes indicated as sensitive, specifically the fifth (Fig. 9) and the sixth (Fig. 10), it revealed a classifier with significantly better performance across all damage conditions. Finally, Fig. 11 represents the best-obtained classifier since it considers the information of both the most sensitive modes of vibration, which even has a considerable second-order influence by combining the two intervals.

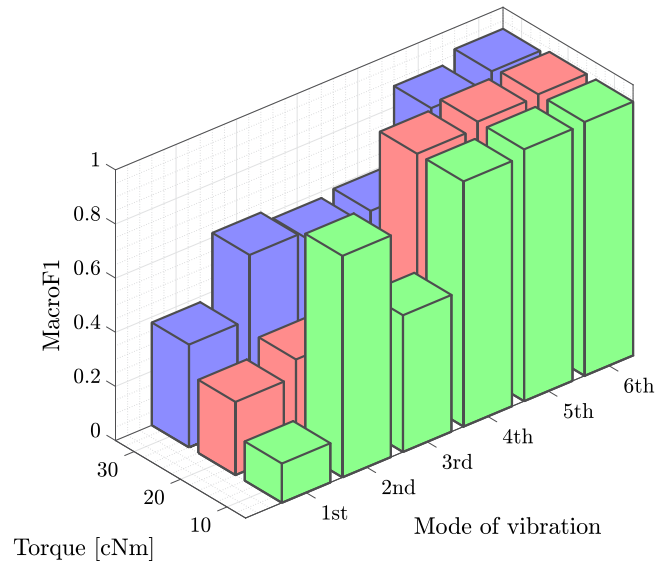


Fig. 7. MacroF1 score computed for all damage levels considering each vibration mode.

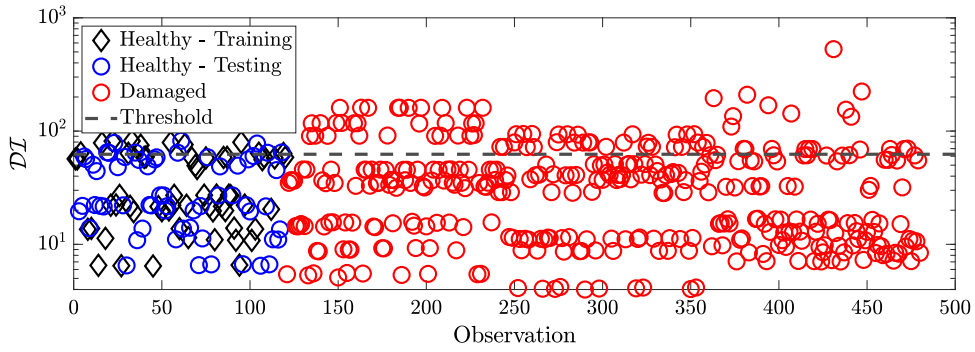


Fig. 8. DI computed by GPR-based MSD using 1st mode. Type I Error are $\approx 6.67\%$, Type II Error are $\approx 75.0\%$, and MacroF1 ≈ 0.38 .

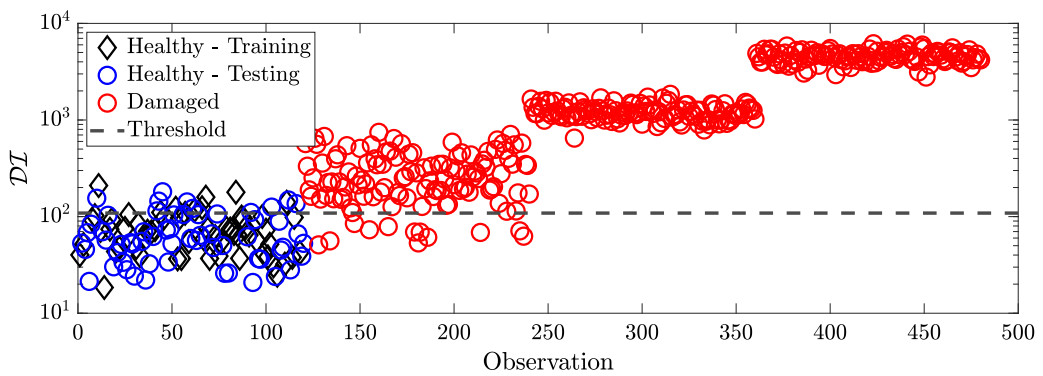


Fig. 9. DI computed by GPR-based MSD using 5th mode. Type I Error are $\approx 15.0\%$, Type II Error are $\approx 10.0\%$, and MacroF1 ≈ 0.88 .

5. Influence of the excitation level in the structural behavior

In identifying damage in nonlinear structures, it can be challenging to determine a direct cause-and-effect relationship between structural changes and damage. Most works address this issue, attempting to isolate the manifestation of damage from other possible variations. However, recent works in the literature have already proposed frameworks to address structural variability in various

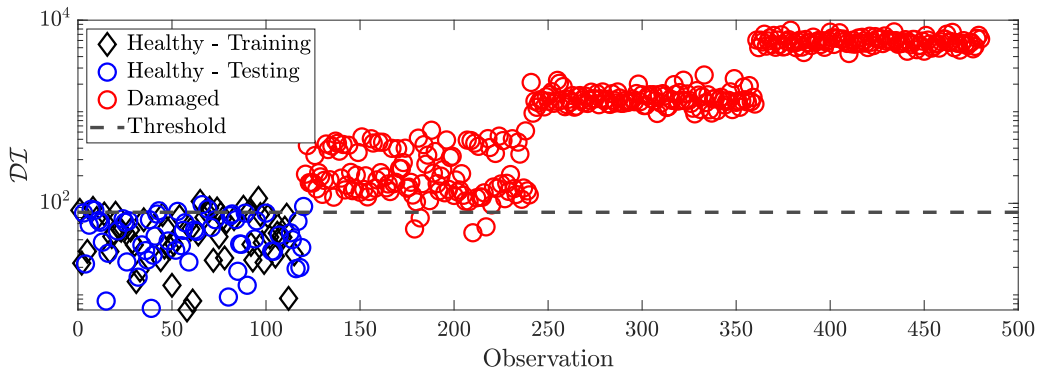


Fig. 10. DI computed by GPR-based MSD using 6th mode. Type I Error are $\approx 13.33\%$, Type II Error are $\approx 3.33\%$, and MacroF1 ≈ 0.92 .

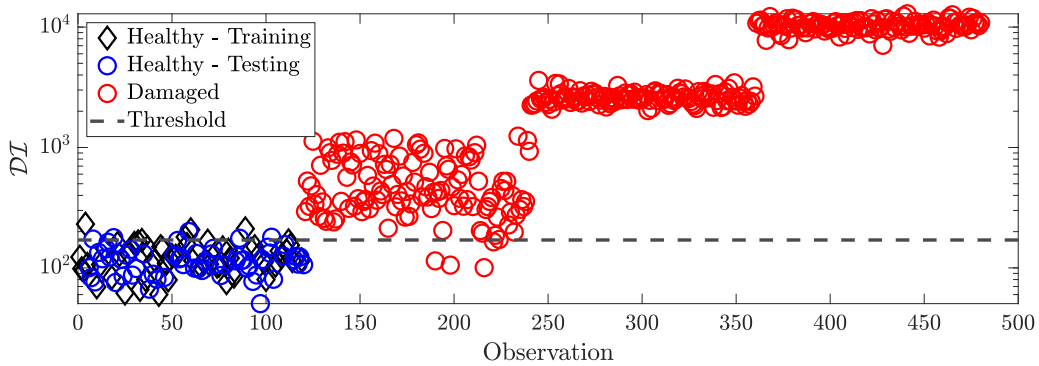


Fig. 11. DI computed by GPR-based MSD using 5th and 6th modes. Type I Error are $\approx 7.5\%$, Type II Error are $\approx 3.33\%$, and MacroF1 ≈ 0.95 .

contexts. For example, [13] used a Gaussian mixture approach to propose an SHM strategy for a dataset with variability induced by bolt disassembly. In [33], a domain adaptation between different structures is presented to transfer the knowledge acquired in a labeled structure to a different, unlabeled one with a different structural behavior.

Another variable source in structural behavior often disregarded in nonlinear SHM is the consideration of different excitation conditions. Building generalized nonlinear models is a challenge task, and modeling is often limited to a specific range of interest. Similarly, it is not expected that a SHM framework designed for a single nonlinear regime would have the same performance for another excitation condition due to the challenging issue of distinguishing whether the nonlinearity variability source is, in fact, damage or just a change in the operation condition. To illustrate this scenario, initial investigations on the variation in the transmissibility curve of the same experimental setup and conditions from a single assembly of the Orion beam presented in Fig. 1 are presented in Fig. 12. For this purpose, three different nonlinear regimes are considered, varying the input level controlled by the acceleration measured in the base attached to the shaker: low (4 m/s^2), medium (8 m/s^2), and high (12 m/s^2) excitation levels. The observed amplitude-dependent effects under the 80 cNm condition reflect intrinsic joint nonlinearities associated to the relative motion between connected surfaces, and should not be interpreted as evidence of insufficient preload. As it has already been observed that there is no significant difference between the 80 cNm and 60 cNm conditions [13], the healthy reference condition is taken as $80\text{--}60 \text{ cNm}$.

Fig. 12 focuses on the fifth and sixth modes of vibration, which were extensively discussed as being more sensitive to the nonlinear effects related to relative motion in the bolted connection, as analyzed through modal features and the GSA in Section 4. As can be seen, the nonlinear damping effect increases with increasing excitation level. This effect can be challenging to interpret in an SHM context, since the increase in excitation stresses the lap joint region, and as consequence, nonlinearities are expected to appear. Conversely, loosening the tightening torque also promotes relative motion between the connected surfaces, which may lead to responses that are difficult to distinguish from those induced by increased excitation. Additionally, the degradation of the resonance peaks increases the estimation errors in modal features, when compared to those of linear responses. Nevertheless, the resonance frequencies for these specific modes have been carefully estimated and are depicted in Fig. 13. This figure illustrates the changes in dynamic behavior resulting from modifications in torque, leading to a loosened state, and variations in the excitation level. As the damage becomes easier to identify as the tightening torque decreases, this section considers only the 30 cNm torque as the loosened condition.

The boxplots in Fig. 13, along with the transmissibility in Fig. 12, demonstrate the difficulty of training a transmissibility-based damage classifier when more than one input level in a nonlinear regime is considered. As can be seen, if only a single input

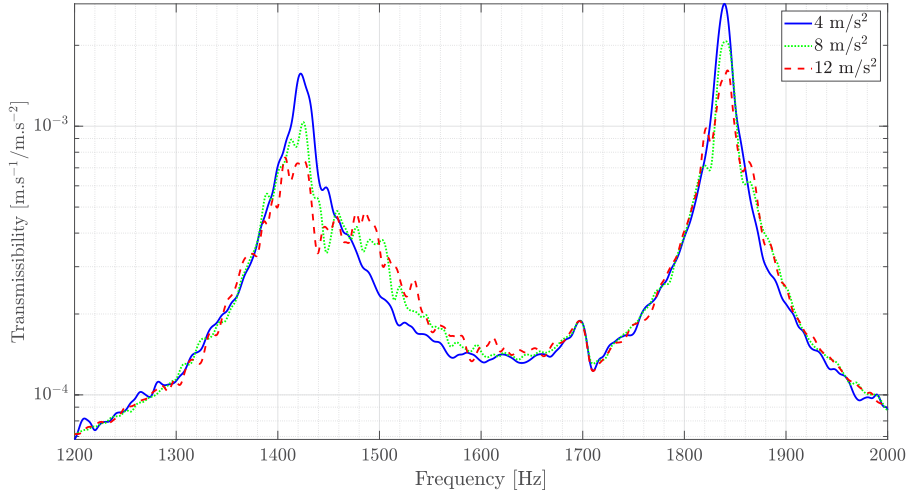


Fig. 12. Transmissibility of the Orion beam under different excitation levels in healthy reference condition.

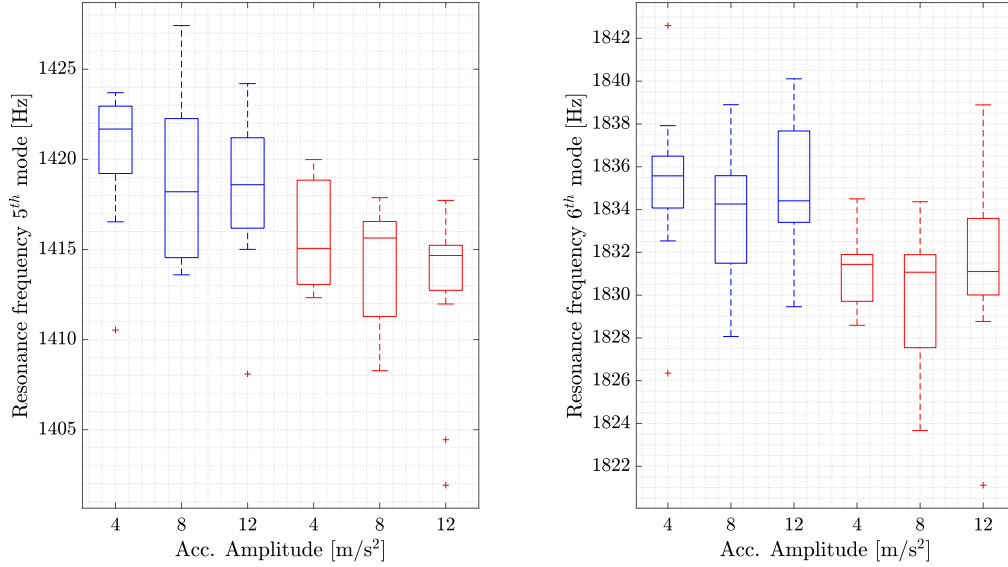


Fig. 13. Boxplot of the fifth and sixth resonance frequencies estimated for the healthy (80-60 cNm) ■ and damaged (30 cNm) ■ condition for different excitation levels. Red dots indicate outliers.

level is considered, particularly with weaker nonlinear behavior due to a low excitation level (4 m/s² acceleration on the base), a simple classifier would be feasible using distance metrics. However, when different amplitudes are taken into account, confusion arises between variations due to damage and input level, necessitating, for example, the use of different statistical models for each amplitude. This challenging issue is evident in Fig. 14, where a damage index is computed by applying a simple MSD that considers both frequencies for all amplitudes, but fails in damage identification. Alternatively, a three-dimensional GPR was trained for damage detection by combining the information on transmissibility, frequency, and acceleration RMS within a stochastic and non-parametric transmissibility surface model.

A three-dimensional version of the GPR follows the same theoretical basis as the two-dimensional one. Taking Eq. (1) as the starting point, the regression function $f(\cdot)$ now has the input matrix

$$\mathbf{X}^{(i)} = \begin{pmatrix} \omega_1^{(i)} & r^{(i)} \\ \vdots & \vdots \\ \omega_{N_f}^{(i)} & r^{(i)} \end{pmatrix}, \quad i = 1, 2, \dots, N_t \quad (30)$$

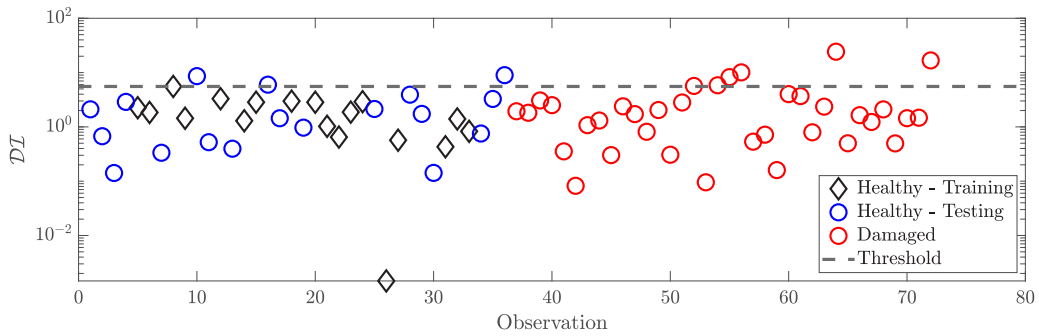


Fig. 14. DI computed by MSD using frequencies of the 5th and 6th modes of vibration.

where $\omega_{(j)}^{(i)}$ is the j th frequency value of the transmissibility i th from the data set, similar to the two-dimensional application. The variable $r^{(i)}$ is the new dimension added and corresponds to the RMS value of the base acceleration signal computed in the time domain. As can be seen, this value is the same for the entire frequency range of the sample i th, and therefore is repeated for all rows of the input $\mathcal{X}^{(i)}$; and $\epsilon_S^{(i)}$ is the stochastic variability of the data, still described as a Gaussian distribution with zero mean. The matrix $\mathcal{X}^{(i)}$ is made up of two independent variables that are used to estimate a transmissibility function $\mathcal{Y}$, that is, the frequency range and the output RMS. In this way, the dataset $\mathcal{D}$ to train the regressor is similar to Eq. (3), but with $\mathcal{Y} \in \mathbb{R}^{(N_f \times N_f) \times 2}$ being the matrix of frequencies and acceleration RMS.

A strategy similar to that proposed in Section 3 was used for damage detection. For this application, only the transmissibility around the sixth vibration mode was considered, which is shown to be more sensitive to damage in Fig. 6. This decision was made to circumvent the deployment of a computationally costly model, as data from three different amplitudes and the newly added dimension related to acceleration RMS were considered. Additionally, since the main results involve a three-dimensional stochastic model, using a more limited frequency spectrum facilitates a more comprehensible presentation and analysis.

To further enrich the dataset, an additional 20 samples were synthesized for each tightening torque condition by superimposing a white noise component on the velocity signal. The purpose of this procedure was to introduce sufficient variability in the transmissibility. However, since the global excitation RMS values (4, 8, and 12 m/s²) remained constant across realizations, directly using them as inputs for the GPR model would not provide meaningful information. To overcome this limitation, the acceleration signals were band-pass filtered around the sixth vibration mode, and the RMS was recomputed from these filtered signals. This yielded RMS values that varied from one realization to another, a consequence of the band-pass filtering around the sixth mode and not of the underlying broadband excitation. In this way, the GPR input captured variability in the relevant frequency range while avoiding the use of constant values. Attempting to induce RMS variability directly at the raw signal level would have completely masked the resonance amplitude of the sixth mode, making it unusable. The adopted approach preserved the physical information in the transmissibility curves while providing a practical level of variability suitable for training the model.

The dataset used in this section comprised 612 transmissibility functions obtained from three excitation levels (4, 8, and 12 m/s²) and three tightening torque conditions (30, 60, and 80 cNm). For each (torque $\times$ excitation-level) combination, 48 experimental realizations were available, complemented by 20 synthetic samples generated by adding white noise, resulting in 68 samples per combination. This yields 204 samples per torque level and 612 samples in total. The 60 cNm and 80 cNm conditions were treated as healthy (408 samples), while 30 cNm was considered as damaged (204 samples). For the 408 healthy samples, 25% were randomly selected for the training set, resulting in a three-dimensional GPR model trained on 102 transmissibility curves, which constitute the transmissibility surface. Following the training phase, the model's confidence intervals were determined by estimating stochastic transmissibility functions across the range between maximum and minimum experimental RMS amplitudes. The confidence bands were defined by the two boundary surfaces of a region containing 95% of the computed functions. Fig. 15 presents the resulting surface model and experimental transmissibility curves.

As illustrated in Fig. 15, the GPR model provides an intuitive and interpretable framework for damage identification. Here, the experimental transmissibility curves for the damaged scenarios lie outside the confidence intervals, whereas most of those for the healthy scenarios stay inside. Fig. 16 illustrates three planned sections for each motion regime controlled by the base acceleration to exemplify performance across different acceleration amplitudes, with frequency on the x -axis and transmissibility on the y -axis. For each regime, two experimental curves are shown: one representing a healthy condition and another depicting a loosened state. Additionally, the mean and confidence interval predictions generated by the GPR model are also included.

The final performance of the model was evaluated using the GPR-based MSD as a damage index, according to Eq. (9), with the statistical parameters μ_* and Σ_*^2 estimated for different RMS acceleration values. The results, shown in Fig. 17, indicate a few false alarms, demonstrating the effectiveness of the proposed framework in detecting damage. The threshold value is empirically obtained considering the maximum damage index computed for the training data samples. Incorporating acceleration RMS as an additional dimension in the training data allows the model to differentiate damage-induced variations and to capture the nonlinear structural behavior expected under varying motion regimes.

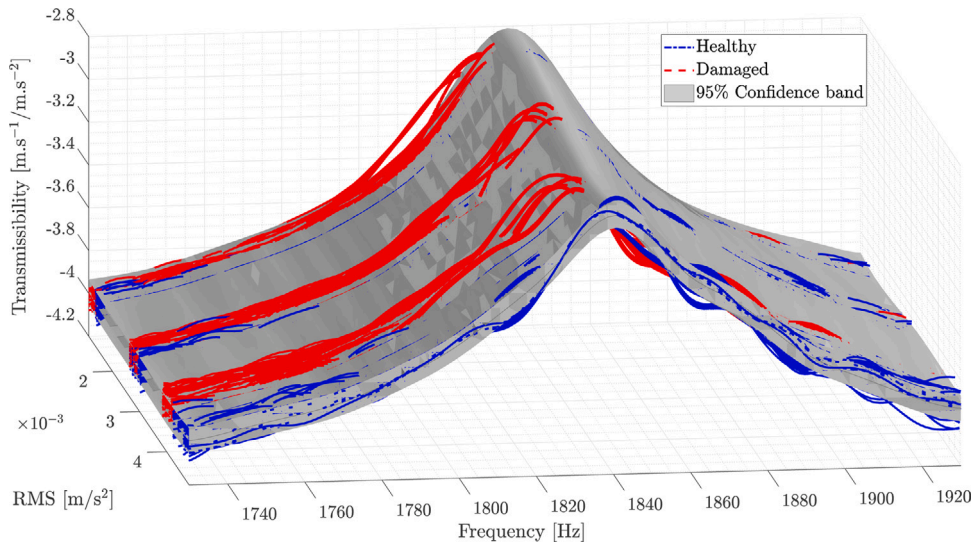


Fig. 15. Transmissibility surface estimated by GPR with the experimental transmissibility functions.

In this study, the 3D analysis is intentionally formulated as a binary classification problem, distinguishing a healthy condition (80–60 cNm) from a loosened condition represented by 30 cNm. More severely loosened states (10 cNm and 20 cNm) were not considered in this analysis. As a consequence, the results presented in this section should be interpreted within a binary damage-detection framework rather than as a multi-severity classification. While the proposed methodology could be extended to discriminate between multiple loosening levels by introducing additional labels or by exploiting complementary frequency-band information, such an extension was beyond the scope of the present work and was not investigated here.

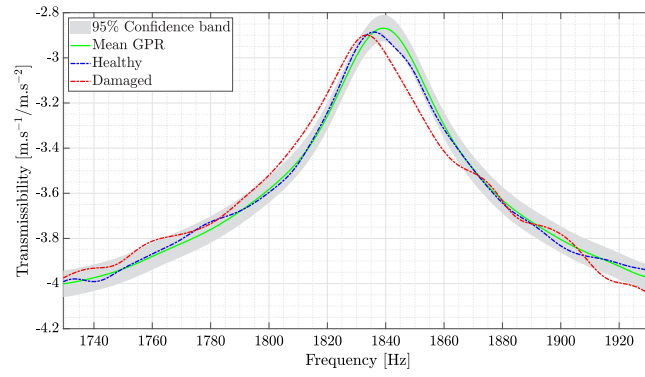
6. Final remarks

The results presented in this work highlight several important considerations regarding the use of transmissibility functions and GPR for vibration-based SHM of bolted joints. The transmissibility function proved to be an effective data for damage detection, as it allows direct use of the measured signals without the need for additional feature extraction algorithms. Nevertheless, due to the large number of spectral lines, classifiers may be trained with redundant or insensitive information, which emphasizes the importance of proper feature selection. The adoption of a GPR-based MSD index proved to be a robust alternative, as it avoids singularities in the covariance matrix that usually appear when the number of features is large. Furthermore, the combination of MSD with a stochastic regression approach provides a reliable surrogate model for the undamaged structural dynamics, enabling a more accurate damage index.

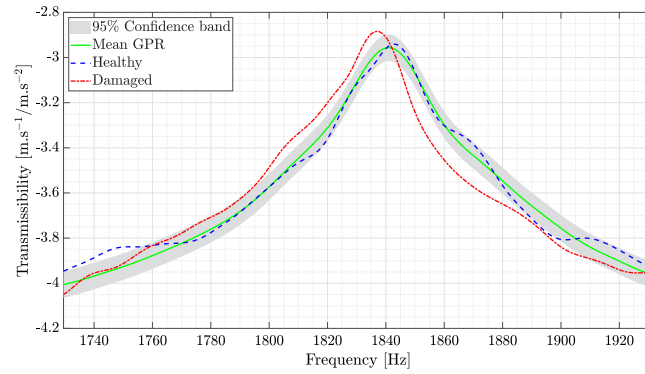
The selection of relevant frequency ranges was shown to be crucial for improving classifier performance while reducing computational cost. The application of GSA with Sobol's indices effectively identified the frequency intervals most sensitive to bolt loosening, allowing less informative regions to be excluded. This procedure improved computational efficiency and reduced the risk of false detections. Extending the framework to a three-dimensional GPR formulation further enhanced the methodology. By incorporating excitation levels into the regression, it became possible to distinguish nonlinear variability caused by operational changes from damage conditions. This capability is particularly valuable in practical scenarios, where structures may operate under a variety of conditions that could otherwise be misinterpreted as structural degradation.

The Orion Beam case study illustrated the applicability of the proposed framework in three progressively challenging scenarios. The first involved low repeatability due to reassembly, which introduced significant variability in the dataset. The second focused on damage detection directly from frequency response functions without any feature extraction procedure. The third addressed the effect of nonlinear dynamics under different excitation levels, where a stochastic transmissibility surface was introduced to represent the baseline condition across multiple input regimes. The results confirmed that transmissibility responses are strongly affected by bolt tightening torque, whether modal parameters are extracted or the full spectrum is analyzed. Although the experimental validation is restricted to a single laboratory structure with fixed geometry and boundary conditions, the considered scenarios already incorporate realistic operational constraints, such as limited experimental realizations, random excitation, and the absence of direct force measurements.

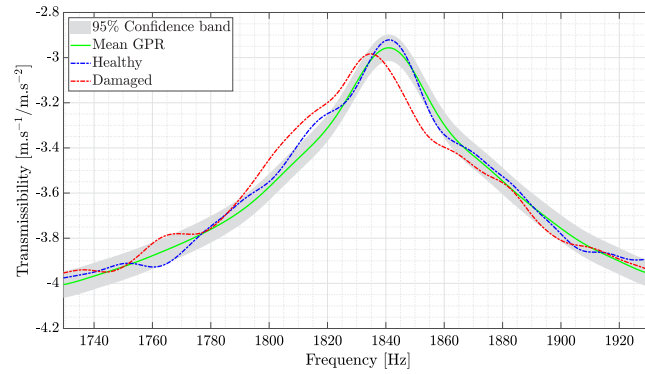
Future research will explore damage detection directly in the time domain using GP-NARX models for metamodeling of time series. Another promising direction is the development of hybrid approaches that integrate physics-based models with machine learning, including reinforcement learning strategies. Physical knowledge about bolted joints, such as stiffness reduction and increased damping associated with loosening, provides valuable information that can guide these hybrid approaches. In this work, the GSA was applied directly on the damage index, which implicitly required knowledge of the structural state to improve the



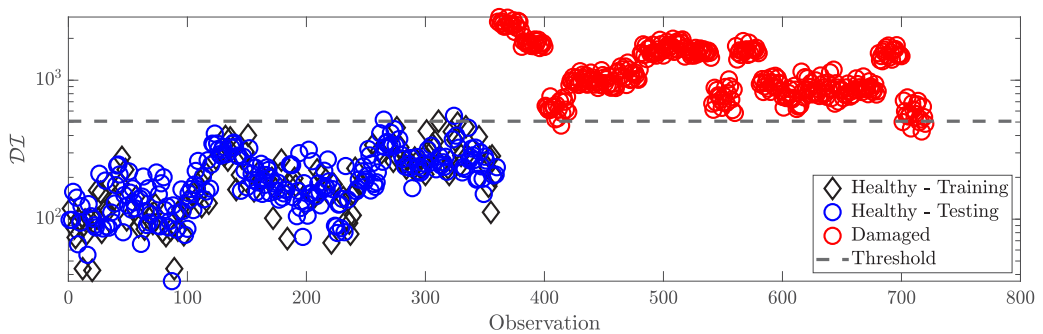
(a) Low acceleration level (4 m/s²).



(b) Medium acceleration level (8 m/s²).



(c) High acceleration level (12 m/s²).

Fig. 16. Experimental and GPR estimated transmissibility functions for different acceleration levels.**Fig. 17.** *DI* computed by GPR-based MSD using 5th and 6th modes. Type I Error are $\approx 0.56\%$, Type II Error are $\approx 1.67\%$, and MacroF1 ≈ 0.98 .

classifier. In real-world scenarios, however, data from damaged conditions are typically scarce. Exploiting the underlying physics directly could offer insights into the most informative frequency ranges to consider, without relying on a priori evaluation of the damage index.

From a computational perspective, improving the scalability of the probabilistic framework through sparse GP formulations and more advanced dimensionality reduction strategies will also be essential for deployment in larger-scale systems.

Extending the proposed methodologies to more complex structures, including multiple bolted joints and additional environmental or operational variability (e.g., temperature changes or boundary condition variations), will be essential to enable simultaneous damage detection and localization under realistic operating conditions.

CRediT authorship contribution statement

Luccas Pereira Miguel: Writing – original draft, Visualization, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Rafael Teloli:** Writing – original draft, Supervision, Investigation, Formal analysis, Data curation. **Samuel da Silva:** Writing – original draft, Supervision, Project administration, Investigation, Funding acquisition, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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