Spectral phase-encoding and vectorial nonlinear propagation of femtosecond pulses for physical computation

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Abstract. Optical fibre-based extreme learning machines with spectral phase encoding are an unconventional computing platform that yields excellent results in machine learning benchmarks. Here we present the first numerical study of the effect of nonlinear polarization dynamics on fibre-ELM performance.

Fibre-based extreme learning machines (ELMs) perform computation by using nonlinear propagation to project information encoded on the spectral phase of a femtosecond input pulse into a higher-dimensional space [1]. Many groups [2–5, 9, 10] have reported experimental and numerical studies of fibre ELM operation, but have not to date considered the impact of polarisation (vectorial) effects during propagation. In this paper, we model polarisation effects in fibre ELM system using coupled nonlinear Schrödinger equations (GNLSE) model including birefringence, Raman scattering, self-steepening, and higher-order dispersion [8, 11]. We determine how ELM classification accuracy for the MNIST digits dataset depends on the input and readout polarisation state, with our results highlighting the importance of polarisation selection for best results.

With all terms having their usual meaning, the coupled GNLSEs are [8, 11, 12]:

$$\begin{aligned}\frac{\partial u_j}{\partial z} &= \left(i\frac{\Delta\beta_0}{2} - \frac{\Delta\beta_1}{2}\frac{\partial}{\partial\tau}\right)u_{3-j} - i\left(\frac{\beta_2}{2!}\frac{\partial^2}{\partial\tau^2} + i\frac{\beta_3}{3!}\frac{\partial^3}{\partial\tau^3} + \cdots\right)u_j \\ &+ i\gamma\left(1 + \frac{i}{\omega_0}\frac{\partial}{\partial\tau}\right)\left\{(1-f_R)\left(\frac{2}{3}|u_j|^2 + \frac{4}{3}|u_{3-j}|^2\right)u_j\right. \\ &+ \left. f_R\, u_j \int_{-\infty}^{\tau} h_R(\tau-\tau')\left[|u_j(\tau')|^2 + |u_{3-j}(\tau')|^2\right] d\tau'\right\}\end{aligned} \tag{1}$$

We model propagation in highly nonlinear fibre with birefringence of $\Delta n = 10^{-5}$. The detailed fibre parameters and propagation distances are different for anomalous and normal dispersion regime propagation, and are described in detail in Ref. [9]. Input pulses have ~150 fs duration and ~30 nm bandwidth centered around 1550 nm, with ~100 W or ~1200 W peak power for anomalous or normal dispersion regime propagation respectively. Each pulse is spectrally phase-encoded with a length-100 compressed representation of an MNIST handwritten digit, scaled to a maximum phase of 0.25π. We study the effect of polarisation by

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varying pump and analyzer angles using Jones matrices. After propagation, a standard ELM approach is used for 60000 training and 10000 testing output spectra using Moore-Penrose regression [9].

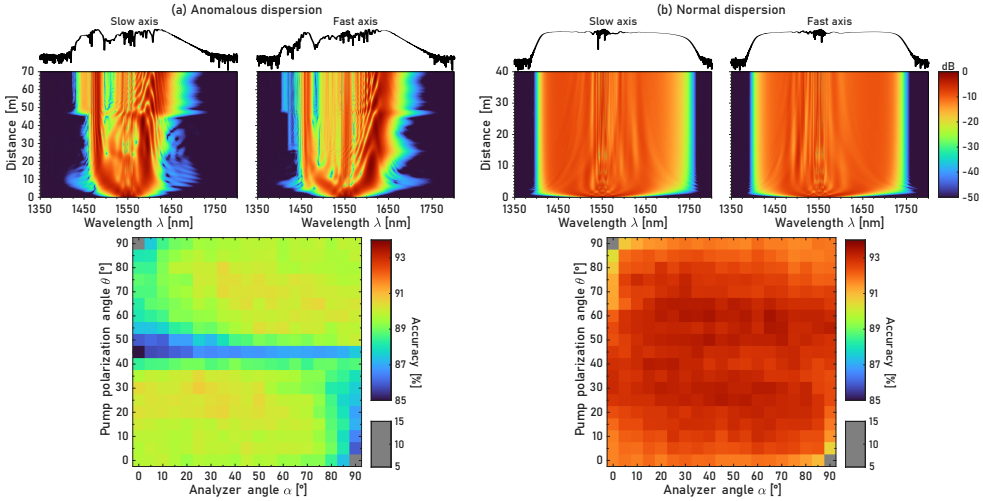


Figure 1. Results for anomalous (a) and normal (b) dispersion. The upper plots show spectral evolution on slow and fast axes for input angle $\theta = 45°$. The heatmaps show the MNIST classification accuracy varying the input angle and the selection readout angle of the output spectra.

Figure 1 (top) shows spectral evolution on the two fibre eigenaxes for (a) anomalous and (b) normal dispersion regime propagation [11]. The impact on ELM performance is examined by plotting the corresponding MNIST classification accuracy as a function of the input angle of the linearly polarised pump θ and the analyzer angle α used to read out the broadened spectra. Both angles are varied over $[0, 90°]$. The heatmaps in Fig. 1 (bottom) show how the ELM accuracy depends on both the input and readout polarisations, with greater variation and a more complex structure for anomalous dispersion propagation. Maximum accuracies of 91% and 93% can be obtained for anomalous and normal dispersion regime propagation, but these results show that careful control of polarisation is necessary for best results.

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