

# On the Construction of Load-Balanced (k,r-hop)-Connected Dominating Set for WSNs

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**Abstract**— Connected Dominating Sets (CDS) are selected to construct a virtual backbone in Wireless Sensor Networks (WSNs). Indeed, CDS are used for fault tolerant broadcasting, routing and even efficient gathering of data. On the other hand, r-hop connected k-dominating sets ((k,r-hop)-CDS) are introduced to reduce the size of CDS and the total amount of information to be transmitted. Unfortunately, no work considers the load-balance factor of (k,r-hop)-CDS. Recently, there is a proposition of constructing load-balanced CDS for WSNs. In this paper we re-use the same concept of selecting load-balanced CDS to construct a Load-Balanced (k,r-hop)-CDS (LB(k,r-hop)-CDS). Then, the allocation of dominatees to the appropriate dominators is done. Through simulations, the proposed method of constructing LB(k,r-hop)-CDS extends network lifetime by an important rate compared with the most recent (k,r-hop)-CDS construction method.

**Keywords**—component; Load-balancing, (k,r-hop)-CDS, Virtual backbone, Fault tolerance, WSNs.

## I. INTRODUCTION

WSNs are ad hoc networks composed of many sensor nodes supervised and controlled by a base station. Each sensor is a small device able to sense, to compute and to communicate via wireless channel. The low cost of these devices and the functionalities of collecting, computing and communicating data have attracted the attention of the community. Thus, WSNs are actually used in several domains such as military, health care, environment and others [1,2]. Nodes in WSNs are deployed without any predefined topologies. Consequently, this network is formed randomly using wireless radio channels. The most important disadvantage of WSNs is their limited energetic capacity, so that many methods, algorithms and protocols were introduced and developed taking in consideration this constraint.

Moreover, the main functionality of WSNs is to carry out a collaborative work in order to observe and supervise nature. That is why, the communication between nodes has a major importance. According to the energy constraint in sensors, many heuristics and strategies were proposed to minimize the broadcasting storm problem, the redundancy [3,4] and to realize a communication with a respectable level of fault tolerance. Selecting a CDS provide a virtual backbone used when broadcasting, routing or even for gathering data. A (k,r-

hop)-CDS is introduced to reduce the size of CDS and the amount of information to be transmitted. In fact, a (k,r-hop)-CDS problem is to select a connected set  $D$  of the network respecting that every vertex  $u$  not in  $D$  is at a distance within r-hop from at list  $k$  vertices in  $D$ . Recently, there is a proposition to find out a load-balanced CDS [12,13,14] that helps to extend the network lifetime with a very high rate. According to the best of our knowledge there is no a proposition of LB(k,r-hop)-CDS. The goal behind our work is to provide a solution to resolve this problem.

This paper is organized as follows. In the next section, we give some important definitions to clarify the vocabularies used in this work. In section III, there is a study of the related work. Section IV describes the problem definition. In section V, the main problem is treated. Section VI contains some measurement and experimental results to illustrate our improvement. Finally, section VII is a recapitulation of this work.

## II. DEFINITIONS

In this section, we present some important definitions that will be used later.

### A. Network model

In this paper we only consider unit disk graph, static WSNs and sensors with the same coverage area. WSNs can be considered as a graph  $G(V;E)$ , where  $V$  is the set of vertex representing nodes and  $E$  is the set of edges between vertexes. An edge exists between two nodes  $A$  and  $B$  in this graph only if  $A$  is in the coverage area of  $B$  and  $B$  is in the coverage area of  $A$ .

### B. $k$ -connected graph

A graph  $G(V;E)$  is said  $k$ -connected graph if and only if it contains  $k$  independent paths between any two vertices.

### C. $k$ -connected $m$ -dominating sets

A subset  $C$  of  $V$  is  $k$ -connected  $m$ -dominating set if every node in  $V \setminus C$  is dominated by at least  $m$  nodes in  $C$  and the sub graph induced by  $C$  is  $k$ -connected, The nodes in  $C$  are denoted as dominators, and nodes in  $V \setminus C$  are denoted as dominatees.

#### D. $r$ -hop connection

Given two vertexes  $u$  and  $v$  of a graph  $G(V,E)$ ,  $u$  is  $r$ -hop connected by  $v$  if we can find a path in  $G$  from  $u$  to  $v$  within  $r$  hops i.e. this path has  $r-1$  intermediates nodes.

#### E. $r$ -hop connected $k$ -dominating sets

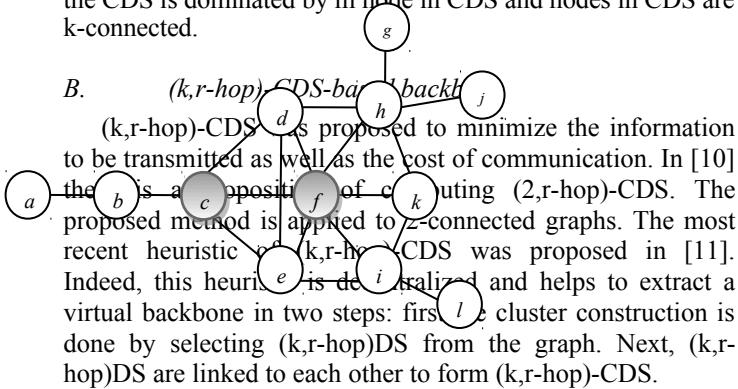
A subset  $D$  of  $V$  is  $r$ -hop connected  $k$ -dominating set if every node in  $V \setminus D$  is  $r$ -hop dominated by at least  $k$  nodes in  $D$  and the sub graph induced by  $D$  is connected. The nodes in  $D$  are denoted as dominators, and node in  $V \setminus D$  are denoted as dominatees.

### III. RELATED WORK

To construct a CDS, lots of efforts have been made and many approaches were used either centralized or decentralized. In what follows, a description of the most important methods of CDS selection are given in addition a short study of the recent LBCDS problem is realized.

#### A. CDS-based backbone

Many heuristic were proposed to resolve the problem of CDS construction. In [5], the authors provide a method of CDS selection based on timer that can be applied to Mobile Ad Hoc Networks (MANET). The construction of CDS can be either centralized or decentralized which mean either that one hope select the CDS or that the whole network participates to the CDS selection. In [6] there is a proposition of a decentralized CDS construction method for MANET. In [8] a method of searching minimum  $m$ -connected  $k$ -tuple CDS is given, this method helps to reduce the consumed energy in routing. The  $(k,m)$ CDS helps to realize the task of routing or broadcasting with high level of fault tolerance [7]. In fact, every node out of the CDS is dominated by  $m$  node in CDS and nodes in CDS are  $k$ -connected.



#### C. Load-balanced CDS-based backbone

Recently, Jing He et al. proposed some methods to construct a load-balanced CDS. In [12], they make use of a genetic algorithm to select LBCDS. In [13] they propose a LBCDS based on a heuristic which is done in two steps. First, the selection of dominators is done. Next, they define a load-balanced method for allocation of the dominatees to the appropriate dominators. This operation allows the increasing of network lifetime. In [14], they used an evolutionary algorithm to resolve the LBCDS problem.

#### D. Synthesis

To the best of our knowledge there is no works that consider the load-balanced factor when constructing  $(k,r)$ -hop-CDS. In this paper we try to re-use the concepts recently defined in [12,13,14] to resolve the problem of LB $(k,r)$ -hop-CDS.

### IV. PROBLEM DEFINITION

In this section, we try to give the definition of our problem by studying a concrete example.

#### A. Symbols and notations

In table I, we introduce the symbols used in this paper and their definitions.

TABLE I. SYMBOLS AND NOTATIONS

Symbol	Signification
$a, b, c, \dots$	Refers to the sensors used in the network
$ND(x)$	Refers to the set of dominatees of a dominator $x$
$ ND(x) $	The number of element in $ND(x)$
$p$ -norm	Defined to compute the load-balanced factor of a subset in WSNs, proposed in [15], and used in [12,13] it is equal to: $ X _p = \left( \sum_{i=1}^n  x_i ^p \right)^{1/p}$
$d(x)$	Refers to the degree of a node $x$ in the graph
$\bar{d}$	Refers to the superior integer part of the average of degrees in a graph $G$ .
$D$	The set of dominators in a graph $G$

#### B. Dominators selection

In fig. 1, an example of WSNs is given. The  $(1,2)$ -hop-CDS from the given network is selected using the latest algorithm of  $(k,r)$ -hop-CDS construction proposed in [11]. The result of this selection is the set  $\{c, f\}$ .

Figure 1. Example of a WSN :  $(1,2)$ -hopCDS is  $\{c, f\}$

In order to measure the load-balancing of the selected set we compute its  $p$ -norm. According to [15], for  $p=2$  the data flow shows an analogy to electrostatic filed. So, computing  $p$ -norm with  $p=2$  helps to measure the load-balance among  $x_i$ . Finally, if the  $p$ -norm of a set becomes small this set will be more load-balanced [12]. According to [12,13] :

$$x_i = |d_i - \bar{d}|$$

Hence,

$$p\text{-norm}(\{c, f\}) = \sqrt{|4-3|^2 + |6-3|^2} = \sqrt{10}$$

In fig. 2, we try to select another (1,2-hop)-CDS which is  $\{c,d,e\}$ . For this set the  $p$ -norm will be:

$$p\text{-norm}(\{c,d,e\}) = \sqrt{|4-3|^2 + |4-3|^2 + |4-3|^2} = \sqrt{3}$$

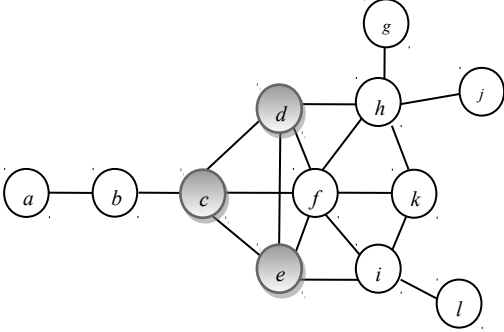


Figure 2. Example of a WSN: (1,2-hop)CDS is  $\{c,d,e\}$

After a simple comparison of the two  $p$ -norms:

$$\sqrt{10} \nless \sqrt{3}$$

To conclude, we can say that the second set is more load-balanced. Consequently, constructing a LB(k,r-hop)-CDS should take into consideration the minimization of its  $p$ -norm value.

#### C. Dominatees association

In this subsection, we give two examples of dominatees' association to dominators. Next, we compare the two possibilities to see which of them is more load-balanced. In fig. 3,  $ND(c)=\{b,a\}$ ;  $ND(d)=\{h,g,j,f,k\}$  and  $ND(e)=\{i,l\}$ .

Then, if we choose

$$x_i = |ND(i) - \overline{ND(i)}|$$

Hence,

$$p\text{-norm}(\{c,d,e\}) = \sqrt{|2-3|^2 + |5-3|^2 + |2-3|^2} = \sqrt{6}$$

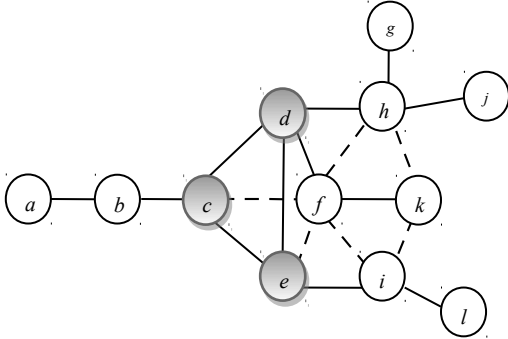


Figure 3. Example of a WSN: a non equilibrate association of dominatees to dominators

In fig. 4, another association of dominatees to dominators is done:  $ND(c)=\{b,a\}$ ;  $ND(d)=\{h,g,j\}$  and  $ND(e)=\{i,l,f,k\}$ . Then the  $p$ -norm will be:

$$p\text{-norm}(\{c,d,e\}) = \sqrt{|2-3|^2 + |3-3|^2 + |4-3|^2} = \sqrt{2}$$

Finally, we can deduce that the second association is more load-balanced.

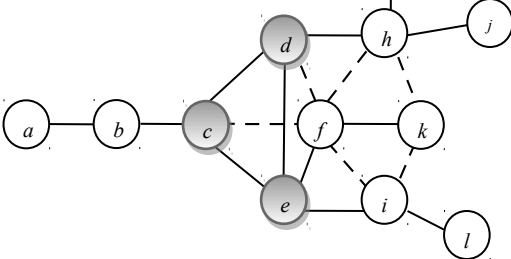


Figure 4. Example of a WSN: an equilibrate association of dominatees to dominators

## V. LB(K,R-HOP)-CDS

In this section, the main problem is formally treated. Two steps are done to resolve LB(k,r-hop)-CDS problem. First, the dominators are selected. Next, the association of dominatees to dominators is done. We treat only LB(1,r-hop)-CDS.

### A. CDS selection

According to the example studied in the previous section the construction of CDS is to select a subset  $D$  of  $V$  ( $V$  is the set of vertices of the WSNs represented by the graph  $G(V,E)$ ). With  $D$  should has the next mentioned proprieties

- $D$  is connected.
- For all node  $u$  of  $V \setminus D$ , there is a node  $v$  in  $D$ , such that  $u$  is at a distance within r-hop from  $v$ .
- Minimize  $|D|_p = (\sum_{i=1}^{|D|} |d(x_i) - \bar{d}|^2)^{1/2}$

Thus, the selection of  $D$  should start by selecting node that minimizes  $|D|_p$  and it ended when  $D$  is connected, and every node not in  $D$  is at a distance within r-hop from at least one node in  $D$ . Finally, we optimize  $D$  i.e. extracting node  $n$  from  $D$ , if  $D \setminus \{n\}$  still respect the above proprieties. Below, the algorithm of  $D$  selection is given:

#### Algorithm1: Dominators selection

**Input:** a connected graph  $G(V,E)$

**Output:** the set of dominators  $D$

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1:  $D \leftarrow \emptyset$ 
2: compute  $\bar{d}$  in  $G$ 
3: for all node  $n$  in  $G$  do
4:   if  $d(n) = \bar{d}$  then  $D \leftarrow D \cup \{n\}$ 
5:   if  $D$  is connected and all nodes in  $V \setminus D$  are within r-hop from at least one node in  $D$  then
6:     break
7:   else  $\bar{d} \leftarrow \bar{d} + 1$ 
8:   end if
9: end for all
10: for all node  $n$  in  $D$  do
11:   if  $D \setminus \{n\}$  is connected and all nodes in  $V \setminus D \setminus \{n\}$  are within r-hop from at least one node in  $D \setminus \{n\}$  then
12:      $D \leftarrow D \setminus \{n\}$ 
13:   end if
14: end for all
```

If we apply *algorithm1* to the network of fig.1 in order to construct (1,2-hop)CDS, we have  $\bar{d}=3$ , there is no node having 3 neighbors. Then, for  $\bar{d}=4$ ,  $D=\{c\}$ . For this set  $D$  there still exist uncovered nodes within 2-hops. Next,  $D=\{c,d\}$  this set also does not cover all nodes within 2-hops. Finally,  $D=\{c,d,e\}$ . This  $D$  is connected and all nodes in  $V \setminus D$  are within 2-hops from at least one node in  $D$ . Consequently, the algorithm terminates. If  $i$  is chosen as an element of  $D$ , it will be extracted in the step of optimization before the end of the algorithm.

#### B. Load-balanced allocation of dominatees to dominators

The problem of load-balanced allocation of dominatees to dominators can be defined as:

- Every dominatees is allocated to only one dominators.
- The distance between a dominatee and his allocated dominator is within  $r$  hops.
- The allocation should minimize

$$|D|_p = \left( \sum_{i=1}^{|D|} ||ND(x_i) - \overline{ND(x_i)}||^2 \right)^{1/2}$$

The algorithm below is defined respecting the above mentioned properties. The algorithm is started by the allocation of the imposed dominatees i.e.:

- A node that can be allocated only to one dominator.
- Or it is a neighbor of only one dominator.
- Or it is a neighbor of only one allocated dominatee.

For other nodes the association is done to the dominators having the minimum  $|ND(x)|$

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#### *Algorithm2: Dominatees allocation*

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**Input:** a connected graph  $G(V,E)$

a set of dominators  $D$  of  $V$ .

**Output:** all dominatees are allocated to dominators.

- 1:  $S \leftarrow V \setminus D$
  - 2:  $Temp \leftarrow G(V,E)$
  - 3: Allocate all imposed domiantees to their appropriate dominators and withdraw them from  $S$ , start by 1-hop domiantees to  $r$ -hop domiantees
  - 4: **for all** node  $n$  in  $S$  **and**  $x$  in  $D$  **do**
  - 5:    $r \leftarrow 1$
  - 6:   select the set  $x$  where we can allocate  $n$  within  $r$ -hop in  $Temp$
  - 7:   choose  $x$  having the minimum  $|ND(x)|$
  - 8:    $ND(x) \leftarrow ND(x) \cup \{n\}$
  - 9:    $S \leftarrow S \setminus \{n\}$
  - 10:   delete edges between other  $x$  and  $n$  from  $Temp$
  - 11:    $r \leftarrow r+1$
  - 12:   **if**  $S$  is empty **break**
  - 13: **end for all**
- 

If we apply *algorithm2* to the network of fig.2 in order to complete the allocation of dominatees to dominators. First,

$ND(c)=\{a,b\}$  because  $a$  and  $b$  are imposed to  $c$ . Same for  $h$  and  $i$  which are imposed respectively to  $d$  and  $e$ . Then,  $l$  is imposed to  $e$  and  $g,j$  are imposed to  $d$ . Consequently,  $ND(d)=\{h,g,j\}$  and  $ND(e)=\{i,l\}$ . The node  $f$  will be linked to  $e$  because  $|ND(e)| < |ND(f)|$ . Finally,  $ND(e)=\{i,l,f,k\}$  and,  $S$  will be empty therefore, the algorithm terminates.

## VI. PERFORMANCE AND EVALUATION

The test and the measurement of the performance are done using NS2 [16]. Identical sensors with the same coverage area (50m) are used. The size of packet to be transmitted between nodes is 200b or 400b. We suppose in this test that the sink node is directly linked to one node of the (k,r-hop)-CDS. In every 10 seconds, the sink node broadcast a packet to all nodes in the network, and after 10 seconds it receives gathered data from all nodes. Finally, we extract periodically the energy of the most worker node in the (k,r-hop)-CDS selected by our load-balanced method. Then, we compare it to the energy of the most worker node in the (k,r-hop)-CDS which is selected using the method proposed in [11]. Initially, every node is alimented by two AA lithium batteries i.e. the initial energy in nodes it is nearly 30kJ. We treat only the consumed energy for communication and we neglect the consumed energy for sensing and internal processing.

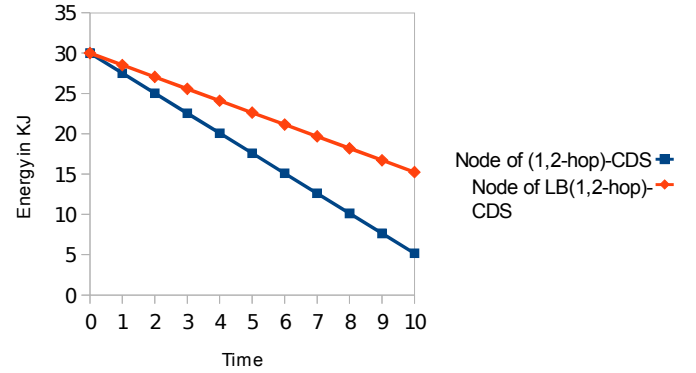


Figure 5. Evolution of energy in the most worker node in the (1,2-hop)CDS with a density of 12 nodes and using packet with 200b as size

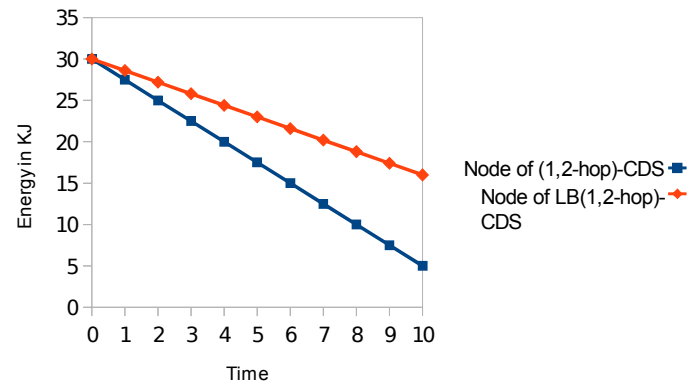


Figure 6. Evolution of energy in the most worker node in the (1,2-hop)CDS with a density of 200 nodes and using packet with 200b as size

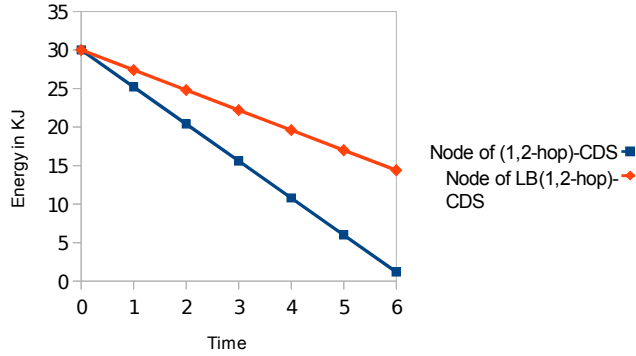


Figure 7. Evolution of energy in the most worker node in the (1,2-hop)CDS with density of 200 nodes and using packet with 400b as size

In fig. 5, we can observe that there is an important improvement of the lifetime of the most worker node in the (1,2-hop)CDS. The two curves are linear. Consequently, we can easily compute the time when nodes will be exhausted. Finally, we can conclude that LB(k,r-hop)-CDS make an extension of node lifetime with 68% if we compare it with the latest proposed (k,r-hop)-CDS construction method. This fact is confirmed in fig. 6 using a network with 200 nodes as density and 200b as size of transmitted packet and in the network of fig. 7, with 200 nodes as density and 400b as size of transmitted packet. The extension of the lifetime of the most worker node in the CDS has an important effect on extending the whole network lifetime. Indeed if a node in CDS will be quickly exhausted the CDS will be disconnected. Then a big set of sensors in the network will be unreachable.

## VII. CONCLUSION

This work treats the problem of LB(k,r-hop)-CDS aiming to select a (k,r-hop)-CDS with taking into consideration the load-balance factor. The problem resolution is centralized, is based on the minimization of  $p$ -norm and done in two steps. First, the set of dominators is selected. Next, the allocation of dominatees to the appropriate dominators is completed in order to guarantee the load-balance propriety. According to the measurement of performance given, LB(k,r-hop)-CDS provides an important improvement in term of increasing the network lifetime compared to the latest proposed method of (k,r-hop)-CDS selection.

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