State space modeling of PEM Fuel Cell with a Moto-Compressor System

A. Saadi[1], M. Becherif[2], D. Hissel[3] and A. Aboubou[4]

[1]University of Mohamed Kheider, Biskra, Algeria
[2]FCLab FR CNRS 3539, FEMTO-ST UMR CNRS 6174, UTBM, Belfort, France.
[3] FCLab FR CNRS 3539, FEMTO-ST UMR CNRS 6174, UFC University, Belfort, France.
[4]Laboratory of Energy Systems Modeling, University of Mohamed Kheider, Biskra, Algeria.
aicha.saadi@utbm.fr, mohamed.becherif@utbm.fr, daniel.hissel@univ-fcomte.fr

Abstract: A proton exchange membrane fuel cell (PEMFC) system is composed of four main subsystems. In this paper, the air supply subsystem with the fuel cell is described by a nonlinear fourth order state space model. The model of the PEMFC is obtained from an equivalent electrical circuit; it combines the simplicity of an electrical circuit and attempts to model the physical phenomenon occurring inside the fuel cell. The effect of temperature variation and relative humidity on the cell are considered in this model. The compressibility of fuel, and oxidant fluids and condensation of water are also taken into account. The most used model in the literature is the nine state model proposed by Pukrushpan, thereafter reduced to the four states by Suh and Stefanopoulou, which was then reduced into three states by Talj and al. Despite the good results for average power, the major disadvantage of this last model is the obligation to start the simulation with the complete states space model due to the presence of a division over the moto-compressor speed in the model. This paper proposes an interesting dynamic model using a simple four order state space models for the moto-compressor group associated to the PEMFC stack which is suitable for purposes of control or diagnosis. The obtained simulation and experimental results for low power are successful and give the first step of the control purpose.

1. Introduction

Fuel Cells (FCs) can offer a highly efficient environment and friendly electrochemical transformation for energy conversion. They are widely considered as a potential alternative power source and have drawn much attention and intensive development for commercial stationary power generation, residential applications, transportation technologies and vehicles. The heart of FC is a device involving multi-physics coupling phenomena: mass transport (in the electrodes and electrolyte), charge transport (in the electrode), and electrochemical kinetics (at reactive sites). Problems are added to these phenomena which are thermal and distribution of reactive gases. Many mathematical models can locally describe these phenomena by means of partial differential equations involving space and time. These models are accurate, but they are, most of time, unusable for a system approach. That is the reason why other models appeared including dynamic models. Some of these models represent FC equivalent electrical circuit. This approach aims at establishing a simple and reliable model of the FC. Over the years, scientists have presented many FC electrical equivalent circuit models. For instance, [1] includes the mass transport effect and the pressure regulator effect which is characterized by two transistors and a resistor. Both form a current limiting circuit that models the polarization of concentration, the parasitic resistance of the diode models the FC ohmic polarization. At the same level, the mass transport effect and the losses in FC are slightly differently presented by another equivalent circuit where two capacitors are used instead of one [2].

The most used model in the literature is the nine states model proposed by Pukrushpan [3]. However, the large number of states restricts its use in control applications, due to the large number of the calculations. Hence, a reduced order is required. [4] is a recent reduced order model for this application. The obtained one is of fourth state spaces order under some assumptions. [5] and [6] propose reduction into four states by replacing the partial pressure of oxygen by the cathode pressure, under the assumptions that molar mass of oxygen, nitrogen and water have almost the same magnitude and the air-flow in the cathode. Then, a higher order sliding mode controller has been designed using super-twisting algorithm. But the major disadvantage of four states model is a division over the compressor angular speed. Thus the system is obliged to start with the complete model and is switched to the four states model [5]. In order to remedy to this and to synthesize a good control law, a comprehensive dynamic model is proposed in this paper. The modeling approach is based on the principles of conservation of mass, charge, momentum and energy. To get a brief overview of the FC electrical equivalent models, one is advised to go through [7] and [8]. Moreover, the air supply subsystem with the FC is described by a nonlinear fourth order model where the state vector is composed of the oxygen and nitrogen partial pressures in cathode and the angular speed manifold. The stack current is traditionally considered as a measurable disturbance to the system while the control input is the compressor motor voltage [3]. The compressibility of fuel and oxidant fluids and condensation of water are also accounted for in this model. [7] proposed it for diagnosis purpose. The model has been validated in [9] on a low power PEMFC. This paper proposes an interesting and simple four order state space model for the group moto-compressor with PEMFC which is suitable for the control or the diagnosis tasks. This paper is organized as follows. The second section gives a complete description of FC system model, which is composed of two parts; the electrical equivalent model of a FC and the moto-compressor model. The third section gives the validation of simulation results by comparing them with experimental results, and the final section is a conclusion.

2. Fuel Cell System Model

In order to produce energy, it is necessary to integrate the fuel stack with other components to form a fuel-based power generator system. Classically, four auxiliary systems can be considered: hydrogen supply system, air supply system, cooling system, and humidification system. In Fig. 1, it is assumed that a compressed hydrogen tank is used. Thus, the control of hydrogen flow is achieved simply by controlling the hydrogen supply valve to reach the desired flow or pressure. The air is assumed to be supplied by an air compressor which is used to increase the power density of the overall system.

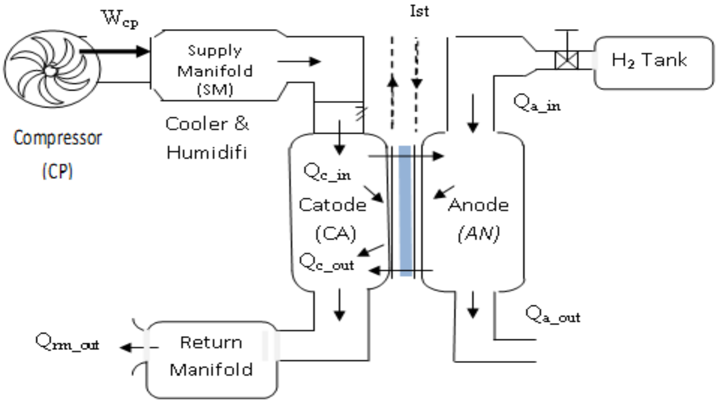


Figure 1. Fuel cell system description [3]

2.1 Fuel cell electrical equivalent circuit model

The FC electrical equivalent model is obtained by setting an equivalent electrical component to account for the pneumatic behavior. This equivalence is based on the conservation principles of mass, charge and energy. Following the Bernoulli's laws of continuity, mass and energy flow are conserved in a pipe for an incompressible fluid; however, there will be losses due to friction and geometry of the pipes. The compressibility of the fluid adds a storing capacity to the pipes. Hence, the molar flow rates in a pneumatic system can be analogous to the flow of charge. The gases that are being passed through the channels are compressible. This later is determined by the volume that the channel can hold under given pressure and temperature conditions. This is analogous to a capacitor in an electrical system which is capable of accumulating charge under transient conditions and acts as an open circuit when steady state is achieved. Based on these information, [7] developed an equivalent electrical circuit which is shown in Fig. 2. See [7], [10] and [11] for more details.

Electrical Environment:

$$I = \frac{dq}{dt} \qquad (1)$$

Pneumatic Environment:

$$Q = \frac{dn}{dt} \qquad (2)$$

With:

$$Q_{m,in} = Q_{m,in} - Q_{m,out} \qquad (3)$$

The gases that are being passed through the channels are compressible. This compressibility is determined by the volume that the channel can hold under given pressure and temperature conditions. This is analogous to a capacitor in an electrical system which is capable of accumulating charge under transient conditions and acts as an open circuit when steady state is achieved.
The model is based on some assumptions listed as follows:
All the gases are ideal gases.
The interaction between different species is negligible ignored.
The principle of superposition for electric circuits is applicable.
The pressure changes produced by the velocity of the flow are negligible.
The changes in the volume of channels are considered negligible.
The flow is assumed to be laminar.
The losses due to fuel and oxidant crossover are considered to be negligible.
The only species that diffuse through the membrane are nitrogen and water.
The water is introduced only on the cathode side.

The channel pressure for anode and cathode is taken to be the average of the inlet and outlet pressures.

$$Q_{in}^{c} + v_w Q_{w,r}^{I} + v_w Q_{w,r-e}^{I} = v_o Q_{O,r}^{I} + Q_{Psat}^{c} + Q_{N,m} + Q_{w,m} + Q_{out}^{c} \qquad (4)$$

$$Q_{in}^{a} + Q_{N,m} + Q_{w,m} + Q_{Cf}^{m} = Q_{Cf}^{c} + v_m Q_{w,r-e}^{I} + v_h Q_{H,r}^{I} + Q_{Psat}^{a} + Q_{out}^{a} \qquad (5)$$

$$V_{Cf}^{i} = \left(R_{out}^{i} + R_{ch}^{i}\right) Q_{out}^{i} + V^{i} \quad ;\ i = a \ or \ c \qquad (6)$$

$$V_{in}^{i} = \left(R_{in}^{i}\right) Q_{in}^{i} + V_{Cf}^{i} \quad ; i = a \ or \ c \qquad (7)$$

$$V_{Cf_{in}^{i}}^{m} = \left(0.5 R_{w,m} + R_{out}^{a} + R_{ch}^{a}\right) Q_{out}^{a} + V^{a} \qquad (8)$$

$$dV_{Cf}^{i} / dt = Q_{Cf}^{i} / C_{f}^{i} \quad ; i = a \ or \ c \qquad (9)$$

$$dV_{Cf}^{m} / dt = Q_{Cf}^{m} / C_{f}^{m} \qquad (10)$$

$$V^{ca} = R_{N,m} \, Q_{N,m} \qquad (11)$$

Where *Vj* denotes the molar volume of *j* where *j*={*w* for water, *o* for oxygen or *h* for hydrogen}. And *i*={*a* for anode, or *c* for cathode}.

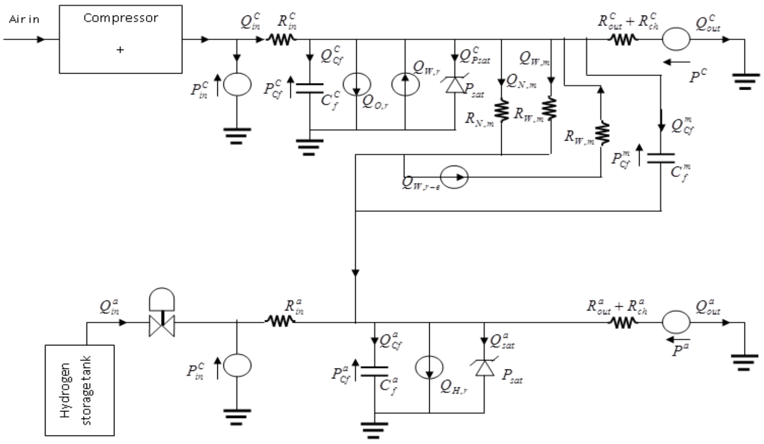


Figure 2. Complete circuit of FC: anode, cathode and membrane

2.2 Moto-Compressor model

PEMFCs can operate from pure oxygen (case of cells used in submarines and spacecraft for example). It is then stored in a pressurized tank. For terrestrial applications, stationary or embedded FCs are mostly supplied with ambient air. The use of a compressor is not without consequences on the overall system performance. It is powered by the FC and it takes a non-negligible power which can represent up to 25% of the electrical power supplied by the FC. The compressor is very important in FC system, and many technological barriers are resolved in this field. It must allow the air supply, dispose produced water without drying the membrane of the cell and ensure good system dynamics. The centrifugal volumetric compressors are among the compressor types commonly used in FC system.

The air supply system, designed to feed in parallel two PEMFC, each is composed of a PMSM driving a volumetric compressor of type double screw.

The air flow control problem is converted into a PMSM speed control problem [13] that is composed of a cascaded control loop, where the outer loop controls the velocity and generates the reference of the quadratic current component I_q^*; and the inner loop controls the three-phase current and generates the desired control input — the voltage that feeds the motor [14].

For the inner loop, the desired direct component of the current is chosen to be $I_d^* = 0$. From $I_d^* = 0$ and I_q^* already defined as the output of the velocity controller, one can reconstruct the desired three-phase current I_a^*, I_b^* *and* I_c^* using the inverse Park transformation. Comparing with the measured three-phase current I_a ,I_b *and* I_c a simple hysteresis can be used to control the current and generates the voltage. The compressor motor torque τ_{cp} is gives by the follow equation:

$$\tau_{cm} = \eta_{cm}.k_t.I_q \tag{16}$$

Where k_t is motor constant and η_{cm} is the motor mechanical efficiency.

The angular speed ω_{cp} dynamics or acceleration is given by the following mechanical equation:

$$\frac{d\omega_{cp}}{dt} = \frac{1}{J_{cp}}\left(\tau_{cm} - \tau_{cp} - \tau_f\right) \tag{17}$$

Where J_{cp} is the compressor motor inertia, $\tau_{cm}$and τ_f respectively denote the compressor motor torque and the friction torque.

$$\tau_f = f.\omega_{cp} \tag{18}$$

$$\tau_{cp} = \frac{C_p}{\omega_{cp}}\frac{T_{atm}}{\eta_{cp}}\left[\left(\frac{p_{sm}}{p_{atm}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]W_{cp} \tag{19}$$

where η_{cp} is the compressor efficiency, γ is the ratio of specific heats of air, C_p is the specific heat capacity of air and Wcp is the compressor mass flow rate. The supply manifold air pressure dynamics are defined by the (21) equation, where V_{sm} is the supply manifold volume and T_{cp} is the temperature of the gas leaving the compressor.

In commercial PEMFC systems, the following variables are available for measurement, and can be defined as output:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} v_{st} \\ p_{sm} \\ W_{cp} \end{bmatrix} \tag{20}$$

Where v_{st}, and p_{sm} are the stack voltage and the supply manifold pressure. The air flow at the output of the compressor is a function of the angular speed of the compressor and the supply manifold pressure, given as follows [14]

$$y_3(x_3) = \frac{1}{2\pi} V_{cpr/tr}.\eta_{v-c}.\rho_a \tag{21}$$

Where η_{v-c} is the volumetric efficiency. $V_{cpr/tr}$ is the compressed volume per turn and ρ_a is the air density.

The net power of the FC system, P_{net}, which is the difference between the power produced by the stack, P_{st}, and the lost powers due mainly to the auxiliaries. The majority of the lost power for an automotive FC system is spent on the air compressor.

$$P_{net} = P_{st} - P_{cm} \tag{22}$$

$$P_{cm} = v_{cm} I_{cm} = \frac{v_{cm}}{R_{cm}}\left(v_{cm} - k_v \omega_{cp}\right) \tag{23}$$

$$P_{st} = v_{st} I_{st} \tag{24}$$

It can be seen that the system state space equations are complicated, based on many parameters. For ease of comprehension, the constant parameters have been regrouped, and the complete dynamic model is of 4-th state space vector

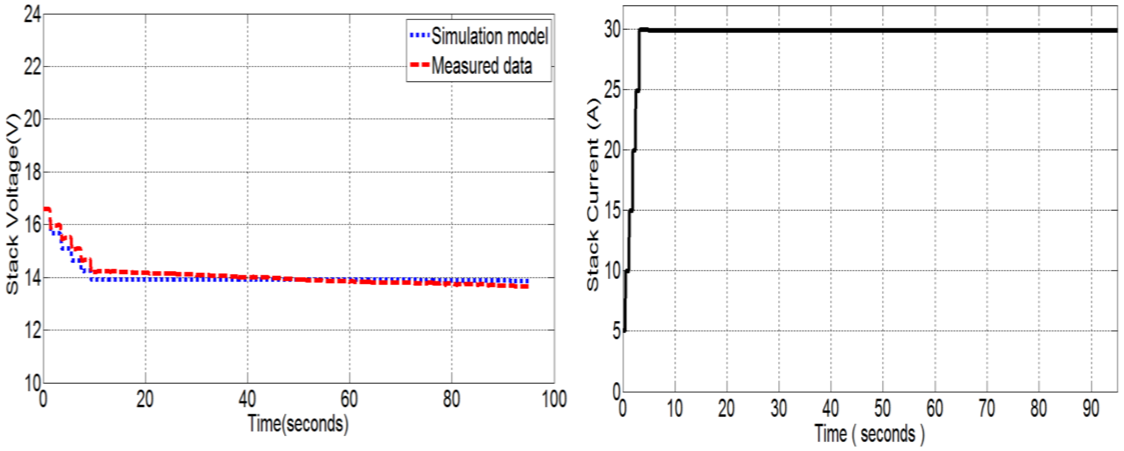
$$x = \left[x_1\ x_2\ x_3\ x_4\right]^T \tag{25}$$

Where x_1 and x_2 are the cathode and membrane pressures, respectively, x_3 is the angular speed of volumetric compressor and x_4 is the air pressure in the supply manifold. The control input u is the motor's quadratic current. All the constant parameters C_i, i=1,...,7 are depending on the FC physical parameters. The reader can refer to [10] for more details on the parameters C_i. The complete system can be written as follows.

$$\begin{aligned}
\dot{x_1} &= \frac{dV_{Cf}^a}{dt} = C_1.Q_{Cf}^a \\
\dot{x_2} &= \frac{dV_{Cf}^m}{dt} = C_2.Q_{Cf}^m \\
\dot{x_3} &= \frac{d\omega_{cp}}{dt} = C_3.u - C_4.[(C_5.x_4)^{C_6} - 1] - C_7.x_3 \\
\dot{x_4} &= \frac{dV_{Cf}^a}{dt} = C_8.Q_{Cf}^c
\end{aligned} \tag{26}$$

3. Validation of simulation results

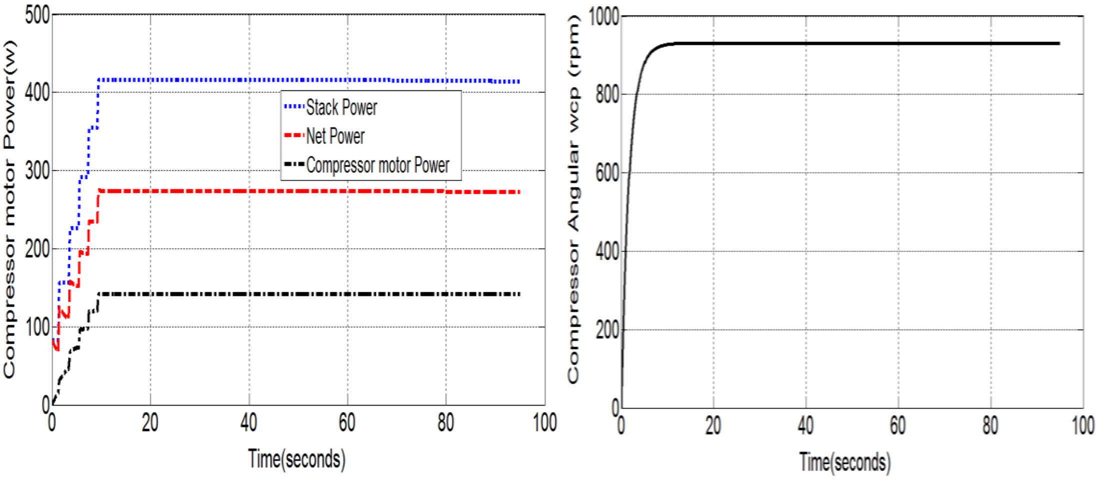
The data used in the simulation are collected from a 20-cell PEMFC stack with 100 cm^2 area of each cell, membrane thickness is 25µm and the nominal power is about 400W. Many physical parameters involved in the stack can be measured. The inlet/exit measured magnitude are flow rates cathode and anode channel, inlet/exit air pressures of cathode and anode, average temperature and humidity of FC, the stack voltage and current which is traditionally considered as a measurable disturbance to the system. The equivalent electrical circuit model is based on modeling of anode, membrane and cathode. The stack current is traditionally considered as a measurable disturbance to the system. Moreover, the losses voltages: activation, concentration and ohmic are depend of the current. The hydrogen pressure is calculated as an intermediate step for calculating the thermodynamic voltage which is obtained by Nernst equation. The stack voltage is also the difference between thermodynamic and losses voltages. During these tests, different current steps are applied to the FC stack to exhibit the steady state and transient behavior. Firstly, the FC is turned on with a power of about 400W with a load that consumes a current of about 30 A shown in Fig. 3b. In the figure (a), the voltage of stack starts from the open circuit voltage (OCV). Apart from this zone, the simulation voltage is considerably close to voltage experimentally obtained, both are about of 14 V, and the error is less than 1.45% in the permanent regime.



(a) (b)

Figure 3. a) Simulated and experimental voltages; b) Simulated and experimental current

The simulation powers are drawn on figure 4. This curve indicates the precision of the system’s state space model. The stack power presents the sum of two powers; net and compressor power respectively. The compressor power is transferred into mechanical power which is provided in the form of motor speed. Figure 4.b presents the speed curve of the motor-compressor group.



(a) (b)

Figure 4. a) Powers (W); b) Simulated Compressor Angular speed

4. Conclusion

In this paper, the state space modeling of PEM fuel cell system with the motor-compressor group is achieved, the obtained nonlinear fourth order model interlaced with electrical equivalent FC circuit model is successfully validated. The obtained model is suitable for the control and the diagnosis purposes, unlike other developed models in the literature [5] and [6]. The motor-compressor speed varies from zero value to desired value and the system is characterized by good performance stability and robustness. The transient and permanent regimes are providing satisfactory results and closeness to the experimental results. The proposed model is simpler compared to the complexity of the nine states model [3]. In addition, the simulation is not time consuming. The obtained results are good, and they establish an important stage towards control.

NOMENCLATURE

Afc	Fuel cell active area	(cm^2)
At	Cathode outlet throttle area	(m^2)
Cd	Cathode outlet throttle discharge coefficient	-
dcp	Compressor diameter	(m^2)
F	Faraday constant	(A.s/mol)
Jcp	Compressor and motor inertia	(kg/m^2)
I	Current	(A)
kt	Motor torque constant	(N.m/A)
n	Number of cells in stack	
$p_{\_atm}$	Air gaz constant	(mbar)
$p_{\_sat}$	Saturation pressure	(mbar)
R	Universal gaz constant	(J/(mol.K))
Ra	Air gaz constant	(J/Kg.K)
ρ_a	Air density	(Kg/m^3)
T	Temperature	(Kelvin)
t_m	Membrane thickness	(m)
V	Voltage	(V)
V_{ca}	Cathode volume	(m^3)
v_{cm}	Voltage compressor-motor	(V)
η	Efficiency	
ω	Rotational speed	(rad/s)
τ	Torque	(N.m)
γ	Ratio of the specific heats of air	

Subscripts

a	Anode
c	Cathode
Cm	Compressor motor
Cp	Compressor
h	Hydrogen
in	Inlet
O	Oxygen
out	Outlet
P	Pressure
sat	Saturation
st	Stack
v	Vapor
W	Mass flow rates

REFERENCES

1. Yuvarajan S., Yu D., 2004, *Characteristics and modeling of PEM fuel cells*, IEEE International Symposium on Circuits and Systems, vol. 5, pp. 880-883.
2. Yu D., Yuvarajan S., 2004, *A novel circuit model for PEM fuel cells*, IEEE Applied Power Electronics Conference and Exposition, vol. 1, pp. 362-366.
3. Pukrushpan J.T., Peng H., 2004, Stefanopoulou A.G., *Control-Oriented Modeling and Analysis for Automotive Fuel Cell Systems*, J. Dyn. Sys, Meas, Control, vol.126, n°1, pp. 14-25.
4. Suh K.W., 2006, *Analysis and control of Fuel Cell Hybrid power system*s, PhD thesis, Department engineering, The University of Michigan.
5. Talj R., Hissel D., Ortega R., Becherif M., Hilairet M., 2009, *A reduced-order model and a higher-order sliding-mode control of the air supply system of a proton-exchange-membrane fuel cell with experimental validation*, Electromotion, 8th International Symposium on, pp. 1- 6.
6. Talj R., Hissel D., Ortega R., Becherif M., Hilairet M., 2010, *Experimental validation of a PEM fuel cell*, IEEE-TIE, Transaction on Industrial Electronics, vol.57, n° 6, pp. 1906-1913.
7. Hernandez A, Hissel D, Outbib R, 2010, *Modeling and Fault diagnosis of a Polymer Electrolyte Fuel Cell Using Electrical Equivalent Analysis*, Energy Conversion, IEEE Transaction on, vol.25, n°1, pp.148-160.
8. Runtz K.J., Lyster M.D., 2005, *Fuel cell equivalent circuit models for passive mode testing and dynamic mode design, Canadian conference on Electrical and Computer Engineering*, pp. 794-797.
9. Becherif M., Hissel D., Gaagat S., Wack M., 2011, *Electrical Equivalent Model of a PEM Fuel Cell with Experimental Validation*, Renewable Energy Journal, vol.36, n°10 ,pp. 2582-2588.
10. Becherif M., Hissel D., Gaagat S., Wack M., 2010, *Three order state space modeling of proton exchange membrane fuel cell with energy function definition*, Journal of Power Sources, vol. 195, n°19: pp. 6645-6651.
11. Hernandez A., 2006, *Diagnostics of PEM Fuel Cells*, PhD thesis, UTBM, France.
12. Hinaje M., Nguyen D., Raël S, Davat B., 2008, Modelling of the Proton Exchange Membrane Fuel Cell in Steady State, IEEE Power Electronics Specialists Conference, pp.3550- 3556.
13. Chang K.T., Low T.S., Lee T.H., 1994, An optimal speed controller for permanent-magnet synchronous motor drives, IEEE Transactions on Industrial Electronics, vol. 41, n° 5, pp. 503-510.
14. Genre-Grandpierre R., 2008, Alimentation en air d’une pile à combustible: Conception du système, Caractérisation, Modélisation, Conservatoire National des Arts et Métiers, Centre Régional Franche-Comté, FEMTOST/ENISYS - FCLAB, Engineering diplôma.