Nonlinear Flatness Control Applied for Energy Management of PV/Batteries/Supercapacitors Hybrid Power Sources for Stand-Alone Applications

M. Benaouadj, M.Y. Ayad, M. Becherif, A. Aboubou, M. Bahri and O. Akhrif

Abstract—This paper presents the hybridization of photovoltaic (PV) panels considered as the main source, with lead-acid batteries and supercapacitors used as storage system, for a stand-alone application. The aim is to supply a domestic load and energy management between different components.

A nonlinear control strategy based on the flatness approach is applied to manage energy flows among the PV source, storage unit and load in the system. Simulation results are presented and discussed when applying different solar illuminations.

I. INTRODUCTION

The production of electrical energy by PV systems is highly dependent on weather conditions. Therefore, this energy should be stored during the day [1].

Presently, the lead-acid batteries are the most common electrochemical storage technology. Their use has been widely demonstrated, but costs and life-cycle characteristics are not yet satisfactory and need to be improved for stand-alone applications [2].

Developed at the end of the seventies for signal applications (for memory back-up for example), supercapacitors present a capacitance of some farads and specific energy of about 0.5 Wh.kg⁻¹. The power supercapacitors appeared during the nineties and brought components of thousands of farads and high specific power of several kW.kg⁻¹.

When lead-acid batteries are subjected to extreme conditions (high discharge current for example), their performance degrades rapidly. So, the use of a hybrid storage source is indispensable: batteries will be dedicated to store and supply the produced photovoltaic energy, while the auxiliary source (supercapacitors) would deal with high solicitations (deep discharge).

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The energy supplied by a PV generator is written as follows [3]:

\[ \text{E}_{\text{PV}} = n_{\text{e}} \text{P}_{\text{c}} \]  

(1)

This paper presents a "PV/Batteries/Supercapacitors" hybrid power sources used to supply a house load. The energy management between the photovoltaic source, storage system and the load is assured by applying a nonlinear control based on the flatness approach.

II. PRESENTATION OF THE AUTONOMOUS HYBRID SYSTEM

The studied autonomous hybrid system combines, as shown in figure 1, a permanent source for energy production, and two intermediate storage elements. It includes:

- A DC link (bus), assuring power segmentation between load, main source (wind and photovoltaic sources), and storage elements (lead-acid batteries and supercapacitors).
- A PV source connected to the DC link using a DC-DC converter.
- Lead-acid batteries (electrochemical storage) connected to the DC bus by a current bidirectional DC-DC converter.
- Supercapacitors (electrostatic storage) connected to the DC link through a current bidirectional DC-DC converter.
- A supervision system used to control the batteries and the supercapacitors "charge/discharge".
- A house load modeled by a daily consumption profile.

![Synoptic of the autonomous hybrid system](image-url)
A MPPT (Maximum Power Point Tracking) controller is used to allow operating the PV source under the maximum available power.

B. Lead-acid batteries

The electrical model of a lead-acid battery has an electromotive force ($E_0$) modelling the battery open circuit voltage, a capacitor ($C_b$) modelling the battery internal capacity, and an internal resistance ($R_s$) [4].

\[ v_b = E_0 - v_{Cb} - R_s i_b \]  

The batteries State of Charge, $SOC_b$, is written in [%] by:

\[ SOC_b = \left( 1 - \frac{Q_{mb}}{C_b} \right) \times 100 \]  

$C_b$: nominal capacity [Ah]
$Q_{mb}$: Quantity of charge missing comparing to $C_b$

C. Supercapacitors

The basic and commonly used equivalent model of a supercapacitor is a capacitor ($C_{elem}$) in series with a resistor ($R_{elem}$) [5].

\[ v_{sc} = v_{Csc} - R_{sc} i_{sc} \]  

The supercapacitors State of Charge, $SOC_{sc}$, is written in [%] as follows [6]:

\[ SOC_{sc} = \left( 1 - \frac{E_{sc}}{E_{sc,max}} \right) \times 100 \]  

$E_{sc}$: stored energy [J]
$E_{sc,max}$: maximal contained energy [J]

D. Supervision system

$P_{ps}$ is the power supplied by the PV source, and $P_{Lo}$ is the consumed power. By adopting the "generator" convention (the current supplied by the storage elements is positive), then:

\[ P_{ps} = P_{Lo} - P_{pv} \]  

$P_{ps}$ represents the available power for charging batteries and supercapacitors. Thus, when it is negative, the corresponding storage element receives power and, is, therefore in charge; it can be said that the principal source production exceeds the load demand. Different thresholds voltages of the supervision system are summarized in Table I.

E. Energy consumption

The energy balance of the chosen house is shown in Table II.

IV. DESIGN OF THE AUTONOMOUS HYBRID SYSTEM

Different characteristics, obtained using the worst month, are summarized in Table III.

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**TABLE I. THRESHOLDS VOLTAGES OF THE SUPERVISION SYSTEM**

<table>
<thead>
<tr>
<th>Threshold</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>High threshold for stop charging batteries</td>
<td>52V</td>
</tr>
<tr>
<td>$V_2$</td>
<td>Low threshold for stop discharging batteries</td>
<td>47.2V</td>
</tr>
<tr>
<td>$V_3$</td>
<td>High threshold for stop charging supercapacitors</td>
<td>30V</td>
</tr>
<tr>
<td>$V_4$</td>
<td>Low threshold for stop discharging supercapacitors</td>
<td>15V</td>
</tr>
</tbody>
</table>

**TABLE II. DAILY ENERGY CONSUMPTION**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluorescent</td>
<td>20</td>
<td>5</td>
<td>4</td>
<td>400</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>70</td>
<td>1</td>
<td>10</td>
<td>700</td>
</tr>
<tr>
<td>Deep freeze</td>
<td>120</td>
<td>1</td>
<td>10</td>
<td>1200</td>
</tr>
<tr>
<td>TV</td>
<td>75</td>
<td>1</td>
<td>4</td>
<td>300</td>
</tr>
<tr>
<td>Washer</td>
<td>300</td>
<td>1</td>
<td>1</td>
<td>300</td>
</tr>
<tr>
<td>PC</td>
<td>40</td>
<td>1</td>
<td>4</td>
<td>160</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>3060</strong></td>
</tr>
</tbody>
</table>

**TABLE III. CHARACTERISTICS OF THE AUTONOMOUS HYBRID SYSTEM**

<table>
<thead>
<tr>
<th>PV source: AEG-40 is chosen</th>
<th>Peak power [W]</th>
<th>Elements number</th>
<th>Surface [m²]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1920</td>
<td>50</td>
<td>19.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lead-acid batteries: Yuasa NP65-12 is chosen</th>
<th>Nominal voltage [V]</th>
<th>Rated capacity [Wh]</th>
<th>Depth of discharge [%]</th>
<th>Autonomy days number</th>
<th>Elements number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>48</td>
<td>4707.7</td>
<td>60</td>
<td>2</td>
<td>20</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>2400</td>
<td>30</td>
<td>44</td>
<td>0.04125</td>
<td>36.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Converters</th>
<th>Inductance $L_p$ [H]</th>
<th>Inductance $L_b$ [H]</th>
<th>Inductance $L_{sc}$ [H]</th>
<th>Capacity $C_p$ [F]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0007</td>
<td>0.0015</td>
<td>0.03</td>
<td>0.0002</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DC link</th>
<th>Nominal voltage [V]</th>
<th>Capacity [F]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>24</td>
<td>0.025</td>
</tr>
</tbody>
</table>
V. FLATNESS CONTROL OF THE AUTONOMOUS HYBRID SYSTEM

A. Structure of the autonomous hybrid system
Structure of the studied system is shown in Figure 4.

\[ y_{bus} = \frac{2y_{bus}}{C_{bus}} f_{v_{bus}}(y_{bus}) \] \hspace{1cm} (7)

\[ P_{sc} = \frac{2y_{bus}}{C_{bus}} i_{Lo} + y_{bus} - P_{pv} - P_{b} \] \hspace{1cm} (8)

\[ v_{bus} = f_{v_{bus}}(y_{bus}) \text{ and } p_{sc} = h_{P_{sc}}(y_{bus}, y_{bus}) \rightarrow \text{The energy system model can be considered as « differentially flat »}[13-14]. \]

- The reference trajectory planning:
  The desired reference trajectory for the electrostatic energy \( y_{bus} \) is:
  \[ y_{busref}(t) = \frac{1}{2} C_{bus} y_{busref}(t)^2 \] \hspace{1cm} (9)

- The flat output variable tracking:
To track \( y_{bus} \) to its reference \( y_{busref} \), the following law is used:
  \[ (\dot{y}_{bus} - \dot{y}_{busref}) + k_{11}(y_{bus} - y_{busref}) + k_{12}(y_{bus} - y_{busref})dt = 0 \] \hspace{1cm} (10)

The choice of \( k_{11}\)and\( k_{12}\)is done by studying roots of the following characteristic equation(roots placement):

\[ s^2 + k_{11}s + k_{12} = 0 \] \hspace{1cm} (11)

With:
\[ \begin{cases} k_{11} = 2\xi \omega_n \\ k_{12} = \omega_n^2 \end{cases} \] \hspace{1cm} (12)

\( \xi \) and \( \omega_n \) are, respectively, the desired dominant damping ratio and the natural frequency.

- The batteries power control:
The power supplied or absorbed by lead-acid batteries is obtained as follows:

\[ \begin{cases} P_{b} = P_{b_{max}} \text{ if } P_{ss} \geq P_{b_{max}} \\ P_{sc} = P_{ss} - P_{b} \end{cases} \] \hspace{1cm} (13)

\[ \begin{cases} P_{b} = P_{ss} \text{ if } 0 < P_{ss} \times P_{b_{max}} \\ P_{sc} = 0 \end{cases} \] \hspace{1cm} (14)

\[ \begin{cases} P_{b} = 0 \text{ if } P_{ss} < 0 \text{ and } v_{sc} < V_{3} \end{cases} \] \hspace{1cm} (15)
\begin{align*}
P_b &= P_{ss} \quad \text{if} \quad P_{ss} < 0 \quad \text{and} \quad v_{sc} \geq V_3 \quad \text{and} \quad v_b < V_1 \\
\end{align*} \quad (16)

- The inductive currents tracking:

To generate signals $u_1, u_2, u_3$, and $u_4$, two hysteresis comparators with fixed bandwidths are used.

![Hysteresis comparator](image)

Figure 4. Principle of a hysteresis comparator.

### E. Simulation results and discussion

The simulation results are shown in the next figures.

![Solar illumination profile](image)

Figure 6. Solar illumination profil.

![Daily PV production](image)

Figure 7. Daily PV production.

![Consumption profile](image)

Figure 8. Consumption profile.

![Power curves of the hybrid storage system](image)

Figure 9. Power curves of the hybrid storage system.

![DC link voltage and reference](image)

Figure 10. DC link voltage and its reference.

![Batteries voltage and state of charge](image)

Figure 11. Batteries voltage and state of charge.

In figures 10, 11 and 12, and after a complete day of system operation, it is noted that the «charging/discharging» process of the lead-acid batteries and the supercapacitors is respected as their states of charge are within admissible intervals, and the capacitive voltage $v_{bus}$ is regulated to its reference:

- When the power $P_{ss}$ is positive and the voltage across the supercapacitors is greater than $V_4$, resulting a decreasing of voltages and states of charge: batteries supply the load power demand, and supercapacitors deliver the necessary power to regulate $v_{bus}$. 
VI. CONCLUSION

In this paper, the flatness control was used to manage energy flows in a hybrid generation system which consists of a photovoltaic source, lead-acid batteries and supercapacitors.

After setting the DC bus voltage and supercapacitors power as state and control variables, respectively, and demonstrated that the system model can be considered as differentially flat, the reference trajectory planning for the electrostatic energy stored in the output capacitor allows to simplifying the system control without having to solve any differential equation.

The simulation results have proved that:
- The DC bus voltage is controlled and regulated to its reference,
- The batteries and supercapacitors voltages and states of charge are maintained in their admissible interval. Therefore, the hybrid system lifetime is theoretically increased.

REFERENCES

