

On Hokusai's *Great Wave off Kanagawa*: localization, linearity and a rogue wave in sub-Antarctic waters

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Abstract

The Hokusai woodcut entitled the *Great Wave off Kanagawa* has been interpreted as an unusually large storm wave, likely to be classed as a rogue wave, and possibly generated from nonlinear wave dynamics [J. H. E. Cartwright and H. Nakamura, Notes Rec. R. Soc. **63** 119-135 (2009)]. In this paper, we present a complementary discussion of this hypothesis, discussing in particular how linear and nonlinear mechanisms can both contribute to the emergence of rogue wave events. By making reference to the *Great Wave's* simultaneous transverse and longitudinal localisation, we show that the purely linear mechanism of directional focussing also predicts characteristics consistent with those of the *Great Wave*. In addition, we discuss the properties of a particular rogue wave photographed on the open ocean in sub-Antarctic waters, which shows two-dimensional localization and breaking dynamics remarkably similar to Hokusai's depiction in the woodcut.

The *Great Wave off Kanagawa* is arguably the most famous of the many woodblock prints made by the Japanese artist Hokusai. The print (shown in Figure 1a) shows an enormous wave on the point of breaking over cargo boats that are being sculled against the direction of the wave's travel. The wave itself is often assumed to be associated with a tsunami but, as examined in detail by Cartwright and Nakamura in this journal [1], the Hokusai print is far more likely to show an extreme plunging breaker, a large storm wave on the point of breaking over the approaching boats. The location of the wave is estimated to be 3 km offshore within Tokyo Bay, and the estimated wave height of around 10 m leads to the conclusion that this would be an exceptionally large amplitude wave for this area and likely to fall into the category of a rogue or freak wave.

Rogue waves are large-amplitude surface waves that represent wave-height outliers for a particular sea state. This means that they possess height and steepness much larger than the average values of other ocean waves in the surrounding environment. It is important to distinguish rogue waves from tsunamis [2]. A rogue wave is a strongly localized wave in space and time on the ocean's surface, whereas a tsunami is caused by massive water displacement. After their initial stage of generation, tsunamis propagate at high speed over long distances at low amplitude, typically as low as tens of centimeters on the open ocean where they are generally unnoticed. Tsunamis only become destructive as they run up to the shore and grow to great height as the water depth decreases.

The dynamics of tsunamis is very well understood: they propagate as linear waves on the ocean [3] until, at a distance from shore of about a seventh of their wavelength some nonlinear processes may play a role [4]. On the other hand, a consensus concerning the physics of oceanic rogue waves is only just beginning to be developed. Rogue waves are now generally considered to form a distinct class of ocean waves, but it seems likely that they arise from a variety of different physical mechanisms [5]. In particular, both linear and nonlinear effects have been shown to lead to unusual wave amplification

under many different conditions. For example, linear effects that can generate rogue waves include: spatial focusing due to refraction in the presence of varying topography; wave-current interactions; dispersive focusing of a chirped wave train; directional focusing of multiple wave trains. Nonlinear effects have also received much attention, especially the process of exponential amplification of random surface noise through modulation instability, and the development of high amplitude coherent structures and related soliton dynamics. Note that whether the generating mechanism is linear or nonlinear does not enter into the definition of a rogue wave; the only criterion is whether the wave is statistically much larger than the other waves in the immediate environment. Reviews of the subject of rogue waves are given in [5-7].

A possible generation mechanism for Hokusai's *Great Wave* in terms of nonlinear soliton collisions was discussed in [1] and indeed, the important role of nonlinear dynamics in wave amplification has been confirmed in a number of recent studies [8-10]. But it is also important to consider the role of linear effects in extreme wave localization, because the initial stage of wave propagation will generally always begin in the linear regime, and nonlinear effects will become important only as the wave amplitude increases to an appropriate level. In the case of the *Great Wave*, a particular clue of how linear effects may play a role is seen by noting the Great Wave's localization both along its direction of travel and transversally – we see the wave rising from the foreground and ending in the middle ground of the print [1]. Whilst such localization has been studied in the framework of nonlinear effects [11, 12], we discuss below how it can also arise from linear propagation.

Before our discussion of this mechanism, however, we first make the important observation that such two-dimensionally localized waves do in fact occur in nature, and can assume large amplitudes and undergo wave breaking as illustrated in the woodcut. In this context, we remark that although breaking waves are often associated with the laterally extended surf waves arriving on a beach, any

water wave can break when the wave crest steepens sufficiently [13]. There is nothing unphysical about a localized wave undergoing breaking at high steepness.

To confirm this explicitly, we show a photograph in Figure 1(b) of an ocean wave with properties remarkably similar to Hokusai's *Great Wave*. The wave is localized in the same manner, is also on the point of breaking, and we also clearly see the dramatic forward ejection of foam. This photograph is significant in that it shows the simultaneous presence of transverse localization and wave breaking on the open sea far from the shore. This provides further confirmation of the accuracy of Hokusai's depiction of nature. The photograph itself was taken in the sub-Antarctic waters of the Southern Ocean from the French research vessel *Astrolabe* during one of its regular voyages between Hobart and the Dumont d'Urville Station in Adélie Land (February 1991). Note that the dimensions of the wave in Figure 1(b) are smaller than that of the great wave in the woodcut, with the wave height in the *Astrolabe* photo estimated between 6-8 m. The height estimation was based on reference relative to the known height of the *Astrolabe*'s helipad deck which was the point of observation. The waves on the background sea were estimated to have much lower heights in the 1-3 m range, consistent with the interpretation of the isolated higher amplitude wave as a rogue wave. We also note that the difference in absolute heights between this sub-Antarctic wave and the *Great Wave* is unimportant for our discussion here because the essential physics of ocean wave breaking remains the same for waves over a large range of amplitudes [13].

We now consider how such dramatic localization as seen in both this photograph and the *Great Wave* woodcut may arise from linear propagation effects. Specifically, we examine the process of directional focusing, which arises whereby periodic wave trains with different directions and phases interfere together at a particular point in space and time to create an extreme rogue wave at their focus. Typical results of numerical modeling are shown in Figure 2 [14]. In fact, the modeling is based on

fully realistic propagation equations that include both linear and nonlinear effects, but the essential concentration of energy at the point of focus arises from linear convergence to a focus. Nonlinearity plays a dominant role only as the wave approaches the linear focus where it increases the wave steepness to the point of breaking. The visual similarity of the numerical modeling of directional focusing with the localization properties seen in the woodcut is immediately apparent, and thus directional focusing is clearly also a mechanism that could underlie the formation of the *Great Wave*.

At this point we can note another feature of the *Great Wave* that suggests the presence of several surrounding waves of large amplitude. Although these other waves appear outside the frame of the image, it is natural to consider whether the fishermen are actually experiencing a “wave packet” of several large rogue waves. This is in fact perfectly plausible. Although each single rogue wave crest is localized in space and time, numerical studies and observation have also reported that several (typically between 2-3) consecutive rogue waves can be generated [7, 15, 16]. The occurrence of multiple rogue waves in this way can arise from nonlinear effects, but is also possible with directional focusing [14].

Our conclusions concerning the *Great Wave off Kanagawa* can be easily stated. When discussing the *Great Wave*, considering its transverse localization is essential to any attempts to understand its origin and to propose plausible underlying physical mechanisms. It is also important to realize that the process of wave breaking itself is not confined to the simple case of extended surf waves on a beach, but occurs with all high amplitude waves. The photographic evidence provided here of a breaking wave in the Southern Ocean is a striking example. Although it is not possible to make any unambiguous conclusions about the particular wave or waves that may have been seen by Hokusai to inspire his celebrated woodcut, we believe that directional focusing is a plausible mechanism that predicts wave characteristics highly similar to the *Great Wave off Kanagawa*. In terms of the artwork of the woodcut itself, highlighting the physics of the transverse localization of the Great Wave provides

room for unexpected optimism when interpreting the scene that is depicted. Transverse localization due to an effect such as directional focusing might suggest that the boats may not be in as much danger as usually believed. Is Hokusai really trying to highlight the skillful way that the Japanese crews are navigating around the wave to avoid it breaking over them?

Our discussion also highlights the generality of wave localization phenomena in nature that can arise from both linear and nonlinear processes, although it is clearly difficult to differentiate which process may be dominant based on field observation alone. Interestingly, this brings to mind a particular problem in the interpretation of water wave phenomena that dates back to the 19th century, and whose resolution in the 20th century is linked to the very beginning of the field of nonlinear science itself. The 19th century event of relevance here is John Scott Russell's 1834 observation of the *Great Wave of Translation* on the Union Canal in Scotland [17]. Although Scott Russell himself was convinced that what he had seen was a fundamentally new class of wave, this interpretation was contentious, and authorities at the time such as Stokes and Airy preferred not to accept the need for any new theoretical ideas beyond the notions of water wave theory at the time [18, 19]. It was only much later in the 1960s with the development of techniques such as inverse scattering and numerical methods for solving nonlinear differential equations, that the full nonlinear theory of solitons provided the context with which to understand Scott Russell's observation completely [20].

Ongoing research in the hydrodynamics community concerning the role of linear and nonlinear effects in rogue wave emergence continues this tradition of debate as to whether observations of novel waves in nature really require new theoretical descriptions [9]. The field of rogue wave study today is of course in a much better position than Scott Russell was, in that the tools of advanced mathematics allow detailed simulations of a wide range of water wave propagation behavior. But even now, we are very often in the situation where we must compare our numerical modeling with the laboratory of the

ocean itself where not all parameters are known or controlled. This serves to highlight the central role of observation above all, and the tremendous importance of talented observers (such as Hokusai) to capture the physics of the natural world.

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(a) Hokusai's Great Wave



(b) Breaking Wave in the Southern Ocean



Figure 1 (a) The Great Wave off Kanagawa (Kanagawa-oki nami-ura) woodcut by Katsushika Hokusai. (b) Photograph of a breaking wave in the sub-Antarctic waters of the Southern Ocean taken from the French research vessel Astrolabe during one of its regular voyages between Hobart and the Dumont d'Urville Station in Adélie Land (V. Sarano, 1991). Note the transverse and longitudinal localization of the wave that is remarkably similar to that depicted by Hokusai.

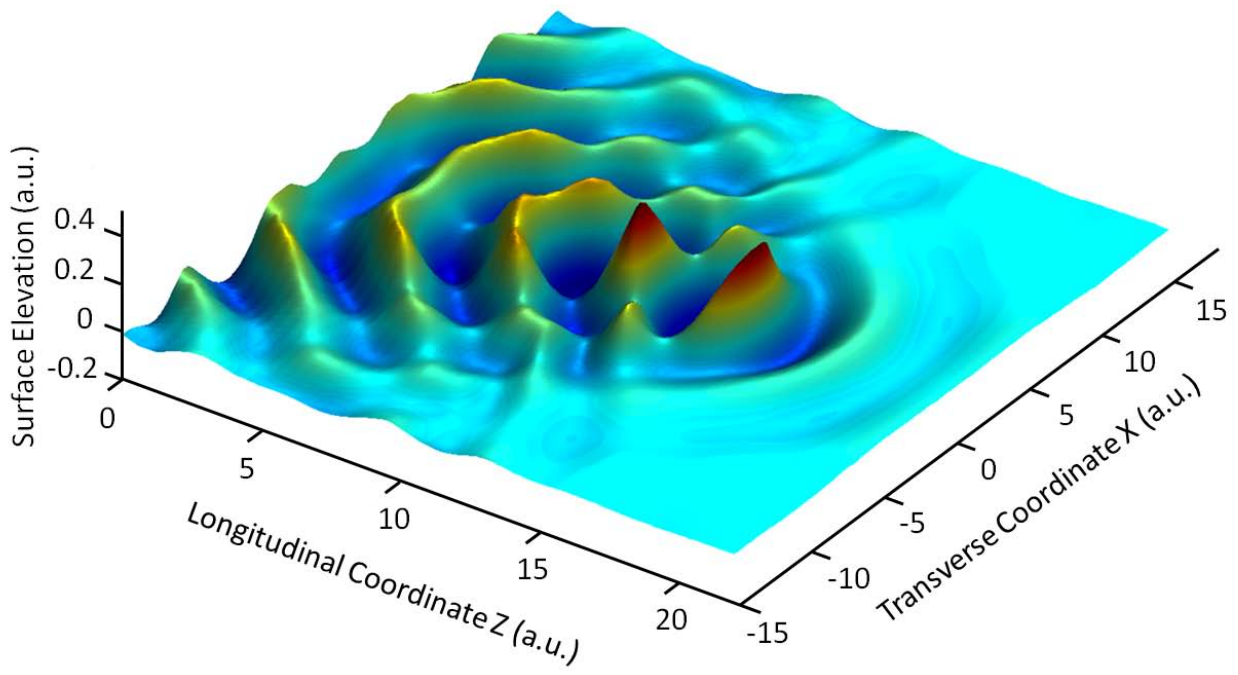


Figure 2 Numerical results showing Directional Focusing of periodic wave trains towards an extreme wave at the focus. The wave steepness at this point is such that we can clearly see the wave crest overturning to the point of breaking (adapted from [10]).