ROM FOR NONLINEAR VIBROACOUSTIC PROBLEMS WITH STRUCTURAL AND ACOUSTIC NONLINEARITIES

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ABSTRACT

This paper presents a reduced order method dedicated to nonlinear vibroacoustic problems. Both structural and acoustics behavior are nonlinear. The structural nonlinearity is due to large displacements while the acoustic nonlinearity is due to the high intensity level in the fluid. The Kuznetsov equation is used to formulate the nonlinear acoustic problem. The reduced order model is based on the Ritz bases of the uncoupled linear problem. Nonlinear behavior is considered as a perturbation of the linear model so that the resolution is compared to a reanalysis problem. The Combined Approximation method dedicated to reanalysis problems is used to enriche the Ritz reduced basis.

1 INTRODUCTION

In this work, structural large displacements nonlinearties are considered. This type of nonlinearity is commonly encountered for thin structures as beams or plates [1]. The linear model yields to decoupling of inplane and bending movement while large displacements hypothesis involve a coupling between both displacements. On the other hand, acoustics become nonlinear with high pressure level $(> 130dB)$. For low levels of nonlinearities where the pressure fluctuation remain small compared to the static pressure, Kuznetsov [2] proposed a formulation describing the fluid movement in term of velocity potential.

Modal superposition principle commonly used in linear cases to build on the reduced order model can not be used in nonlinear problems. The most common reduced order models in nonlinear problems are based on the proper orthogonal decomposition (POD) or the nonlinear normal modes (NNM). Both techniques have been compared in fluid-structure interaction [3]. In this paper, the combined approximation method (CA) [4] developed for reanalysis problems is used.

2 PROBLEM FORMULATION

Let us consider a fluid filled domain Ω_f bounded by a rigid boundary Γ_{fr} and a flexible boundary Γ_{fs}, coupled with a thin structure filling the domain Ω_s. The structure is holded on the boundary Γ_{sr}.

The three equations describing the fluid behavior are the momentum conservation, the mass conservation and the isentropic equation of state. Linearizing those equations at the first order leads to the wave equation where the fluid behavior is described with a single variable In the case of nonlinear behavior, the linearization step is no more admitted but we always aim to get an equation with one variable describing the fluid behavior. Kuznetsov has proposed a formulation with the velocity potential (Φ) as a state variable. The nonlinear wave equation is written as follow:

$$\Delta\phi - \frac{1}{c_0^2}\frac{\partial^2\phi}{\partial t^2} = \frac{1}{c_0^2}\frac{\partial}{\partial t}\left[(\nabla\phi)^2 + \frac{1}{c_0^2}\frac{(\gamma-1)}{2}\left(\frac{\partial\phi}{\partial t}\right)^2\right], \tag{1}$$

where γ is the ratio of the heat capacities.

Using the finite element method, the coupled problem can be written as follows:

$$\begin{bmatrix} \frac{1}{\rho_f}M_s & 0 \\ 0 & -M_f - \Delta M_f \end{bmatrix}\begin{bmatrix} \ddot{U} \\ \ddot{\Phi} \end{bmatrix} + \begin{bmatrix} 0 & C^T \\ C & -\Delta B_f \end{bmatrix}\begin{bmatrix} \dot{U} \\ \dot{\Phi} \end{bmatrix} + \begin{bmatrix} \frac{1}{\rho_f}K_s + \Delta K_s & 0 \\ 0 & -K_f \end{bmatrix}\begin{bmatrix} U \\ \Phi \end{bmatrix} = \begin{bmatrix} \frac{1}{\rho_f}F \\ 0 \end{bmatrix}. \tag{2}$$

M_s, K_s are the mass and stiffness structure matrices, M_f, K_f are the mass and stiffness fluid matrices, C is the coupling matrix, ΔK_s is a nonconstant matrix describing the nonlinear effect of the structure, ΔM_f and ΔB_f are a nonconstant matrices describing the nonlinear effect in the fluid.

3 REDUCED ORDER METHOD

The combined approximation method (CA) was developed for reanalysis problems with reduced order models. It can be applied to nonlinear problems if the nonlinearity can be separated in a linear term and nonlinear term. Reduced order basis is written as follow:

$$T_{CA} = \left(I - B_0 + B_0^2 + \cdots + (-B_0)^{\nu-1}\right) r_0. \tag{3}$$

I is the identity matrix, r_0 and B_0 are the parameters of the reduction written as follow:

$$\begin{cases} r_0 = K_0^{-1} \left(M_0 + \Delta M\right) \varphi_0, \\ B_0 = K_0^{-1} \Delta K. \end{cases} \tag{4}$$

φ_0 is the modal basis of the initial problem.

The starting point or the reduced model is the modal basis verifying the eigenvalue problem of the uncoupled fluid-structure model (T_0). This initial basis is composed of two sub-bases representing the structural basis (T_0^s) and the fluid basis $\left(T_0^f\right)$. In order to take into account the coupling effect and the nonlinear behavior, these bases should be enriched by informations that are detailed hereinafter:

- The combined approximation method is applied to the structural basis where $\varphi_0 = T_0^s$ and $\Delta K = \Delta K_s$. For thin structures, T_0^s should include inplane and bending modes.
- Considering that the acoustic fluctuation still small compared to the static case, T_0^f can be used to predict the nonlinear uncoupled acoustic problem. With the coupling case, this basis is enriched by the static response of the fluid due to the structural contribution so that $T^f = \left[T_0^f | \Delta T_{fs}\right]$, where :

$$\Delta T_{fs} = \left(K_f - \omega^2 M_f\right)^{-1} C T_0^s. \tag{5}$$

Finally, the reduced order basis can be written as follow:

$$T = \begin{bmatrix} T_{CA}^s & 0 \\ 0 & T^f \end{bmatrix}. \tag{6}$$

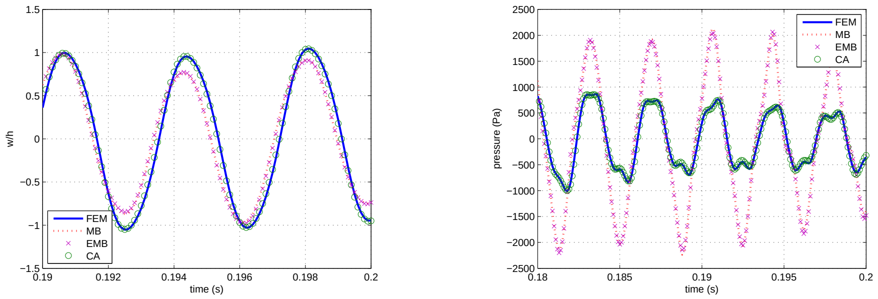
4 APPLICATION

Let us consider a thin plate supported on a parallelepiped air filled cavity. The structure is excited with a harmonic excitation near the first acoustic mode. A temporal response is computed to illustrate the method. The full model (**FEM**) is compared to the reduced model using different reduced method:

- a reduced order model using the modal bases T_0^s and T_0^f of uncoupled problems (**MB**),
- a reduced order model using the modal basis T_0^s and enriching T_0^f by the structure contribution (cf. equation (5), **EMB**),
- a reduced order model using the CA for T_0^s and enriching T_0^f by the structure contribution (**CA**).

Figure 1a shows the displacement on the center of the structure for different models and figure 1b shows the pressure of different models at the corner of the cavity near the structure. To quantify the error, the energy of each signal is calculated. The relative error amplitude of the reduced order models are compared to the full model. Let $x(t)$ be a signal, the energy of $x(t)$ is written as:

$$E = \int_{-\infty}^{+\infty} x(t)^2 dt. \tag{7}$$



(a) Relative displacements (b) Pressure

Figure 1: Response of the cavity.

Model	U	$\dot{U}$	Φ	P
MB	26.5	27.1	12.9	22.4
EMB	26.9	27.6	13.4	22.9
CA	0.2	0.4	2.9	3.1

Table 1: Amplitude relative error (%)

Table 1 shows the displacements, the velocity,the velocity potential ant the pressure relative errors of the reduced order models. The **MB** and the **EMB** models predict badly both structure and fluid behavior. This is explained by the fact that the response is dominated by the nonlinear behavior that is neglected in the reduced order models. Using the combined approximation method and taking into account the structure contribution on the fluid describe with a high precison the structure behavior. Nevertheless, the fluid prediction can not be considered as optimum, this is due to the reduced order basis that does not take into account the nonlinear behavior.

5 CONCLUSION

A reduced order model dedicated to nonlinear vibroacoustic problems has been presented. It is based on the combined approximation method enriched by the structure contribution on the fluid. The application shows a good prediction on the structure behavior but the fluid behavior need a better prediction. A reduced order method taking into account the nonlinear fluid behavior should improve this estimation.

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